COMP 251 - Fall 2017 - Assignment 5

Due: 11:59pm Dec 3rd

General rules: In solving these questions you may consult your book; You can discuss high level ideas with each other, but each student must find and write his/her own solution. You should upload the pdf file (either typed, or a clear scan) of your solution to mycourses.

1. (15 points) Consider the closest pair of points problem discussed in the class (Section 5.4), but now suppose that the distance between two points (x_i, y_i) and (x_j, y_j) is measured as $|x_i - x_j| + |y_i - y_j|$ instead of the Euclidean distance. Modify the algorithm discussed in the class to solve this problem.

2. (15 points) Consider an $n \times n$ table where some cells are marked as forbidden. A frog is placed in the $(1, 1)$-cell (that is lowest and left most cell). At every step, the frog can jump from its current cell (i, j) to any cell (i', j') that satisfies $i' + j' = i + j + 1$ provided that the cell is not forbidden. We assume that $(1, 1)$ and (n, n) are never forbidden.

   Design and analyze an efficient algorithm that given the coordinates of the forbidden cells, finds the number of different paths that the frog can take to move from $(1, 1)$ to (n, n).

3. (15 points) Design and analyze a $O(n^3)$ algorithm that given a sequence of n distinct numbers, finds the largest subsequence, such that each number is between the previous two numbers in the subsequence. In other words if a, b, c are three consecutive numbers in the subsequence, then $\min(a, b) < c < \max(a, b)$. For example if the input is $1, 3, 9, 8, 5, 7, 6$, then the output is $1, 9, 5, 7, 6$

4. (15 points) We are given a set of n jobs. The i-th job has a deadline d_i, requires processing time of t_i, and creates a profit of p_i only if it is finished before the deadline. The numbers d_i, t_i, and p_i are all positive integers. We have a single processor (that can start processing at time 0), and we want to select a subset of jobs that will create the maximum possible profit. Design and analyze an algorithm that performs this task.

5. (20 points) We are given a directed graph with n nodes, and each node is labeled with the currency of a country (e.g. Canadian dollar, Yuan, Kyat, Euro, Ariary, etc). Every (directed) edge uv is labeled with a real number $0 < \alpha_{uv}$ signifying the fact that 1 unit of the currency u can be exchanged with α_{uv} units of the currency v. We want to see if we can make money by just buying and selling currencies. In other words, we want to find a sequence of currencies $u_1, \ldots, u_k$ such that, we can start from one unit of u_1, and exchange it with $\alpha_{u_1 u_2}$ units of u_2, and exchange that with $\alpha_{u_2 u_3}\alpha_{u_1 u_2}$ units of u_3, etc, and eventually exchange the amount of u_k that we obtained with u_1, so that we will end up with more than one unit of u_1.

   Design and analyze a polynomial-time algorithm that tells us whether this is possible.

6. (20 points) Design and analyze an efficient algorithm that given an undirected graph $G = (V, E)$, and two nodes s, t finds the number of shortest paths from s to t.