MATH 3503: Tashfeen's Discrete Mathematics

Homework 3

Question 1. Please read chapters 3 and 4 of Chartrand et al. and write a couple sentences about a topic/example/concept that you found difficult or interesting and why?

Question 2. Consider the following quantified statement: For every real number x, there exists a positive real number y such that $y < x^2$.

(a) Express this quantified statement in symbols.
(b) Express the negation of this quantified statement in symbols.
(c) Express the negation of this quantified statement in words.

Question 3. Prove that if r and s are rational numbers, then $r - s$ is a rational number.

Question 4. Let x and y be integers. Prove that if $x + y \geq 9$, then either $x \geq 5$ or $y \geq 5$.

Question 5. Let m and n be two integers. Prove that mn and $m + n$ are both even if and only if m and n are both even.

Question 6. Disprove: Let A, B and C be sets. If $A \cup B = A \cup C$, then $B = C$.

Question 7. Prove that if a and b are positive real numbers, then $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$.

Question 8. Let $r \geq 2$ be an integer. Prove that $1 + r + r^2 + \cdots + r^n = \frac{r^{n+1}-1}{r-1}$ for every positive integer n.

Question 9. Prove that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n+1}$ for every integer $n \geq 3$.

Question 10. A sequence $a_1, a_2, a_3, \cdots$ is defined recursively by $a_1 = 3$ and $a_n = 2a_{n-1} + 1$ for $n \geq 2$.

(a) Determine a_2, a_3, a_4, and a_5.
(b) Based on the values obtained in (a), make a guess for a formula for a_n for every positive integer n and use induction to verify that your guess is correct.

Question 11. In Example 4.36, we saw that n^{th} Fibonacci number $F_n \leq 2^n$. Prove that $F_n \leq (\frac{5}{3})^n$ for every positive integer n.

Question 12. A sequence $\{a_n\}$ is defined recursively by $a_1 = 5$, $a_2 = 7$ and $a_n = 3a_{n-1} - 2a_{n-2} - 2$ for $n \geq 3$. Prove that $a_n = 2n + 3$ for every positive integer n.

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