# Homework 3

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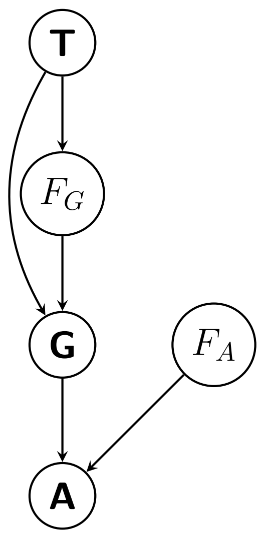
## 1 Designing a Bayesian Network

### 1.1 a.




### 1.2 b.

No, it is not a polytree as there is an undirected cycle, or there are two paths that lead from **T** to **G**.

### 1.3 c.

$$x = P(T = G|F_G = 0)$$
$$y = P(T = G|F_G = 1)$$

| G | $F_G$ | T=Normal | T=High |
|---|---|---|---|
| N | 1 | y | 1-y |
| N | 0 | x | 1-x |
| H | 1 | 1-y | y |
| H | 0 | 1-x | x |

### 1.4 d.

| A | $F_A$ | G=Normal | G=Exceeded |
|---|---|---|---|
| 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 |

### 1.5 e.

We want to calculate $P(T = High|F_A = 0, F_G = 0, A = 1)$ but with Bayes Ball we can simplify it to $P(T = High|F_G = 0)$ as $G$ "blocks".

$$
\begin{aligned}
P(T = High|F_G = 0, G = g) &= \sum_g P(T = High, F_G = 0, G = g) \\
&= \sum_g P(T = High) \times P(F_G = 0|T = High) \times P(G = g|F_G = 0, T = High)
\end{aligned}
$$

## 2 Inference in Bayesian Networks

### 2.1 a. $P(s, r)$

$$
\begin{aligned}
P(s, r) &= P(S = 1, R = 1) \\
&= \sum_{b,a,t} P(r, B = b, A = a, T = t, s) \\
&= \sum_{b,a,t} P(r) \times P(T = t|r, B = b, A = a) \times P(B = b) \times P(A = a|B = b) \times P(s|A = a)
\end{aligned}
$$

| B | A | T | P(r) | P(T=t\|r,B=b,A=a) | P(B=b) | P(A=a\|B=b) | P(s\|A=a) | Product | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0.15 | 0.95 | 0.2 | 0.6 | 0.8 | 0.01368 | |
| 1 | 0 | 1 | 0.15 | 0.9 | 0.2 | 0.4 | 0.2 | 0.00216 | |
| 0 | 1 | 1 | 0.15 | 0.92 | 0.8 | 0.4 | 0.8 | 0.035328 | |
| 0 | 0 | 1 | 0.15 | 0.85 | 0.8 | 0.6 | 0.2 | 0.01224 | |
| 1 | 1 | 0 | 0.15 | 0.05 | 0.2 | 0.6 | 0.8 | 0.00072 | |
| 1 | 0 | 0 | 0.15 | 0.1 | 0.2 | 0.4 | 0.2 | 0.00024 | |
| 0 | 1 | 0 | 0.15 | 0.08 | 0.8 | 0.4 | 0.8 | 0.003072 | |
| 0 | 0 | 0 | 0.15 | 0.15 | 0.8 | 0.6 | 0.2 | 0.00216 | |
| | | | | | | | | 0.0696 | =P(s,r) |

