1. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Solution: Let $P = A \cup (B \cup C)$ and $Q = (A \cup B) \cap (A \cup C)$.
Consider an element a $\in \mathrm{P}$ and $\mathrm{b} \in \mathrm{Q}$.
As $P = A \cup (B \cap C)$ and $a \in P$, it implies that

$$\begin{aligned}
& a \in A \text{ or } a \in (B \cap C) \\
\Longrightarrow\ & a \in A \text{ or } \{a \in B \text{ and } a \in C\} \\
\Longrightarrow\ & \{a \in A \text{ or } a \in B\} \text{ and } \{a \in A \text{ or } a \in C\} \\
\Longrightarrow\ & a \in (A \cup B) \text{ and } a \in (A \cup C) \\
\Longrightarrow\ & a \in (A \cup B) \cap (A \cup C)
\end{aligned}$$

It is proved that P is a subset of Q.
As $Q = (A \cup B) \cap (A \cup C)$ andb $\in Q$, it implies that

$$\begin{aligned}
& b \in (A \cup B) and b \in (A \cup C) \\
\Longrightarrow\ & \{b \in A \text{ or } b \in B\} and \{b \in A \text{ or } b \in C\} \\
\Longrightarrow\ & b \in A \text{ or } \{b \in B \text{ and } b \in C\} \\
\Longrightarrow\ & b \in A \text{ or } b \in (B \cap C)
\end{aligned}$$

It is proved that Q is a subset of P.
As P is a subset of Q and Q is a subset of P, it implies that $P \equiv Q$.
Thus $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ is proved.

2. Prove the following identity:
r(s+t)=r s+r t

Solution: Let $x \in r(s+t)$,

$$\begin{aligned} &\Longrightarrow x \in r \text{ and } x \in (s+t) \\ &\Longrightarrow x \in r \text{ and } (x \in s \text{ or } x \in t) \\ &\Longrightarrow (x \in r \text{ and } x \in s) or (x \in r \text{ and } x \in t) \\ &\Longrightarrow x \in (rs+rt) \end{aligned}$$

That is $r(s+t) \subseteq (\text{isasubsetof})(rs+rt)$.

Let $x \in (rs + rt)$

$$
\begin{aligned}
&\Longrightarrow (x \in rs \text{ or } x \in rt)\\
&\Longrightarrow (x \in r \text{ and } x \in s) \text{ or } (x \in r \text{ and } x \in t)\\
&\Longrightarrow x \in r \text{ and } (x \in \text{ s or } x \in t)\\
&\Longrightarrow x \in r(s + t)
\end{aligned}
$$

That is $(rs + rt) \subseteq r(s + t)$.
So, $r(s + t) = rs + rt($ proved $)$.