Let $x \in (rs + rt)$

$$\begin{aligned}
&\Longrightarrow (x \in rs \; or \; x \in rt) \\
&\Longrightarrow (x \in r \; and \; x \in s) or (x \in r \; and \; x \in t) \\
&\Longrightarrow x \in r \; and \; (x \in s \; or \; x \in t) \\
&\Longrightarrow x \in r(s + t)
\end{aligned}$$

That is $(rs + rt) \subseteq r(s + t)$.
So, $r(s + t) = rs + rt(proved)$.

3. Prove that

$(A \cup B) \cap (B \cup C) \cap (C \cup A) = (A \cap B) \cup (B \cap C) \cup (C \cap A)$.
Solution: From the distributive property, we know $A \cup (B \cap C) = (A \cup B) \cap (B \cup C)$.

$$\longrightarrow (B \cup A) \cap (B \cup C) \rightarrow B \cup (A \cap C) \rightarrow B \cup (C \cap A)$$

LHS $(A \cup B) \cap (B \cup C) \cap (C \cup A)$

$$\begin{aligned}
&\Longrightarrow [B \cup (C \cap A)] \cap (C \cup A) \\
&\Longrightarrow [B \cap (C \cup A)] \cup [(C \cap A) \cup (C \cup A)][As \; A \cap A = A] \\
&\Longrightarrow (B \cap C) \cup (B \cap A) \cup (C \cap A) \\
&\Longrightarrow (A \cap B) \cup (B \cap C) \cup (C \cap A) = RHS(proved).
\end{aligned}$$

4. Let R be an equivalence relation in $\{0\}^*$ with the following equivalence classes:

$[\,]R = \{0\}^0$
$[0]RR = 0^1$
$[00]R = \{0\}^2 \cup \{0\}^3 \cup \{0\}^4 \ldots\ldots\ldots$
Show that R is right invariant .

Solution: From the defi nition of right invariant, we know that if R is right invariant then for all x, y, and z,

$$xRy \Rightarrow xzRyz.$$

From the given equivalence classes,
$[\,]_R$ means a null string.
$[0]_R$ has only string 0.
$[00]_R$ has strings 00, 000, 0000, ..., etc.

Three cases may occur:
i) If x and y are null string, i.e., if x, y $\in [\,]_R$. xz = z and yz = z and zRz holds good.
ii) If x, y $\in [0]_R, andz \in [\,]_R$, then xz = x and yz = y. So, xRy = xzRyz.
If z $\in [0]_R$, then xz = 00 and yz = 00, Hence, xz = yz, which implies that xzRyz holds good.
If z $\in [00]_R$, then z has at least 2 zeros. This means that xz and yz have at least 3 zeros which belongs to $[00]_R$. This means that xzRyz holds good.
iii) If x, y $\in [00]_R$, then x and y each have at least 2 zeros. z may belong to any one of the three, but xy and yz produce strings of 2 or more zeros. That is, both of them belong to $[00]_R$. This means that xzRyz holds good.