Replication: Two Linear Regression Models

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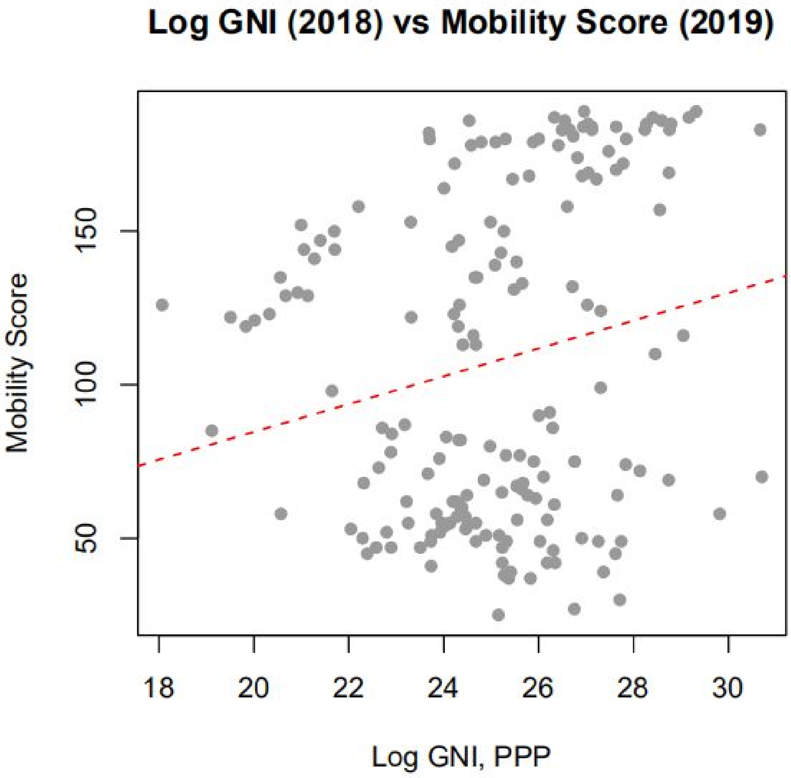
1 Checking the Assumptions

Our client plans to use two simple linear regression to see the relationship between Mobility and GNI&FSI. For the simple linear regression model works appropriately, the datasets must satisfy the following conditions.

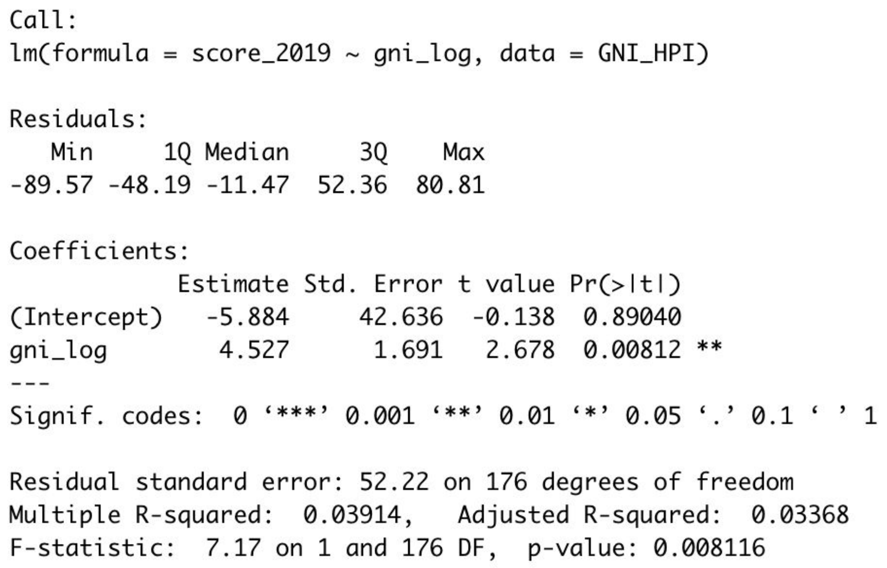
Next, we need to ensure that the outcome variable Y has a roughly linear relationship with the explanatory variable X. More simply, make a plot based on the dataset we used, then check with eyes if they seem to be linear. We have put the codes in the Appendix. For the Mobility~Log(GNI) plot, we found a weak positive relation. And for the Mobility~FSI plot, we found a strong negative linear relation. Thus, both of them satisfied the SLM conditions.