Figure 1: $P(s, r)$

## 2.2 b. $P(a, \neg t)$

$$
\begin{aligned}
P(a, \neg t) &= P(A = 1, T = 0) \\
&= \sum_{r,b,s} P(R = r, B = b, a, \neg t, S = s) \\
&= \sum_{r,b,s} P(R = r) \times P(\neg t | R = r, B = b, a) \times P(a | B = b) \times P(S = s | a)
\end{aligned}
$$

| R | B | S | P(R=r) | P(¬t\|R=r,B=b,a) | P(a\|B=b) | P(S=s\|a) | P(B=b) | Product | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0.15 | 0.05 | 0.6 | 0.8 | 0.2 | 0.00072 | |
| 1 | 0 | 1 | 0.15 | 0.08 | 0.4 | 0.8 | 0.8 | 0.003072 | |
| 0 | 1 | 1 | 0.85 | 0.65 | 0.6 | 0.8 | 0.2 | 0.05304 | |
| 0 | 0 | 1 | 0.85 | 0.4 | 0.4 | 0.8 | 0.8 | 0.08704 | |
| 1 | 1 | 0 | 0.15 | 0.05 | 0.6 | 0.2 | 0.2 | 0.00018 | |
| 1 | 0 | 0 | 0.15 | 0.08 | 0.4 | 0.2 | 0.8 | 0.000768 | |
| 0 | 1 | 0 | 0.85 | 0.65 | 0.6 | 0.2 | 0.2 | 0.01326 | |
| 0 | 0 | 0 | 0.85 | 0.4 | 0.4 | 0.2 | 0.8 | 0.02176 | |
| | | | | | | | | 0.17984 | =P(a,¬t) |

Figure 2: $P(a, \neg t)$

## 2.3 c. $P(t|s)$

$$
\begin{aligned}
P(t|s) &= P(T = 1 | S = 1) \\
&= \frac{P(t, s)}{P(s)} \\
P(t, s) &= \sum_{r,b,a} P(R = r, B = b, A = a, t, s) \\
&= \sum_{r,b,a} P(R = r) \times P(t | R = r, B = b, A = a) \times P(A = a | B = b) \times P(s | A = a) \\
P(s) &= P(s|a)P(a) + P(s|\neg a)P(\neg a) = P(s|a)P(a) + P(s|\neg a)(1 - P(a)) \\
P(a) &= P(a|b)P(b) + P(a|\neg b)P(\neg b) = P(a|b)P(b) + P(a|\neg b)(1 - P(b))
\end{aligned}
$$

$$
\frac{P(t, s)}{P(s)} \approx 0.5
$$

| | | | | | | | | | P(t) | P(t,s) |
|---|---|---|---|---|---|---|---|---|---|---|
| R | B | A | S | P(R=r) | P(t\|R=r,B=b,A | P(B=b) | P(A=a\|B=b) | P(S=s\|A=a) | Product | Product |
| 1 | 1 | 1 | 1 | 0.15 | 0.95 | 0.2 | 0.6 | 0.8 | 0.01368 | 0.01368 |
| 1 | 1 | 1 | 0 | 0.15 | 0.95 | 0.2 | 0.6 | 0.2 | 0.00342 | 0 |
| 1 | 1 | 0 | 1 | 0.15 | 0.9 | 0.2 | 0.4 | 0.2 | 0.00216 | 0.00216 |
| 1 | 1 | 0 | 0 | 0.15 | 0.9 | 0.2 | 0.4 | 0.8 | 0.00864 | 0 |
| 1 | 0 | 1 | 1 | 0.15 | 0.92 | 0.8 | 0.4 | 0.8 | 0.035328 | 0.035328 |
| 1 | 0 | 1 | 0 | 0.15 | 0.92 | 0.8 | 0.4 | 0.2 | 0.008832 | 0 |
| 1 | 0 | 0 | 1 | 0.15 | 0.85 | 0.8 | 0.6 | 0.2 | 0.01224 | 0.01224 |
| 1 | 0 | 0 | 0 | 0.15 | 0.85 | 0.8 | 0.6 | 0.8 | 0.04896 | 0 |
| 0 | 1 | 1 | 1 | 0.85 | 0.35 | 0.2 | 0.6 | 0.8 | 0.02856 | 0.02856 |
| 0 | 1 | 1 | 0 | 0.85 | 0.35 | 0.2 | 0.6 | 0.2 | 0.00714 | 0 |
| 0 | 1 | 0 | 1 | 0.85 | 0.4 | 0.2 | 0.4 | 0.2 | 0.00544 | 0.00544 |
| 0 | 1 | 0 | 0 | 0.85 | 0.4 | 0.2 | 0.4 | 0.8 | 0.02176 | 0 |
| 0 | 0 | 1 | 1 | 0.85 | 0.6 | 0.8 | 0.4 | 0.8 | 0.13056 | 0.13056 |
| 0 | 0 | 1 | 0 | 0.85 | 0.6 | 0.8 | 0.4 | 0.2 | 0.03264 | 0 |
| 0 | 0 | 0 | 1 | 0.85 | 0.05 | 0.8 | 0.6 | 0.2 | 0.00408 | 0.00408 |
| 0 | 0 | 0 | 0 | 0.85 | 0.05 | 0.8 | 0.6 | 0.8 | 0.01632 | 0 |
| | | | | | | | | | 0.37976 | 0.232048 |

