Data Structures
HW1

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Problem 1

Show that $3n^2 + 25$ is $O(n^2)$:

There must exist n_0 and c such that :

$$3n^2 + 25 \leq c.n^2$$
$$n \geq n_0$$

So with $c = 4$:

$$3n^2 + 25 \leq 4n^2$$
$$25 \leq n^2$$
$$5 \leq n$$

$$\rightarrow c = 4, n_0 = 5$$

Problem 2

if $4n^3 + 7n^2 + 12$ is $O(n^3)$ find its corresponding n_0 and k according to the Big O notation formula. :

There must exist n_0 and c such that :

$$4n^3 + 7n^2 + 12 \leq k.n^3$$
$$n \geq n_0$$

So with $k = 4 + 7 + 12 = 23$:

$$4n^3 + 7n^2 + 12 \leq 23n^3$$
$$19n^3 - 7n^2 - 12 \geq 0$$
$$(n-1)(19n^2 + 12n + 12) \geq 0$$
$$(19n^2 + 12n + 12) \text{ always } \geq 0 \rightarrow (n-1) \geq 0$$
$$n \geq 1$$

$$\rightarrow k = 23, n_0 = 1$$

Problem 3

Show that $nlog(n) - 2n + 13$ is $\Omega(nlog(n))$. :

There must exist n_0 and c such that :

$$nlog(n) - 2n + 13 \geq cnlog(n)$$
$$n \geq n_0$$

So with $c = 0.1$:

$$0.9nlog(n) \geq 2n - 13$$
$$9nlog(n) \geq 20n - 130$$
$$9log(n) \geq 20 - \frac{130}{n}$$
$$log(n) \geq \frac{20}{9} - \frac{130}{9n}$$
$$n \geq 10^{\frac{20}{9} - \frac{130}{9n}}$$
$$n \geq 0 \rightarrow 10^{\frac{20}{9}} > 10^{\frac{20}{9} - \frac{130}{9n}}$$
$$\rightarrow n \geq 10^{\frac{20}{9}} \rightarrow n \geq 1000$$

$$\rightarrow c = 0.1, n_0 = 1000$$

Problem 4

Show that $P(n) = n^2 + 5n + 7$ is $\Theta(n^2)$. :

Big O :

There must exist n_0 and c such that :

$$n^2 + 5n + 7 \leq cn^2$$
$$n \geq n_0$$

So with $c = 2 + 5 + 7 = 13$:

$$n^2 + 5n + 7 \leq 13n^2$$
$$12n^2 - 5n - 7 \geq 0$$
$$(n-1)(12n+7) \geq 0$$
$$n \geq 1$$
$$\rightarrow c = 13, n_0 = 1$$
$$\Rightarrow P(n) \in O(n^2)$$

Big Ω :

There must exist n_0 and c such that :

$$n^2 + 5n + 7 \geq cn^2$$
$$n \geq n_0$$

So with $c = 1$ *and* $n_0 = 0$ the unequality is obviously satisfied.

$$\Rightarrow P(n) \in \Omega(n^2)$$
$$\Rightarrow P(n) \in \Theta(n^2)$$

Problem 5

Show that $P(n) = 0.5n^2 - 3n$ is $\Theta(n^2)$. :

Big O :

There must exist n_0 and c such that :

$$0.5n^2 - 3n \leq cn^2$$
$$n \geq n_0$$

So with $c = 0.5$:

$$0.5n^2 - 3n \leq 0.5n^2$$
$$3n \geq 0$$
$$n \geq 0$$
$$\rightarrow c = 0.5, n_0 = 0$$
$$\Rightarrow P(n) \in O(n^2)$$

Big Ω :

There must exist n_0 and c such that :

$$0.5n^2 - 3n \geq cn^2$$
$$n \geq n_0$$

So with $c = 0.4$:

$$0.5n^2 - 3n \geq 0.4n^2$$
$$0.1n^2 - 3n \geq 0$$
$$n(0.1n - 3) \geq 0$$
$$n \geq 30$$
$$\rightarrow c = 0.4, n_0 = 30$$
$$\Rightarrow P(n) \in \Omega(n^2)$$
$$\Rightarrow P(n) \in \Theta(n^2)$$

Problem 6

Show that $P(n) = 3n^2 + 8nlog(n)$ is $\Theta(n^2)$. :

Big O :

There must exist n_0 and c such that :

$$3n^2 + 8nlog(n) \leq cn^2$$
$$n \geq n_0$$

So with $c = 3 + 8 = 11$:

$$3n^2 + 8nlog(n) \leq 11n^2$$
$$8n^2 - 8nlog(n) \geq 0$$
$$n^2 - nlog(n) \geq 0$$
$$n(n - log(n)) \geq 0$$
$$(n - log(n)) \text{ is always } > 0 \rightarrow n \geq 0$$
$$log(0) \text{ is undefined} \rightarrow n > 0$$
$$\rightarrow c = 11, n_0 = 1$$
$$\Rightarrow P(n) \in O(n^2)$$

Big Ω :

There must exist n_0 and c such that :

$$3n^2 + 8nlog(n) \geq cn^2$$
$$n \geq n_0$$

So with $c = 1, n_0 = 1$ the unequality is obviously satisfied.

$$\Rightarrow P(n) \in \Omega(n^2)$$
$$\Rightarrow P(n) \in \Theta(n^2)$$