2 Model

2.1 Log GNI (2018) vs Mobility Score (2019)

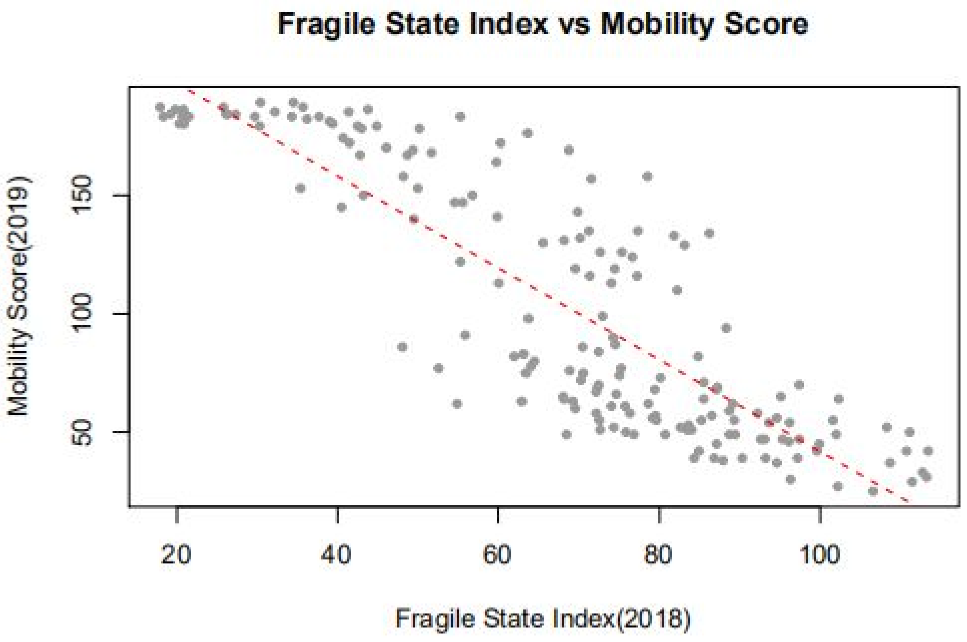


Result:

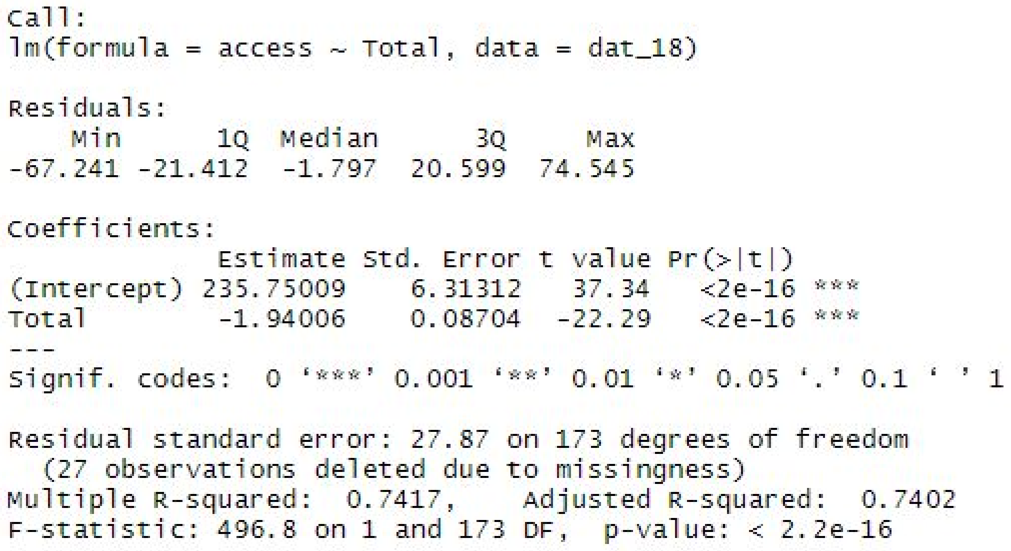


```
Call:
lm(formula = score_2019 ~ gni_log, data = GNI_HPI)

Residuals:
   Min     1Q Median     3Q    Max 
-89.57 -48.19 -11.47  52.36  80.81 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)   
(Intercept)   -5.884     42.636  -0.138  0.89040   
gni_log        4.527      1.691   2.678  0.00812 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 52.22 on 176 degrees of freedom
Multiple R-squared:  0.03914,	Adjusted R-squared:  0.03368 
F-statistic:  7.17 on 1 and 176 DF,  p-value: 0.008116
```

