Math 1 Answers - *Chapter Two*

1.

2.

3.

4.

$$\lim_{x\to 1}\frac{\ln x}{x-1}=\frac{0}{0}$$
$$\xrightarrow{Hopital}\lim_{x\to 1}\frac{\frac{1}{x}}{1}=1=f(1)$$

So $f(x)$ is continues in $x=1$.

5. We must show the function is derivative in $x=0$

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{x^2\sin\frac{1}{x}}{x}$$
$$=\lim_{x\to 0}x\sin\frac{1}{x}=0$$

And $f'(x)$ equal to:

$$f'(x)=2x\sin\frac{1}{x}+x^2\frac{1}{x^2}\cos\frac{1}{x}$$
$$=2x\sin\frac{1}{x}-\cos\frac{1}{x}$$

Part B:

$$\lim_{x\to 0}f'(x)=\lim_{x\to 0}2x\sin\frac{1}{x}-\cos\frac{1}{x}$$
$$=\lim_{x\to 0}-\cos x=-1\neq f'(0)$$

6.

7. A) Deform equation to:

$$f(x)=\frac{f(x+y)}{f(y)}$$

Calculate $x=0$:

$$f(0)=\frac{f(0+y)}{f(y)}=1(f(y)\neq 0)$$

B)

8.

$$\xrightarrow{Hopital} \lim_{x \to a} \frac{f(a) - 0 - af'(x))}{1} = f(a) - af'(x)$$

9.

$$g'(x) = \frac{\cos x}{2\sqrt{\sin x}} + \frac{1}{2\sqrt{x}} \cos \sqrt{x}$$
$$g''(x) =$$

10. A)

$$\begin{aligned} h'(x) &= \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \\ &= \frac{2-0}{4} \\ &= \frac{1}{2} \end{aligned}$$

B)

$$\begin{aligned} h'(x) &= \frac{g'(x)(1 + f(x)) - g(x)f'(x)}{(1 + f(x)) \times 2} \\ &= \frac{1}{2} \end{aligned}$$

C)

$$\begin{aligned} h'(t) &= \frac{(f(t) + g(t)) - t(f'(t) + g'(t))}{(f(t) + g(t))^2} \\ &= \frac{3-1}{3^2} = 9 \end{aligned}$$

11.

12. $f(x)$ is derivative, so it is continues in $x = 2$:

$$\lim_{x \to 2^+} 2a + b = \lim_{x \to 2^-} 4 = f(2) = 4$$

So:

$$\begin{aligned} a &= 4 \\ 2a + b &= 4 \to b = -4 \\ f'(x) &= \text{if} x \leqslant 2 \text{then} 2x | \text{if} x > 2 \text{then} 4 \end{aligned}$$

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13.

14.

$$\begin{aligned} F'(x) = (f(x) - xf'(x))(f'(xf(x))) \\ F'(0) = 1 \times 2 = 2 \end{aligned}$$

15.

16.

17.

$$\begin{aligned} (f(g(x)))' &= 1 \\ &= g'(x)f'(g(x)) \\ &= g'(x)(1 + g^2(x)) = 1 \end{aligned}$$

So:

$$g'(x) = \frac{1}{1 + g^2(x)}$$

18.

$$\begin{aligned} \lim_{x \to a} \frac{f'(x)}{\frac{1}{2\sqrt{x}}} &= \lim_{x \to a} 2\sqrt{x}f'(x) \\ &= 2\sqrt{a}f'(a) \end{aligned}$$

19.

20.