COMP 251 - Fall 2017 - Assignment 2

Due: 11:59pm Oct 13th

General rules: In solving these questions you may consult your book; You can discuss high level ideas with each other, but each student must find and write his/her own solution. You should upload the pdf file (either typed, or a clear scan) of your solution to mycourses.

1. (10 points) How can the number of strongly connected components of a directed graph change if we add a new edge? Explain your answer.

2. (10 points) A celebrity among a group of n people is a person who knows nobody but is known by everyone else. The task is to identify a celebrity by only asking questions to people of the form: "Do you know him/her?" Design an efficient algorithm to identify a celebrity or determine that the group has no such person. How many questions does your algorithm need in the worst case?

3. (20 points) Consider a list of n airports, and a list of m flights. The information of the k-th flight is given to us as a quadruple (a_k, b_k, t_k, d_k), where a_k is the name of the origin airport, b_k is the destination airport, t_k is the time of departure, and d_k is the time of arrival.

   Given four parameters a, b, s, t, we want to see that starting at time s at airport a, we can take a sequence of flights to get to b no later than time t. Here we assume that transferring from one flight to the next takes no time.

   Give an algorithm that solves this problem in $O(n + m \log m)$.

4. (20 points) Suppose that a black stone is placed on a vertex s and a white stone is placed on a different node t of an undirected graph G, where there is no edge between s and t. At every step, we have to move both stones simultaneously to two non-adjacent (different) vertices (the stones can visit a node several times during the algorithm if needed but they have to move at every step). A stone can only be moved from a node to a neighbouring node. The ultimate goal is to switch the places of the two stones. That is to have the black stone on t and the white stone on s. Design an algorithm that takes as input G and s and t, and tells whether this is possible, and if it is, then what is the minimum number of steps required to achieve this.

5. (20 points) The distance between two nodes in an undirected graph G is the number of edges in the shortest path between them. The diameter of a graph G is the maximum distance between any pair of nodes in the graph. Give an $O(n)$ algorithm that computes the diameter of a *tree* T on n nodes. Analyze the running time of your algorithm.

6. (20 points) Consider a directed graph G with n vertices and m edges, where every node is labeled with a unique number from the set $\{1, \ldots, n\}$. For every node u, let $\min(u)$ be the smallest label among all the vertices that can be reached from u. Give an $O(m + n)$ algorithm that computes $\min(u)$ for all the vertices of G.