2.2 Fragile State Index vs Mobility Score



Result:



```
Call:
lm(formula = access ~ Total, data = dat_18)

Residuals:
    Min      1Q  Median      3Q     Max 
-67.241 -21.412  -1.797  20.599  74.545 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 235.75009    6.31312   37.34   <2e-16 ***
Total        -1.94006    0.08704  -22.29   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 27.87 on 173 degrees of freedom
  (27 observations deleted due to missingness)
Multiple R-squared:  0.7417,    Adjusted R-squared:  0.7402 
F-statistic: 496.8 on 1 and 173 DF,  p-value: < 2.2e-16
```

3 Interpretation

3.1 Log GNI (2018) vs Mobility Score (2019)

- We create gni_log to take the log of the gni coefficient because the original data are too large for model fitting; the score_2019 represents the mobility score of this country in 2019.
- The Intercept -5.884 implies that when the gni_log equals to zero, which means GNI is 1, then the mobility score in the 2019 will be -5.884
- The coefficient 4.527 implies that with every 1 unit increase in the log of the gni coefficients, there will be a 4.527 unit increase in the mobility score in 2019.

3.2 Fragile State Index vs Mobility Score

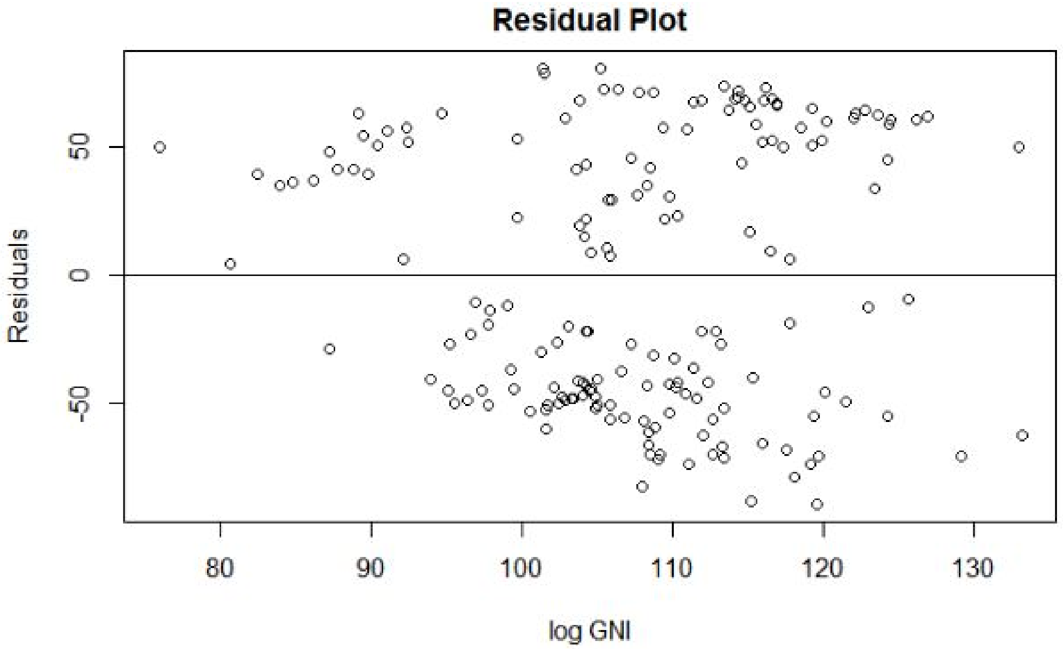
- The Total variable represents the fsi score of this country.
- The intercept 235.75 implies that when the Total variable equals 0, the access number would be 236.
- The coefficient -1.94 implies that with every 1 unit increase in the Total variable, the mobility score would decrease by two units in the number of access, in other words, the number of access will decrease by 2.

