

Homework #1: Propositional Logic

Instructions: Answer the following questions using this week's readings. Write them up and submit through Moodle using either .docx or .pdf format. For your convenience a list of logic symbols has been provided below:

Logic Symbols: \rightarrow \leftrightarrow \neg \wedge \vee \forall \exists

Problem 1

Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of the following propositions.

- Quixote Media had the largest annual revenue.
- Nadir Software had the lowest net profit and the Acme Computer had the largest annual revenue.
- Acme Computer had the largest net profit or Quixote Media had the largest net profit.
- If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.

Problem 2

Express these system specifications using the propositions p "The message is scanned for viruses" and q "The message was sent from an unknown system" together with logical connectives.

- "The message is scanned for viruses whenever the message was sent from an unknown system."
- "The message was sent from an unknown system but it was not scanned for viruses."
- "It is necessary to scan the message for viruses whenever it was sent from an unknown system."
- "When a message is not sent from an unknown system it is not scanned for viruses."

Problem 3

Show that each of the following conditional statements is a tautology by using truth tables.

- $(p \wedge q) \rightarrow p$
- $p \rightarrow (p \vee q)$
- $\neg p \rightarrow (p \rightarrow q)$
- $(p \wedge q) \rightarrow (p \rightarrow q)$
- $\neg(p \rightarrow q) \rightarrow p$
- $\neg(p \rightarrow q) \rightarrow \neg q$

Problem 4

Determine the truth value of each of these statements if the domain consists of all integers.

- a) $\forall n(n + 1 > n)$
- b) $\exists n(2n = 3n)$
- c) $\exists n(n = -n)$
- d) $\forall n(3n \leq 4n)$

Problem 5

Translate the following statements into English, where the domain for each variable consists of all real numbers.

- a) $\forall x \exists y (x < y)$
- b) $\forall x \forall y ((x \geq 0) \vee (y \geq 0)) \rightarrow (xy \geq 0)$
- c) $\forall x \forall y \exists z (xy = z)$