Figure 3: $P(t)$ and $P(t,s)$. Anything for P(t) should be ignored.

# 3 Variable Elimination

$P(T|a) = P(T=t|A=1)$

## 3.1 Variable ordering

With ordering $R, S, B, A, T$ and $T$ as the query.

## 3.2 Factor list

$P(R=r), P(T=t|R=r, B=b, A=a), P(B=b), P(A=a|B=b), P(S=s|A=a), \delta(A,1)$

## 3.3 Marginalizing

$$m_R(T,B,A) = \sum_r P(r) \times P(T|r,B,A) \times \delta(A,1)$$

$$m_S(A) = \sum_s P(s|A)$$

$$m_B(A) = \sum_b P(b) \times P(A|b)$$

$$m_A(T) = \sum_a m_R(T,B,A) \times m_S(A)$$

We get 0.26016 for $T=1$ and 0.17984 for $T=0$.

| | | | | | | | | | | m_R(T,B,a) | m_S(a) | m_B(a) | m_A(T=1) | m_A(T=0) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| R | B | A | T | S | P(R=r) | P(T\|R=r,B=b,a | P(B=b) | P(A=a\|B=b | P(S=s\|A=a) | Product | Product | Product | Product | Product |
| 1 | 1 | 1 | 1 | 1 | 0.15 | 0.95 | 0.2 | 0.6 | 0.8 | 0.1425 | 0.8 | 0.12 | 0.01368 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0.15 | 0.95 | 0.2 | 0.6 | 0.2 | 0.1425 | 0.2 | 0.12 | 0.00342 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0.15 | 0.05 | 0.2 | 0.6 | 0.8 | 0.0075 | 0.8 | 0.12 | 0 | 0.00072 |
| 1 | 1 | 1 | 0 | 0 | 0.15 | 0.05 | 0.2 | 0.6 | 0.2 | 0.0075 | 0.2 | 0.12 | 0 | 0.00018 |
| 1 | 1 | 0 | 1 | 1 | 0.15 | 0.95 | 0.2 | 0.4 | 0.2 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0.15 | 0.95 | 0.2 | 0.4 | 0.8 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0.15 | 0.05 | 0.2 | 0.4 | 0.2 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0.15 | 0.05 | 0.2 | 0.4 | 0.8 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0.15 | 0.92 | 0.8 | 0.4 | 0.8 | 0.138 | 0.8 | 0.32 | 0.035328 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0.15 | 0.92 | 0.8 | 0.4 | 0.2 | 0.138 | 0.2 | 0.32 | 0.008832 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0.15 | 0.08 | 0.8 | 0.4 | 0.8 | 0.012 | 0.8 | 0.32 | 0 | 0.003072 |
| 1 | 0 | 1 | 0 | 0 | 0.15 | 0.08 | 0.8 | 0.4 | 0.2 | 0.012 | 0.2 | 0.32 | 0 | 0.000768 |
| 1 | 0 | 0 | 1 | 1 | 0.15 | 0.92 | 0.8 | 0.6 | 0.2 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0.15 | 0.92 | 0.8 | 0.6 | 0.8 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0.15 | 0.08 | 0.8 | 0.6 | 0.2 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0.15 | 0.08 | 0.8 | 0.6 | 0.8 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0.85 | 0.35 | 0.2 | 0.6 | 0.8 | 0.2975 | 0.8 | 0.12 | 0.02856 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0.85 | 0.35 | 0.2 | 0.6 | 0.2 | 0.2975 | 0.2 | 0.12 | 0.00714 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0.85 | 0.65 | 0.2 | 0.6 | 0.8 | 0.5525 | 0.8 | 0.12 | 0 | 0.05304 |