4 Discussion

4.1 Weakness and Proposal

1. Residual Plot

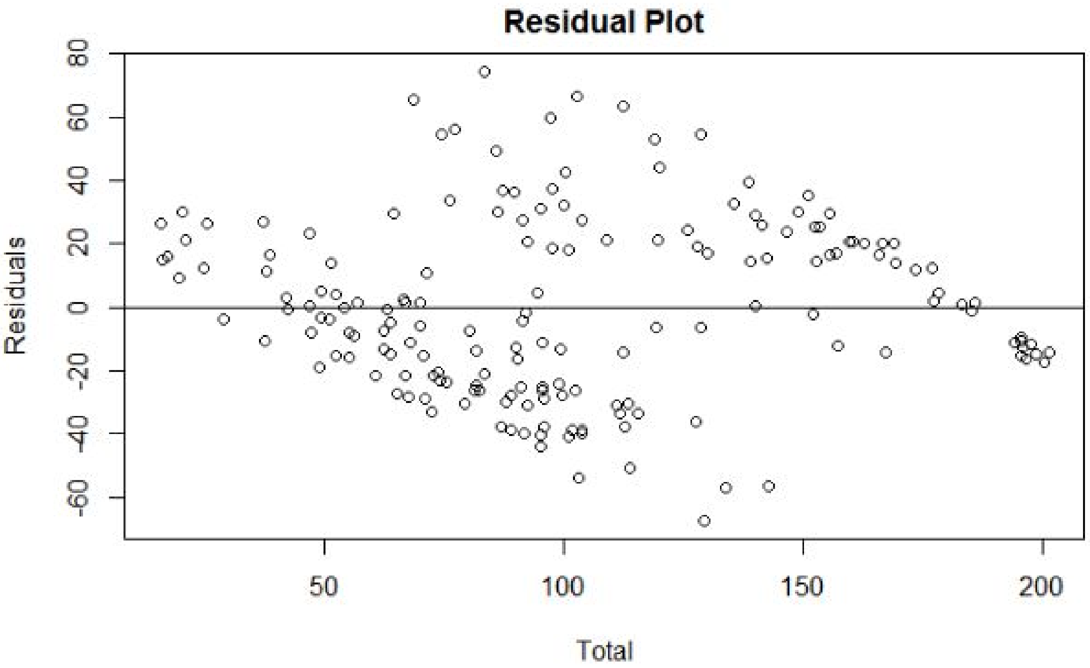
- Mobility Score ~ log(GNI)



We can use the residual plot to visually confirm the validity of the model.

From the residual vs predicted model plot of the first model, we can see that most of the points do not follow a straight line so the linearity is violated. And points are not equally spread. We can clearly see that there are many points clustered between the log GNI values of 100 and 120. Therefore, the simple regression model we used there is not suitable.

