

# Testing the Bekenstein-Hawking Area Law at Cosmological Scales: A Data-Driven Reconstruction of the Horizon Entropy Current

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The Bekenstein-Hawking area law implies a constant horizon entropy slope,  $\mu(A) \propto dS/dA = \text{const.}$  We test this assumption directly at cosmological scales using a data-driven reconstruction of the apparent-horizon entropy current from late-time background geometry (Pantheon+, BAO, and cosmic chronometers), in a framework designed to avoid GR-anchored circularity. The reconstruction prefers a time-evolving entropy slope with a negative mean gradient,  $\langle d\log \mu / d\log A \rangle \approx -0.24,$  with posterior weight  $P(d\log \mu / d\log A < 0) \approx 0.79.$  Function-space proximity metrics favor non-extensive templates (Tsallis/Barrow-like) over strict Bekenstein-Hawking behavior, with representative distances  $D^2 \sim 10^{-8}$  versus  $D^2 \sim 10^{-4}$  for  $\mu = 1.$  Synthetic-closure and simulation-based calibration checks are used to verify that the recovered trend is not a reconstruction artifact. These results support a thermodynamic interpretation in which the effective gravitational sector evolves with horizon area, providing a direct theoretical bridge to modified-propagation (friction) signatures in dark-siren analyses.

## I. INTRODUCTION

The thermodynamic interpretation of gravity links horizon geometry to entropy. In standard General Relativity with Bekenstein-Hawking entropy, the horizon entropy is proportional to area,  $S \propto A,$  so  $dS/dA$  is constant. A central question for late-time cosmology is whether this area law remains valid on the apparent horizon traced by the observed expansion history.

This paper is framed as a direct test of that assumption. Rather than starting from a fixed gravitational field equation and fitting parameters, we reconstruct the entropy-current proxy  $\mu(A)$  from observed background geometry and then ask whether  $\mu(A) = 1$  is supported. In this sense, the analysis is not only a reconstruction exercise; it is a fundamental consistency test of space-time thermodynamics at cosmological scales.

The same thermodynamic degree of freedom is also the ingredient that controls modified gravitational-wave propagation in dark-siren sectors. The present work therefore serves as the theory-facing companion to dark-siren propagation tests: here we infer the background thermodynamic current, while dark sirens test its propagation consequence.

## II. THERMODYNAMIC LINK AND INFERENCE STRATEGY

We define the entropy-slope modification

$$\mu(A) = \frac{(dS/dA)_{\text{BH}}}{dS/dA}, \quad (1)$$

so that  $\mu = 1$  corresponds to strict Bekenstein-Hawking behavior. In the apparent-horizon mapping used in this pipeline, the late-time background obeys

$$\frac{dH^2}{dz} = 3H_0^2\Omega_{m0}(1+z)^2\mu(A), \quad A(z) = 4\pi \left( \frac{c}{H(z)} \right)^2 \quad (2)$$

for the flat mapping variant.

The inference is performed by forward modeling: we sample a spline representation of  $\log \mu$  and solve for  $H(z),$  then evaluate the likelihood in data space. The baseline geometry stack is Pantheon+, BAO, and cosmic chronometers. Inference products are stress-tested with synthetic closure and simulation-based calibration (SBC), including nominal 68% and 95% coverage diagnostics, to ensure that apparent trends are not driven by inversion pathologies.

## III. RESULTS

### A. Area-law test from reconstructed entropy current

The reconstructed posterior favors a negative entropy-current slope over a constant-slope area law. A representative summary is

$$\left\langle \frac{d\log \mu}{d\log A} \right\rangle \approx -0.24, \quad P\left(\frac{d\log \mu}{d\log A} < 0\right) \approx 0.79. \quad (3)$$

Interpreted physically, this corresponds to a decreasing  $\mu(A)$  as the horizon area grows, i.e., an evolving entropy density on cosmological horizons.

Function-space proximity tests provide a complementary model-comparison view. Relative to a strict Bekenstein-Hawking template ( $\mu = 1$ ), the reconstructed curve is substantially closer to non-extensive families:

$$D_{\text{BH}}^2 \sim 10^{-4}, \quad D_{\text{Tsallis/Barrow}}^2 \sim 10^{-8}, \quad (4)$$

indicating that non-extensive effective descriptions capture the reconstructed shape more efficiently.

## 55 B. Method calibration and identifiability gate

To verify that sign-level statements are identifiable under finite Monte Carlo noise, we retain the power-map calibration used in the production pipeline. To quantify the pipeline's ability to distinguish a true slope signal from noise, we compute true-positive rates (TPR) for injected slope anomalies of strength  $\nu$ . With fixed threshold  $\tau = 0.06459924567842923$  (false-alarm control), the baseline run gives

$$\text{TPR}_{\nu=0.2} = 0.3432, \quad \text{TPR}_{\nu=0.5} = 0.3805, \quad \text{TPR}_{\nu=0.8} = \frac{105}{(5)}$$

$$\text{FPR}_{\nu=0.0} = 0.2983, \quad \Delta_{\text{sep}} = 0.1059. \quad (6)$$

65 Three additional full-core replicate seeds yield  $\Delta_{\text{sep}} = \{0.1000, 0.1067, 0.1089\}$ , giving

$$\langle \Delta_{\text{sep}} \rangle = 0.1054, \quad \sigma_{\text{run}} = 0.0038. \quad (7)$$

This establishes stable, nonzero discrimination power for the mapping-choice gate used upstream of decision-grade runs.<sup>115</sup>

### C. Hubble-scale anchors and robustness context

For transfer-map stress tests, we track the conventional anchor scales  $H_0^{\text{Planck}} = 67.4 \pm 0.5$  and  $H_0^{\text{local}} = 73.0 \pm 1.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$  as external references. These anchors are used for sensitivity mapping, not as hard priors in the core area-law test, to preserve modified-gravity neutrality in the reconstruction stage.

