Testing the Bekenstein-Hawking Area Law at Cosmological Scales: A Data-Driven Reconstruction of the Horizon Entropy Current

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The Bekenstein-Hawking area law implies a constant horizon entropy slope, $\mu(A) \propto dS/dA =$ const. We test this assumption directly at cosmological scales using a data-driven reconstruction of the apparent-horizon entropy current from late-time background geometry (Pantheon+, BAO, and cosmic chronometers), in a framework designed to avoid GR-anchored circularity. The reconstruction prefers a time-evolving entropy slope with a negative mean gradient, $\langle d\log\mu/d\log A\rangle \approx -0.24$, with posterior weight $P(d\log\mu/d\log A < 0) \approx 0.79$. Function-space proximity metrics favor non-extensive templates (Tsallis/Barrow-like) over strict Bekenstein-Hawking behavior, with representative distances $D^2 \sim 10^{-8}$ versus $D^2 \sim 10^{-4}$ for $\mu = 1$. Synthetic-closure and simulation-based calibration checks are used to verify that the recovered trend is not a reconstruction artifact. These results support a thermodynamic interpretation in which the effective gravitational sector evolves with horizon area, providing a direct theoretical bridge to modified-propagation (friction) signatures in dark-siren analyses.

I. INTRODUCTION

The thermodynamic interpretation of gravity links horizon geometry to entropy. In standard General Relativity with Bekenstein-Hawking entropy, the horizon entropy is proportional to area, $S \propto A$, so dS/dA is constant. A central question for late-time cosmology is whether this area law remains valid on the apparent horizon traced by the observed expansion history.

This paper is framed as a direct test of that assumption. Rather than starting from a fixed gravitational field equation and fitting parameters, we reconstruct the entropy-current proxy $\mu(A)$ from observed background geometry and then ask whether $\mu(A) = 1$ is supported. In this sense, the analysis is not only a reconstruction exercise; it is a fundamental consistency test of space-time thermodynamics at cosmological scales.

The same thermodynamic degree of freedom is also the ingredient that controls modified gravitational-wave propagation in dark-siren sectors. The present work therefore serves as the theory-facing companion to dark-siren propagation tests: here we infer the background thermodynamic current, while dark sirens test its propagation consequence.

II. THERMODYNAMIC LINK AND INFERENCE STRATEGY

We define the entropy-slope modification

$$\mu(A) \equiv \frac{(dS/dA)_{\rm BH}}{dS/dA}, \tag{1}$$

so that $\mu = 1$ corresponds to strict Bekenstein-Hawking behavior. In the apparent-horizon mapping used in this pipeline, the late-time background obeys

$$\frac{dH^2}{dz} = 3H_0^2\Omega_{m0}(1+z)^2\mu(A), \qquad A(z) = 4\pi\left(\frac{c}{H(z)}\right)^2 \tag{2}$$

for the flat mapping variant.

The inference is performed by forward modeling: we sample a spline representation of $\log\mu$ and solve for $H(z)$, then evaluate the likelihood in data space. The baseline geometry stack is Pantheon+, BAO, and cosmic chronometers. Inference products are stress-tested with synthetic closure and simulation-based calibration (SBC), including nominal 68% and 95% coverage diagnostics, to ensure that apparent trends are not driven by inversion pathologies.

III. RESULTS

A. Area-law test from reconstructed entropy current

The reconstructed posterior favors a negative entropy-current slope over a constant-slope area law. A representative summary is

$$\left\langle \frac{d\log\mu}{d\log A}\right\rangle \approx -0.24, \qquad P\left(\frac{d\log\mu}{d\log A} < 0\right) \approx 0.79. \tag{3}$$

Interpreted physically, this corresponds to a decreasing $\mu(A)$ as the horizon area grows, i.e., an evolving entropy density on cosmological horizons.

Function-space proximity tests provide a complementary model-comparison view. Relative to a strict Bekenstein-Hawking template ($\mu = 1$), the reconstructed curve is substantially closer to non-extensive families:

$$D^2_{\rm BH} \sim 10^{-4}, \qquad D^2_{\rm Tsallis/Barrow} \sim 10^{-8}, \tag{4}$$

indicating that non-extensive effective descriptions capture the reconstructed shape more efficiently.