- Mobility Score ~ Fragile State Index



By looking at the residuals plot of the second model, although the points here look close to a straight line, we can see that the residuals of this model has a pattern of two clusters of data, which means the original global_ranking_2019 data might need to be divided into two groups and fit two models separately. And we can also find that the constant variance is violated because the points here are not equally spread, which means the model is not appropriate for the situation.

2. Based on the first model, we can find that the regression line just passes through the middle of the graph, and the data scatters randomly which means that the regression line doesn't reflect the relationship between log(GNI) and mobility score. According to the scatterplot, the data has been divided into two clusters so that we can make regression models for two clusters respectively.

3. To make models more clear, it's better to mark data with countries' names and different colors which represent different continents.

4. As the weakness part mentioned above, a simple linear regression model suffers from many flaws. Besides, the data has been divided into small clusters which means that these countries are not independent. Therefore, it is not appropriate to make a simple linear regression model. To fix this problem, we can try to use a different regression model for data analysis. In addition, GNI measures a country's income level and FSI measures a country's level of fragility. What we are considering is the correlation between these two variables. For a country, a high GNI value may correspond to a low FSI value. That means, both the two variables can be fitted in a single regression model. After preliminary consideration, we suggest using a multivariate regression model.

4.2 Next Step

For further data analysis, we are going to do it from two aspects. Time Series and Machine Learning.

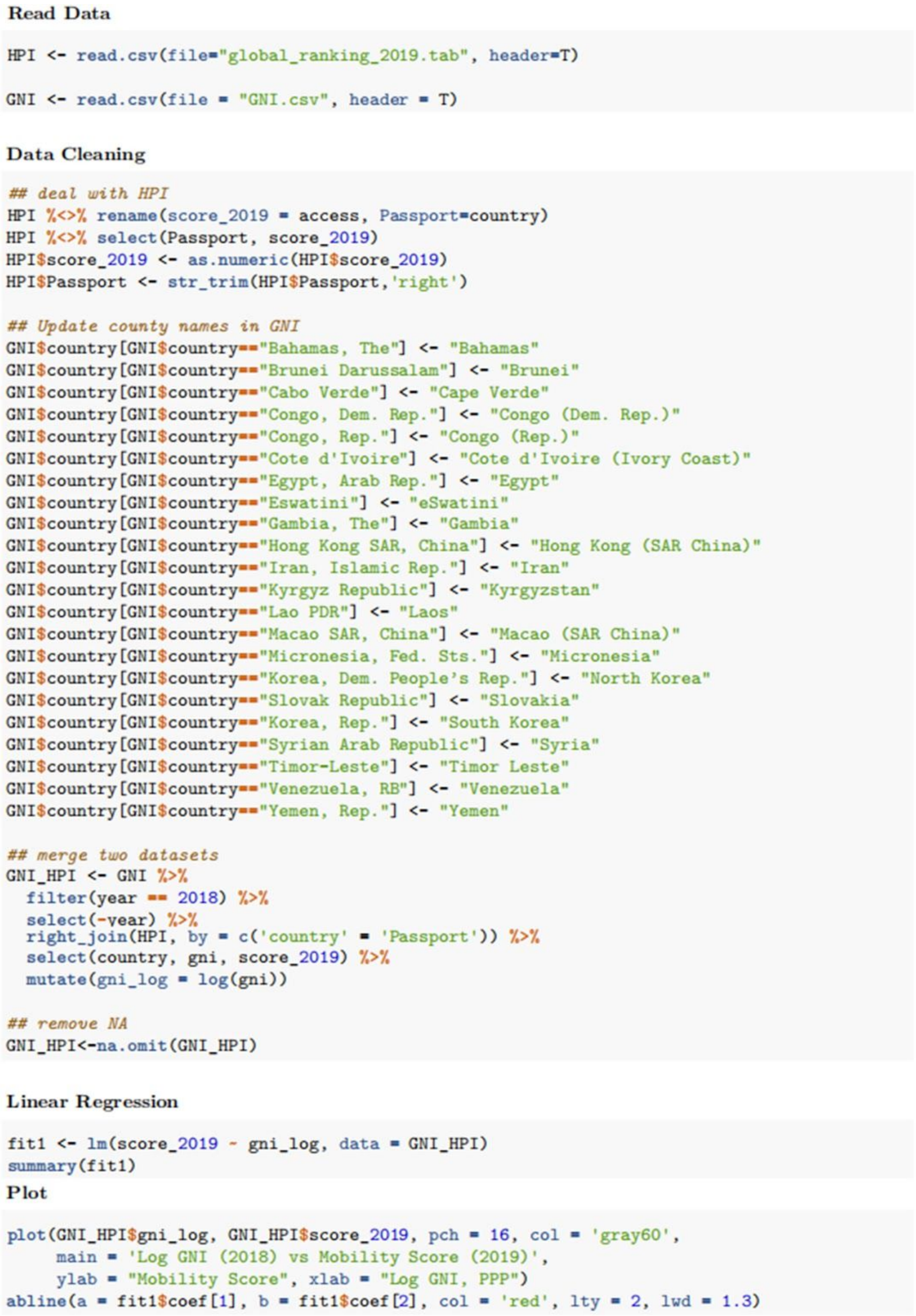
First of all, we plan to use the ARIMA model based on countries to predict the future mobility scores. We will try the AR(1) or AR(2) model which means regression from value in the past 1 or 2 years. To make sure which one is more fit, we're going to break the data into two parts- train&test. We'll use the data from the train section to train the model, and then bring the model back to the test section for prediction. Then we will compare the predict data and the real data to see the accuracy.

