MATH 3503: Tashfeen's Discrete Mathematics

Homework 6

Please give your final answer as a number where possible as well as show work for full credit.

Question 1. Please read chapter 9 of Chartrand et al. and write a couple sentences about a topic/example/concept that you found difficult or interesting and why?

Question 2. What is the exact term in $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^{10}$ containing x?

Question 3. Use the Binomial Theorem to simplify the sum,

$$2^{n^2} + \binom{n}{1}2^{n(n-1)} + \binom{n}{2}2^{n(n-2)} + \cdots + \binom{n}{n-1}2^n + 1$$

Question 4. How many different 8-digit numbers can be obtained by permuting the digits in the number $70,440,704$? (Since each number is an 8-digit number, the first digit cannot be 0.)

Question 5. A professor has 10 identical new pens that he no longer needs. In how many ways can these pens be given to 3 students if

    (a) there are no other conditions?
    (b) every student must receive at least one pen?
    (c) every student must receive at least two pens?
    (d) every student must receive at least three pens?

Question 6. How many 4-digit numbers are there, the sum of whose digits is 11?

Question 7. A man enters Ben's Bagels to buy a dozen bagels only to learn that even though Ben has a large supply of plain bagels and blueberry bagels, he only has two cinnamon bagels left and no other kinds.

    (a) Express the number of selections of r bagels ($r \geq 0$) the man can make as a product of polynomials and/or power series[1].
    (b) Determine the coefficient of x^{12} of the product in (a). What information does this coefficient provide?

Question 8. Suppose that we are interested in the number of non-negative integer solutions a, b and c of the equation $a + 2b + 3c = r$, where $r \in \mathbb{N}$.

    (a) What is this number when $r \in \{1, 2, 3, 4\}$?
    (b) Use generating functions to describe this number for a general $r \in \mathbb{N}$.

Question 9. A man buys 6 doughnuts, each of which is a plain doughnut, a powdered doughnut or a glazed doughnut. How many possible selections are there if he buys at least one doughnut of each kind?

    (a) Answer the question above by listing all possible selections in a table.
    (b) Answer the question above by determining the coefficient of x^6 in a product of polynomials and/or power series.
    (c) Answer the question above by computing $\binom{s+t-1}{s}$ for an appropriate choice of s and t.

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[1]The book has a list of common power series in figure 9.8 on page 347.