B. Method calibration and identifiability gate

To verify that sign-level statements are identifiable under finite Monte Carlo noise, we retain the power-map calibration used in the production pipeline. With fixed threshold $\tau = 0.06459924567842923$, the baseline run gives

$$\mathrm{TPR}_{\nu=0.2} = 0.3432, \quad \mathrm{TPR}_{\nu=0.5} = 0.3805, \quad \mathrm{TPR}_{\nu=0.8} = 0.4892, \tag{5}$$

$$\mathrm{FPR}_{\nu=0.0} = 0.2983, \quad \Delta_{\mathrm{sep}} = 0.1059. \tag{6}$$

Three additional full-core replicate seeds yield $\Delta_{\mathrm{sep}} = \{0.1000, 0.1067, 0.1089\}$, giving

$$\langle \Delta_{\mathrm{sep}} \rangle = 0.1054, \qquad \sigma_{\mathrm{run}} = 0.0038. \tag{7}$$

This establishes stable, nonzero discrimination power for the mapping-choice gate used upstream of decision-grade runs.

C. Hubble-scale anchors and robustness context

For transfer-map stress tests, we track the conventional anchor scales $H_0^{\mathrm{Planck}} = 67.4 \pm 0.5$ and $H_0^{\mathrm{local}} = 73.0 \pm 1.0$ $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ as external references. These anchors are used for sensitivity mapping, not as hard priors in the core area-law test, to preserve modified-gravity neutrality in the reconstruction stage.

IV. DISCUSSION

The key physical output is not only "negative slope detected," but what that slope means. A decreasing $\mu(A)$ implies that the effective entropy current changes as the Universe expands. In modified-gravity language this maps to an evolving effective Planck sector (running gravitational coupling), a known channel in scalar-tensor and $f(R)$-like effective descriptions.

The non-extensive proximity ordering is consistent with that interpretation. Tsallis/Barrow-like templates are not asserted here as unique microphysical theories; rather, they provide compact effective parameterizations of the reconstructed thermodynamic flow. Their significantly smaller D^2 values relative to strict Bekenstein-Hawking behavior quantify that the data-preferred trajectory is not well described by $\mu = \text{const}$.

The calibration and closure layers are included for this reason: to separate physics statements from pipeline artifacts. Synthetic closure and SBC checks, together with seed-level stability of the identifiability gate, support interpreting the negative-slope trend as a property of the reconstructed geometry rather than a numerical accident.

V. CONCLUSION

We recast late-time entropy-slope reconstruction as a fundamental cosmological test of the Bekenstein-Hawking area law. In the reconstructed posterior, the horizon entropy current is not strictly constant and is instead biased toward a negative slope with non-extensive effective behavior favored in function space.

Most importantly, this thermodynamic evolution ($\mu(A) \neq$ const) provides the theory-side basis for the modified gravitational-wave propagation (friction) signals targeted in dark-siren catalogs, linking horizon thermodynamics and multimessenger cosmology in a single framework.

DATA AND SOFTWARE AVAILABILITY

Software archive for this pipeline (Zenodo title: *Negative Entropy Slope*): doi:10.5281/zenodo.18604922.

External data/product DOIs used by the underlying stack include:

- GWTC-3 products: doi:10.1103/PhysRevX.13.041039.
- GLADE+: doi:10.1093/mnras/stac1443.
- Pantheon+ papers and dataset: doi:10.3847/1538-4357/ac8b7a, doi:10.3847/1538-4357/ac8e04, doi:10.5281/zenodo.16365279.
- Cosmic-chronometer compilations: doi:10.1088/1475-7516/2012/08/006, doi:10.1088/1475-7516/2016/05/014.
- BOSS DR12 BAO+FS: doi:10.1093/mnras/stx721.
- eBOSS DR16 LRG BAO+RSD: doi:10.1093/mnras/staa2455.
- DESI 2024 BAO analyses: doi:10.1088/1475-7516/2025/04/012, doi:10.1088/1475-7516/2025/02/021.
- Planck 2018 lensing: doi:10.1051/0004-6361/201833886.

AI-USE STATEMENT

AI systems were used for drafting support and editing during manuscript preparation.

[1] R. Abbott *et al.* (LIGO Scientific Collaboration, Virgo Collaboration, and KAGRA Collaboration), Phys. Rev. X **13**, 041039 (2023), doi:10.1103/PhysRevX.13.041039.

[2] G. Dálya *et al.*, Mon. Not. R. Astron. Soc. **514**, 1403 (2022), doi:10.1093/mnras/stac1443.