Another way we think is good to predict the future MS for a certain country is to use the Xgboost model. To do this, we need to communicate with our client first to see what's the possible factors that may influence the final results besides FSI and GNI. Then we will do the data cleaning things to put all observed values and their eigenvalues into one dataset to train the model.

After that, we can predict all countries' MS scores with their current or future(predicted) FSI and GNI data, and etc.

5 Appendix

5.1 Mobility Score ~ Log (GNI)



Read Data

```
HPI <- read.csv(file="global_ranking_2019.tab", header=T)

GNI <- read.csv(file = "GNI.csv", header = T)
```

Data Cleaning

```
## deal with HPI
HPI %<>% rename(score_2019 = access, Passport=country)
HPI %<>% select(Passport, score_2019)
HPI$score_2019 <- as.numeric(HPI$score_2019)
HPI$Passport <- str_trim(HPI$Passport,'right')

## Update county names in GNI
GNI$country[GNI$country=="Bahamas, The"] <- "Bahamas"
GNI$country[GNI$country=="Brunei Darussalam"] <- "Brunei"
GNI$country[GNI$country=="Cabo Verde"] <- "Cape Verde"
GNI$country[GNI$country=="Congo, Dem. Rep."] <- "Congo (Dem. Rep.)"
GNI$country[GNI$country=="Congo, Rep."] <- "Congo (Rep.)"
GNI$country[GNI$country=="Cote d'Ivoire"] <- "Cote d'Ivoire (Ivory Coast)"
GNI$country[GNI$country=="Egypt, Arab Rep."] <- "Egypt"
GNI$country[GNI$country=="Eswatini"] <- "eSwatini"
GNI$country[GNI$country=="Gambia, The"] <- "Gambia"
GNI$country[GNI$country=="Hong Kong SAR, China"] <- "Hong Kong (SAR China)"
GNI$country[GNI$country=="Iran, Islamic Rep."] <- "Iran"
GNI$country[GNI$country=="Kyrgyz Republic"] <- "Kyrgyzstan"
GNI$country[GNI$country=="Lao PDR"] <- "Laos"
GNI$country[GNI$country=="Macao SAR, China"] <- "Macao (SAR China)"
GNI$country[GNI$country=="Micronesia, Fed. Sts."] <- "Micronesia"
GNI$country[GNI$country=="Korea, Dem. People’s Rep."] <- "North Korea"
GNI$country[GNI$country=="Slovak Republic"] <- "Slovakia"
GNI$country[GNI$country=="Korea, Rep."] <- "South Korea"
GNI$country[GNI$country=="Syrian Arab Republic"] <- "Syria"
GNI$country[GNI$country=="Timor-Leste"] <- "Timor Leste"
GNI$country[GNI$country=="Venezuela, RB"] <- "Venezuela"
GNI$country[GNI$country=="Yemen, Rep."] <- "Yemen"

## merge two datasets
GNI_HPI <- GNI %>%
  filter(year == 2018) %>%
  select(-year) %>%
  right_join(HPI, by = c('country' = 'Passport')) %>%
  select(country, gni, score_2019) %>%
  mutate(gni_log = log(gni))

## remove NA
GNI_HPI<-na.omit(GNI_HPI)
```