| 0 | 1 | 1 | 0 | 0 | 0.85 | 0.65 | 0.2 | 0.6 | 0.2 | 0.5525 | 0.2 | 0.12 | 0 | 0.01326 |
| 0 | 1 | 0 | 1 | 1 | 0.85 | 0.35 | 0.2 | 0.4 | 0.2 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0.85 | 0.35 | 0.2 | 0.4 | 0.8 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0.85 | 0.65 | 0.2 | 0.4 | 0.2 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0.85 | 0.65 | 0.2 | 0.4 | 0.8 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0.85 | 0.6 | 0.8 | 0.4 | 0.8 | 0.51 | 0.8 | 0.32 | 0.13056 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0.85 | 0.6 | 0.8 | 0.4 | 0.2 | 0.51 | 0.2 | 0.32 | 0.03264 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0.85 | 0.4 | 0.8 | 0.4 | 0.8 | 0.34 | 0.8 | 0.32 | 0 | 0.08704 |
| 0 | 0 | 1 | 0 | 0 | 0.85 | 0.4 | 0.8 | 0.4 | 0.2 | 0.34 | 0.2 | 0.32 | 0 | 0.02176 |
| 0 | 0 | 0 | 1 | 1 | 0.85 | 0.6 | 0.8 | 0.6 | 0.2 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0.85 | 0.6 | 0.8 | 0.6 | 0.8 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0.85 | 0.4 | 0.8 | 0.6 | 0.2 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0.85 | 0.4 | 0.8 | 0.6 | 0.8 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | | | | | 4 | 8 | 3.52 | 0.26016 | 0.17984 |

Figure 4: $P(T|a).A$ is given (=1).

# 4 Learning with Bayesian Networks

## 4.1 a.

### 4.1.1 i.

$$\theta_A = P(A)$$
$$\theta_B = P(B|A=1)$$
$$\theta_B = P(B|A=1)$$
$$\theta_C = P(C|A=1)$$
$$\theta_C = P(C|A=1)$$
$$\theta_D = P(D|B=1, C=1)$$
$$\theta_D = P(D|B=1, C=0)$$
$$\theta_D = P(D|B=0, C=1)$$
$$\theta_D = P(D|B=0, C=0)$$

### 4.1.2 ii.

$$
\begin{aligned}
P(A=1) &= \frac{\#(A)}{\#(A=1)+\#(A=0)} = 49/146 && \approx 0.335616 \\
P(B=1|A=1) &= \frac{\#(B=1,A=1)}{\#(B=1,A=1)+\#(B=0,A=1)} \\
&\approx 0.87755102 \\
P(B=1|A=0) &= \frac{\#(B=1,A=0)}{\#(B=1,A=0)+\#(B=0,A=0)} \\
&\approx 0.701030928 \\
P(C=1|A=1) &= \frac{\#(C=1,A=1)}{\#(C=1,A=1)+\#(C=0,A=1)} \\
&\approx 0.387755102 \\
P(C=1|A=0) &= \frac{\#(C=1,A=0)}{\#(C=1,A=0)+\#(C=0,A=0)} \\
&\approx 0.577319588 \\
P(D=1|B=1,C=1) &= \frac{\#(D=1,B=1,C=1)}{\#(D=1,B=1,C=1)+\#(D=0,B=1,C=1)} \\
&\approx 0 \\
P(D=1|B=1,C=0) &= \frac{\#(D=1,B=1,C=0)}{\#(D=1,B=1,C=0)+\#(D=0,B=1,C=0)} \\
&\approx 0.323076923 \\
P(D=1|B=0,C=1) &= \frac{\#(D=1,B=0,C=1)}{\#(D=1,B=0,C=1)+\#(D=0,B=0,C=1)} \\
&\approx 0.275862069 \\
P(D=1|B=0,C=0) &= \frac{\#(D=1,B=0,C=0)}{\#(D=1,B=0,C=0)+\#(D=0,B=0,C=0)} \\
&\approx 0.666666667
\end{aligned}
$$