#### D. Directional null test (Pantheon+ hemispheres and Planck lensing)

Because the entropy-current reconstruction is built from low-redshift geometry, it is important to check that<sup>130</sup> the recovered trend is not dominated by a particular sky region or survey footprint. We therefore perform an explicit statistical-isotropy diagnostic using a hemisphere-split scan on the Pantheon+ sky: for each axis on a coarse HEALPix grid, we compute the slope-scar statistic  $s$  separately in opposite hemispheres and form  $\Delta s$  and its uncertainty.<sup>135</sup>

To suppress look-elsewhere effects, we use a five-fold crossfit procedure in which the “best” axis is chosen on a training split (maximizing  $|\Delta s|/\sigma_{\Delta s}$ ) and then evaluated only once on held-out data. The held-out confirmations are consistent with noise, with fold-level  $z_{\text{test}} \in \{-0.354, -0.254, -0.028, +0.488, +0.065\}$  and a convenient aggregate diagnostic  $z_{\text{Stouffer}} \approx -0.037$ .<sup>140</sup>

As an external cross-check, we evaluate a hemispherical Planck 2018 CMB lensing convergence variance statistic along the strongest training axis, finding  $\Delta \log \text{Var}(\kappa) \approx 0.032$  with a random-axis calibrated fixed-axis  $p$ -value

$p \approx 0.22$  ( $z \approx 1.23$ ). Within current sensitivity, these diagnostics provide no evidence for a robust dipolar modulation of the inferred slope statistic.

#### IV. DISCUSSION

The key physical output is not only “negative slope detected,” but what that slope means. A decreasing  $\mu(A)$  implies that the effective entropy current changes as the Universe expands. In modified-gravity language this<sup>4892</sup> maps to an evolving effective Planck sector (running gravitational coupling), a known channel in scalar-tensor and  $f(R)$ -like effective descriptions.

The non-extensive proximity ordering is consistent with that interpretation. Tsallis/Barrow-like templates are not asserted here as unique microphysical theories; rather, they provide compact effective parameterizations of the reconstructed thermodynamic flow. Their significantly smaller  $D^2$  values relative to strict Bekenstein-Hawking behavior quantify that the data-preferred trajectory is not well described by  $\mu = \text{const.}$

The calibration and closure layers are included for this reason: to separate physics statements from pipeline artifacts. Synthetic closure and SBC checks, together with seed-level stability of the identifiability gate, support interpreting the negative-slope trend as a property of the reconstructed geometry rather than a numerical accident.

## V. CONCLUSION

We recast late-time entropy-slope reconstruction as a fundamental cosmological test of the Bekenstein-Hawking area law. In the reconstructed posterior, the horizon entropy current is not strictly constant and is instead biased toward a negative slope with non-extensive effective behavior favored in function space.

Most importantly, this thermodynamic evolution ( $\mu(A) \neq \text{const}$ ) provides the theory-side basis for the modified gravitational-wave propagation (friction) signals targeted in dark-siren catalogs, linking horizon thermodynamics and multimessenger cosmology in a single framework.

## DATA AND SOFTWARE AVAILABILITY

Software archive for this pipeline (Zenodo title: *Negative Entropy Slope*): doi:10.5281/zenodo.18604922.

External data/product DOIs used by the underlying stack include:

- GWTC-3 products:  
doi:10.1103/PhysRevX.13.041039.
  - GLADE+: doi:10.1093/mnras/stac1443.

- 145 • Pantheon+ papers and dataset: doi:10.3847/1538-4357/ac8b7a, doi:10.3847/1538-4357/ac8e04,  
doi:10.5281/zenodo.16365279.

- 155 • Cosmic-chronometer compilations:  
doi:10.1088/1475-7516/2012/08/006,  
doi:10.1088/1475-7516/2016/05/014.

- 150 • BOSS DR12 BAO+FS: doi:10.1093/mnras/stx721.

- DESI 2024 BAO analyses: doi:10.1088/1475-7516/2025/04/012,  
doi:10.1088/1475-7516/2025/02/021.

- Planck 2018 lensing: doi:10.1051/0004-6361/201833886.

#### AI-USE STATEMENT

- 160 • eBOSS DR16 LRG BAO+RSD: AI systems were used for drafting support and editing  
doi:10.1093/mnras/staa2455. during manuscript preparation.

[1] R. Abbott *et al.* (LIGO Scientific Collaboration, Virgo Collaboration, and KAGRA Collaboration), Phys. Rev. X **13**, 041039 (2023), doi:10.1103/PhysRevX.13.041039.

[2] G. Dálya *et al.*, Mon. Not. R. Astron. Soc. **514**, 1403 (2022), doi:10.1093/mnras/stac1443.