Linear Regression

```
fit1 <- lm(score_2019 ~ gni_log, data = GNI_HPI)
summary(fit1)
```

Plot

```
plot(GNI_HPI$gni_log, GNI_HPI$score_2019, pch = 16, col = 'gray60',
     main = 'Log GNI (2018) vs Mobility Score (2019)',
     ylab = "Mobility Score", xlab = "Log GNI, PPP")
abline(a = fit1$coef[1], b = fit1$coef[2], col = 'red', lty = 2, lwd = 1.3)
```

5.2 Mobility Score ~ Fragile State Index

```r
# Read the data
fsi_18 <- read.csv("E:/Boston University/Statistics Practicum/Project 1/fsi-2018.csv")
ms_19 <- read.csv("E:/Boston University/Statistics Practicum/Project 1/global_ranking_2019.tab")

# Update country names
fsi_18$Country[fsi_18$Country=="Brunei Darussalam"] <- "Brunei"
fsi_18$Country[fsi_18$Country=="Congo Democratic Republic"] <- "Congo (Dem. Rep.)"
fsi_18$Country[fsi_18$Country=="Congo Republic"] <- "Congo (Rep.)"
fsi_18$Country[fsi_18$Country=="Cote d'Ivoire"] <- "Cote d'Ivoire (Ivory Coast)"
fsi_18$Country[fsi_18$Country=="Guinea Bissau"] <- "Guinea-Bissau    "
fsi_18$Country[fsi_18$Country=="Israel and West Bank"] <- "Israel"
fsi_18$Country[fsi_18$Country=="Kyrgyz Republic"] <- "Kyrgyzstan"
fsi_18$Country[fsi_18$Country=="Lao PDR"] <- "Laos"
fsi_18$Country[fsi_18$Country=="Russia"] <- "Russian Federation"
fsi_18$Country[fsi_18$Country=="Timor-Leste"] <- "Timor Leste"
fsi_18$Country[fsi_18$Country=="Cape Verde"] <- "Cape Verde Islands"
fsi_18$Country[fsi_18$Country=="Comoros"] <- "Comores Islands"
fsi_18$Country[fsi_18$Country=="Slovak Republic"] <- "Slovakia"
fsi_18$Country[fsi_18$Country=="Timor Leste"] <- "Timor-Leste"



# data cleaning
fsi_data <- filter(fsi_18,Year==2018) %>%
  select(Country,Year,Total)
fsi_data$Country <- str_c(fsi_data$Country,' ')

ms_19 <- rename(ms_19,Country=country)

# Merge two dataset
dat_18 <- merge(fsi_data,ms_19,all=T)

# Linear regression
fit_18 <-lm(access~Total,data = dat_18)
summary(fit_18)
```

```r
# Plot
plot(dat_18$Total,dat_18$access,pch = 20, col = 'gray60',main = 'Fragile State Index vs Mobility Score'
abline(coef(fit_18[1]),coef(fit_18[2]), col = 'red', lty = 2, lwd = 1.3)
```