

Kinetic Energy for Pendulum-1 is;

$$T = \frac{1}{2}m_1L_1^2\dot{\theta}_1^2$$

Potential Energy for Pendulum-1 is;

$$V = -m_1GL_1\cos(\theta_1)$$

Kinetic Energy for Pendulum-2 is;

$$T = \frac{1}{2}m_2L_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2L_2^2\dot{\theta}_2^2 + m_2L_1L_2\cos(\theta_1 - \theta_2)\dot{\theta}_1\dot{\theta}_2$$

Potential Energy for Pendulum-2 is;

$$V = -m_1GL_1\cos(\theta_1) - m_2GL_2\cos(\theta_2)$$

Kinetic Energy for Pendulum-3 is;

$$T = \frac{1}{2}m_3L_1^2\dot{\theta}_1^2 + \frac{1}{2}m_3L_2^2\dot{\theta}_2^2 + \frac{1}{2}m_3L_3^2\dot{\theta}_3^2 + m_3L_1L_2\cos(\theta_1 - \theta_2)\dot{\theta}_1\dot{\theta}_2 + m_3L_1L_3\cos(\theta_1 - \theta_3)\dot{\theta}_1\dot{\theta}_3 + m_3L_2L_3\cos(\theta_2 - \theta_3)\dot{\theta}_2\dot{\theta}_3$$

Potential Energy for Pendulum-3 is;

$$V = -m_3GL_1\cos(\theta_1) - m_3GL_2\cos(\theta_2) - m_3GL_3\cos(\theta_3)$$

The Lagrangian(T-V) becomes;

$$L = \frac{1}{2}(m_1 + m_2 + m_3)L_1^2\dot{\theta}_1^2 + \frac{1}{2}(m_2 + m_3)L_2^2\dot{\theta}_2^2 + \frac{1}{2}m_3L_3^2\dot{\theta}_3^2 + (m_2 + m_3)L_1L_2\cos(\theta_1 - \theta_2)\dot{\theta}_1\dot{\theta}_2 + m_3L_1L_3\cos(\theta_1 - \theta_3)\dot{\theta}_1\dot{\theta}_3 + m_3L_2L_3\cos(\theta_2 - \theta_3)\dot{\theta}_2\dot{\theta}_3 + (m_1 + m_2 + m_3)GL_1\cos(\theta_1) + (m_2 + m_3)GL_2\cos(\theta_2) + m_3GL_3\cos(\theta_3)$$

$$\frac{\partial L}{\partial \theta_1} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_1} \right) = 0$$

*Euler – Lagrange differential equation for  $\theta_1$  becomes;*

$$\frac{(m_1+m_2+m_3)L_1^2}{a_1}\ddot{\theta}_1 + \frac{(m_2+m_3)L_1L_2\cos(\theta_1-\theta_2)}{a_2}\ddot{\theta}_2 + \frac{m_3L_1L_3\cos(\theta_1-\theta_3)}{a_3}\ddot{\theta}_3 = \frac{-(m_1+m_2+m_3)GL_1\sin(\theta_1)-(m_2+m_3)L_1L_2\sin(\theta_1-\theta_2)\dot{\theta}_2^2-m_3L_1L_3\sin(\theta_1-\theta_3)\dot{\theta}_3^2}{d_1}$$

$$\frac{\partial L}{\partial \theta_2} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_2} \right) = 0$$

*Euler – Lagrange differential equation for  $\theta_2$  becomes;*

$$\frac{(m_2+m_3)L_1L_2\cos(\theta_1-\theta_2)}{b_1}\ddot{\theta}_1 + \frac{(m_2+m_3)L_2^2}{b_2}\ddot{\theta}_2 + \frac{m_3L_2L_3\cos(\theta_2-\theta_3)}{b_3}\ddot{\theta}_3 = \frac{-(m_2+m_3)GL_2\sin(\theta_2)+(m_2+m_3)L_1L_2\sin(\theta_1-\theta_2)\dot{\theta}_1^2-m_3L_2L_3\sin(\theta_2-\theta_3)\dot{\theta}_3^2}{d_2}$$

$$\frac{\partial L}{\partial \theta_3} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_3} \right) = 0$$

*Euler – Lagrange differential equation for  $\theta_3$  becomes;*

$$\frac{m_3L_1L_3\cos(\theta_1-\theta_3)}{c_1}\ddot{\theta}_1 + \frac{m_3L_2L_3\cos(\theta_2-\theta_3)}{c_2}\ddot{\theta}_2 + \frac{m_3L_3^2}{c_3}\ddot{\theta}_3 = \frac{-m_3GL_3\sin(\theta_3)+m_3L_1L_3\sin(\theta_1-\theta_3)\dot{\theta}_1^2+m_3L_2L_3\sin(\theta_2-\theta_3)\dot{\theta}_2^2}{d_3}$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \times \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\ddot{\theta}_1 = \left( \begin{aligned} & -\frac{d_3(-a_2(a_1b_3-a_3b_1)+a_3(a_1b_2-a_2b_1))}{(a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1)} \\ & + \frac{d_1(a_1b_2((a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1))-(a_2(a_1b_3-a_3b_1)+a_3(a_1b_2-a_2b_1))(b_1(a_1c_2-a_2c_1)-c_1(a_1b_2-a_2b_1)))}{a_1(a_1b_2-a_2b_1)((a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1))} \\ & + \frac{d_2(-a_1a_2((a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1))+a_1(a_1c_2-a_2c_1)(-a_2(a_1b_3-a_3b_1)+a_3(a_1b_2-a_2b_1)))}{a_1(a_1b_2-a_2b_1)((a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1))} \end{aligned} \right)$$

$$\ddot{\theta}_2 = \left( \begin{aligned} & -\frac{a_1d_3(a_1b_3-a_3b_1)}{(a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1)} \\ & + \frac{d_1(-b_1((a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1))-(a_1b_3-a_3b_1)(b_1(a_1c_2-a_2c_1)-c_1(a_1b_2-a_2b_1)))}{(a_1b_2-a_2b_1)((a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1))} \\ & + \frac{d_2(a_1(a_1b_3-a_3b_1)(a_1c_2-a_2c_1)+a_1((a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1)))}{(a_1b_2-a_2b_1)((a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1))} \end{aligned} \right)$$

$$\ddot{\theta}_3 = \left( \begin{aligned} & -\frac{a_1d_2(a_1c_2-a_2c_1)}{(a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1)} \\ & + \frac{a_1d_3(a_1b_2-a_2b_1)}{(a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1)} \\ & + \frac{d_1(b_1(a_1c_2-a_2c_1)-c_1(a_1b_2-a_2b_1))}{(a_1b_2-a_2b_1)(a_1c_3-a_3c_1)-(a_1b_3-a_3b_1)(a_1c_2-a_2c_1)} \end{aligned} \right)$$