Question 7 – Coordinates-to-index & Index-to-coordinates

7.1 2-dimension

Size of 2-D grid: $(L_1,\ L_2)$

L_1: length of grid in x_1 direction

L_2: length of grid in x_2 direction

Coordinates $(x_1,\ x_2)$ to Index (I):

$$I = x_1 + L_1 * x_2$$

Index (I) to Coordinates $(x_1,\ x_2)$:

$$x_2 = I\ //\ L_1$$

$$x_1 = I\ \%\ L_1$$

7.2 d-dimension

Size of d-D grid: $(L_1,\ L_2,\ L_3, \dots,\ L_d)$

L_1: length of grid in x_1 direction

L_2: length of grid in x_2 direction

L_3: length of grid in x_3 direction

...

L_d: length of grid in x_d direction

Coordinates $(x_1,\ x_2,\ x_3, \dots, x_d)$ to Index (I):

$$I = x_1 + (L_1) * x_2 + (L_1 * L_2) * x_3 + \cdots + (L_1 * L_2 * L_3 * \dots * L_{d-1}) * x_d$$

$$= x_1 + \sum_{i=2}^{d} \left(x_i * \prod_{j=1}^{i-1} L_j \right)$$

Index (I) to Coordinates $(x_1,\ x_2,\ x_3, \dots, x_d)$:

$$x_d = I\ //\ (L_1 * L_2 * L_3 * \dots * L_{d-1}) = I\ //\ \prod_{i=1}^{d-1} L_i$$

$$x_{d-1} = \left(I - \left(\prod_{i=1}^{d-1} L_i \right) * (x_d) \right) // \left(\prod_{i=1}^{d-2} L_i \right)$$

$$x_{d-2} = \left(I - \left(\prod_{i=1}^{d-1} L_i \right) * (x_d) - \left(\prod_{i=1}^{d-2} L_i \right) * (x_{d-1}) \right) // \left(\prod_{i=1}^{d-3} L_i \right)$$

...

$$x_1 = I\ \%\ L_1$$