pions, and events with at least one combination satisfying $M^{\rm recoil}_{\pi^+\pi^-} \in [3.45,\ 3.65]$ GeV/c^2 and $M^{\rm recoil}_{\gamma\pi^+\pi^-} \in [2.8,\ 3.2]$ GeV/c^2 are kept for a further analysis. Here $M^{\rm recoil}_{\pi^+\pi^-}$ ($M^{\rm recoil}_{\gamma\pi^+\pi^-}$) is the mass recoiling from the $\pi^+\pi^-$ ($\gamma\pi^+\pi^-$) pair, which should be in the mass range of the h_c (η_c).

To determine the species of final state particles and to select the best photon when additional photons (and π^0 or η candidates) are found in an event, the combination with the minimum value of $\chi^2 = \chi^2_{\rm 4C} + \sum_{i=1}^{N} \chi^2_{\rm PID}(i) + \chi^2_{\rm 1C}$ is selected for a further analysis, where $\chi^2_{\rm 4C}$ is the χ^2 from the initial-final four-momentum conservation (4C) kinematic fit, $\chi^2_{\rm PID}(i)$ is the χ^2 from particle identification using the energy loss in the MDC and the time measured with the Time-of-Flight system. N is the number of the charged tracks in the final states, and $\chi^2_{\rm 1C}$ is the sum of the 1C (mass constraint of the two daughter photons) χ^2 of the π^0 and η in each final state. There is also a $\chi^2_{\rm 4C}$ requirement, which is optimized using the figure-of-merit, $S/\sqrt{S+B}$, where S and B are the numbers of MC simulated signal and background events, respectively, and $\chi^2_{\rm 4C} < 35$ (efficiency is about 80% from MC simulation) is required for final states with only charged or K^0_S particles, while $\chi^2_{\rm 4C} < 20$ (efficiency is about 70% from MC simulation) is required for those with π^0 or η [15]. A similar optimization procedure determines the η_c candidate mass window around the nominal η_c [14] mass to be ± 50 MeV/c^2 with efficiency about 85% from MC simulation (± 45 MeV/c^2 with efficiency about 80% from MC simulation) for final states with only charged or K^0_S particles (those with π^0 or η).

Figure 1 shows as an example the scatter plot of the mass of the η_c candidate versus that of the h_c candidate at the CM energy of 4.26 GeV, as well as the projection of the invariant mass distribution of $\gamma\eta_c$ in the η_c signal region, where a clear $h_c \to \gamma\eta_c$ signal is observed. To extract the number of $\pi^+\pi^- h_c$ signal events, the $\gamma\eta_c$ mass spectrum is fitted using the MC simulated signal shape convolved with a Gaussian function to reflect the mass resolution difference (around 10%) between data and MC simulation, together with a linear background. The fit to the 4.26 GeV data is shown in Fig. 1. The tail in the high mass side is due to the events with initial state radiation (ISR) which is simulated well in MC, and its fraction is fixed in the fit. At the energy points with large statistics (4.23, 4.26, and 4.36 GeV), the fit is applied to the 16 η_c decay modes simultaneously, while at the other energy points, we fit the mass spectrum summed over all the η_c decay modes. The number of signal events ($n^{\rm obs}_{h_c}$) and the measured Born cross section at each energy are listed in Table I. The $\pi^+\pi^- h_c$ cross section appears to be constant above 4.2 GeV with a possible local maximum at around 4.23 GeV. This is in contrast to the observed energy dependence in the $e^+e^- \to \pi^+\pi^- J/\psi$ channel which revealed a decrease of cross sections at higher energies [2, 17].

Systematic errors in the cross section measurement mainly come from the luminosity measurement, the branching fraction of $h_c \to \gamma\eta_c$, the branching fraction of $\eta_c \to X_i$, the detection efficiency, the ISR correction factor, and the fit. The integrated luminosity at each energy point is measured using large angle Bhabha events, and it has an estimated uncertainty of 1.2%. The branching fractions of $h_c \to \gamma\eta_c$ and $\eta_c \to X_i$ are taken from Refs. [12, 13]. The uncertainties in the detection efficiency are estimated in the same way as described in Refs. [13, 16], and the error in the ISR correction is estimated as described in Ref. [1]. Uncertainties due to the choice of the signal shape, the background shape, the mass resolution, and fit range are estimated by varying the h_c and η_c resonant parameters and line shapes in MC simulation, varying the background function from linear to a second-order polynomial, varying the mass resolution difference between data and MC simulation by one standard deviation, and by extending the fit range. Assuming all of the sources are independent, the total systematic error in the $\pi^+\pi^- h_c$ cross section measurement is determined to be between 7% and 9% depending on the energy, and to be conservative we take 9%