| #(A=1) | #(B=1) | #(C=1) | #(D=1) | #(A=0) | #(B=0) | #(C=0) | #(D=0) |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 4 | 4 | 4 | 4 | 0 |
| 0 | 0 | 20 | 0 | 20 | 20 | 0 | 20 |
| 0 | 0 | 4 | 4 | 4 | 4 | 0 | 0 |
| 0 | 34 | 0 | 0 | 34 | 0 | 34 | 34 |
| 0 | 2 | 0 | 2 | 2 | 0 | 2 | 0 |
| 0 | 32 | 32 | 0 | 32 | 0 | 0 | 32 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 4 | 0 | 4 | 4 | 0 | 4 | 0 | 0 |
| 10 | 10 | 0 | 0 | 0 | 0 | 10 | 10 |
| 19 | 19 | 0 | 19 | 0 | 0 | 19 | 0 |
| 14 | 14 | 14 | 0 | 0 | 0 | 0 | 14 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 49 | 111 | 75 | 33 | 97 | 35 | 71 | 113 |
| | | | | | | | |
| P(A=1) | | | | | | | |
| 0.335616 | | | | | | | |

Figure 5

| #(B=1,A=1) | #(B=1,A=0) | #(C=1,A=1) | #(C=1,A=0) | #(B=0,A=1) | #(B=0,A=0) | #(C=0,A=1) | #(C=0,A=0) |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 4 | 0 | 4 |
| 0 | 0 | 0 | 20 | 0 | 20 | 0 | 0 |
| 0 | 0 | 0 | 4 | 0 | 4 | 0 | 0 |
| 0 | 34 | 0 | 0 | 0 | 0 | 0 | 34 |
| 0 | 2 | 0 | 0 | 0 | 0 | 0 | 2 |
| 0 | 32 | 0 | 32 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 4 | 0 | 4 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 10 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 19 | 0 |
| 14 | 0 | 14 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 43 | 68 | 19 | 56 | 6 | 29 | 30 | 41 |
| | | | | | | | |
| P(B=1\|A=1) | P(B=1\|A=0) | P(C=1\|A=1) | P(C=1\|A=0) | | | | |
| 0.87755102 | 0.701030928 | 0.387755102 | 0.577319588 | | | | |

Figure 6

| #(D=1,B=1,C=1) | #(D=0,B=1,C=1) | #(D=1,B=1,C=0) | #(D=0,B=1,C=0) | #(D=1,B=0,C=1) | #(D=0,B=0,C=1) | #(D=1,B=0,C=0) | #(D=0,B=0,C=0) |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 20 | 0 | 0 |
| 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 |
| 0 | 0 | 0 | 34 | 0 | 0 | 0 | 0 |
| 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 |
| 0 | 32 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 |
| 0 | 0 | 0 | 10 | 0 | 0 | 0 | 0 |
| 0 | 0 | 19 | 0 | 0 | 0 | 0 | 0 |
| 0 | 14 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 46 | 21 | 44 | 8 | 21 | 4 | 2 |
| | | | | | | | |
| P(D=1,B=1,C=1) | | P(D=1,B=1,C=0) | | P(D=1,B=0,C=1) | | P(D=1,B=0,C=0) | |
| 0 | | 0.323076923 | | 0.275862069 | | 0.666666667 | |

Figure 7