Boğaziçi University
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Poisson Distribution

Experiment 6

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1 Abstract

The main purpose of this experiment was to explore the statistical properties of counting events in a random process and examine the difference between Poisson and Gaussian distributions by observing radioactive decay.We counted the number of γrays from radioactive sources by using Geiger Tube. In the end of experiment we conclude that Poisson distribution can be used for radioactive decays and Poisson Distribution can fit better than normal fits.However, in the situation of larger mean, Poisson and Gaussian fit can be similar.

2 Introduction

Poisson distribution is developed by Simeon Denis Poisson and was named after him, who introduced the distribution in 1837[1].Poisson statistics describe random, independent events that occur at a fixed mean rate λ.[2].In this experiment each radioactive decay event is random ,independent and occurs at a fixed mean rate λ. For this reason,it can be modelled that this radioactive process by using Poisson statistics.

3 Theory

3.1 Probabilistic Relation of Radioactive Decay

The radioactive decay equation can be derived, as an exercise in calculus and probability, as a consequence of two physical principles: a radioactive nucleus has no memory, and decay times for any two nuclei of the same isotope are governed by the same probability distribution. The first principle implies that this distribution has a continuous exponential probability density function. Then, knowing the probability of survival of a single nucleus to a specified time, the second principle allows the number of a collection of nuclei surviving to this time to be treated as a random variable governed by the discrete binomial distribution. The familiar radioactive decay equation does not give the actual number remaining at this time, but rather the expected value of this distribution. With this proper interpretation of the radioactive decay equation, the number of nuclei need not be a large. Algebraic and numerical experiments illustrate, however, that as the number of nuclei grows to the large values associated with geochronological studies, the probability of significant departure from the expected value becomes negligibly small,thus Poisson Distribution becomes available to perform in calculation of probability distribution for radioactive decay situations [4].

3.2 Derivation of Poisson Distribution from Binomial Distribution

The formula of Poisson Distribution as

$$P(\lambda, n) = \frac{\lambda^n e^{-\lambda}}{n!} \tag{1}$$

where λ is the event occurs at an average rate in an interval and n is number of occurrence. We can obtain Poisson Distribution by using binomial distribution. The binomial distribution can be formulate as

$$\binom{N}{k} \cdot p^k q^{N-k} \tag{2}$$

x events occur with probability p each,and the $(n-x)$ events occur with probability $(1-p)$ each, and $\binom{N}{k}$ is number of ways to choose x of n items.

The Poisson distribution can be derived from the binomial distribution by $n \to \infty$ and we can let $p \to 0$

$$\frac{n!}{(n-x)!} = n(n-1)...(n-x-2)(n-x-1) = n^x \tag{3}$$

$$(1-p)^{n-x} = (1-p)^{-x}(1-p)^n = (1+px)^{-x}(1-p)^{\frac{\lambda}{p}} = e^{-\lambda} \tag{4}$$

$$P_p(x;\lambda) = \frac{1}{x!}\frac{n!}{(n-x)!}p^x(1-p)^{-x} = \frac{\lambda^x}{x!}e^{-\lambda} \tag{5}$$

where λ equals to

$$\lambda = np \tag{6}$$

3.3 Normalization

Sum of all possibilities yields;

$$\sum_{n=0}^{\infty}\frac{\lambda^n e^{-\lambda}}{n!} = e^{-\lambda}\sum_{n=1}^{\infty}\frac{\lambda^n}{n!} = e^{-\lambda}e^{\lambda} = 1 \tag{7}$$

3.4 Mean and Standard Deviation

To calculate mean;

$$\mu = \sum_{n=0}^{\infty} n\frac{\lambda^n e^{-\lambda}}{n!} = \lambda e^{-\lambda}\sum_{n=1}^{\infty}\frac{\lambda^{n-1}}{(n-1)!} \tag{8}$$

$$= \lambda e^{-\lambda}\sum_{k=0}^{\infty}\frac{\lambda^k}{(k)!} = \lambda e^{-\lambda}e^{\lambda} = \lambda \tag{9}$$

To calculate standard deviation;

$$\sigma^2 = \left\langle n^2 \right\rangle - \left\langle n \right\rangle^2 \tag{10}$$

Where, $\langle\rangle$ is the expected value. We calculated $\langle n \rangle = \mu = \lambda$. As a next step, calculation of $\langle n^2 \rangle$ as;

$$\left\langle n^2 \right\rangle = \sum_{n=0}^{\infty}(n^2)\frac{\lambda^n e^{-\lambda}}{n!} = e^{-\lambda}\sum_{n=1}^{\infty}[n(n-1)+n]\frac{\lambda^n}{n!} \tag{11}$$

$$= e^{-\lambda}\sum_{n=2}^{\infty}[n(n-1)]\frac{\lambda^n}{n!} + e^{-\lambda}\sum_{n=1}^{\infty}\frac{\lambda^n}{(n-1)!} \tag{12}$$

$$= e^{-\lambda}\lambda^2\sum_{n=2}^{\infty}\frac{\lambda^{(n-2)}}{(n-2)!} + e^{-\lambda}\lambda\sum_{n=0}^{\infty}\frac{\lambda^n}{(n)!} \tag{13}$$

$$= e^{-\lambda}\lambda^2\sum_{n=0}^{\infty}\frac{\lambda^n}{(n)!} + e^{-\lambda}\lambda\sum_{n=0}^{\infty}\frac{\lambda^n}{(n)!} \tag{14}$$

$$= \lambda^2 + \lambda \tag{15}$$

Thus, we obtain that variance σ^2 and standard deviation σ will be;

$$\sigma^2 = \lambda^2 + \lambda - \lambda^2 \tag{16}$$

$$\sigma^2 = \lambda \tag{17}$$

$$\sigma = \sqrt{\lambda} \tag{18}$$

3.5 Probability of Observing n Counts During Time Interval t

$\alpha = \lambda/t$, where t is any given interval, Poisson distribution becomes;

$$P(\lambda, n) = P(\alpha, t, n) = \frac{(\alpha t)^n e^{-\alpha t}}{n!} \tag{19}$$

Probability of having one event in a time interval dt is;

$$P(\alpha, dt, 1) = \frac{(\alpha dt) e^{-\alpha dt}}{1!} \tag{20}$$

;

$$P_P(n+1, t)dt = P(\alpha, t, n) P(\alpha, dt, 1) \tag{21}$$

$$P_P(n+1, t)dt = \frac{(\alpha t)^n e^{-\alpha t}}{n!} \frac{(\alpha dt) e^{-\alpha dt}}{1!} \tag{22}$$

Because of $dt << 0$, $e^{-\alpha dt}$ approaches to 1;

$$P_P(n+1, t)dt \approx \frac{(\alpha t)^n e^{-\alpha t} \alpha dt}{n!} \tag{23}$$

By dividing dt ,equation will be ;

$$P_P(n+1, t) = \frac{(\alpha t)^n e^{-\alpha t} \alpha}{n!} \tag{24}$$

Also, Poisson distribution behaves Gaussian distribution as λ gets larger. Gaussian distribution is;

$$P_G(\lambda, \sigma, n) = \frac{1}{\sigma\sqrt{2\pi}} exp[-\frac{(n-\lambda)^2}{2\sigma^2}] \tag{25}$$

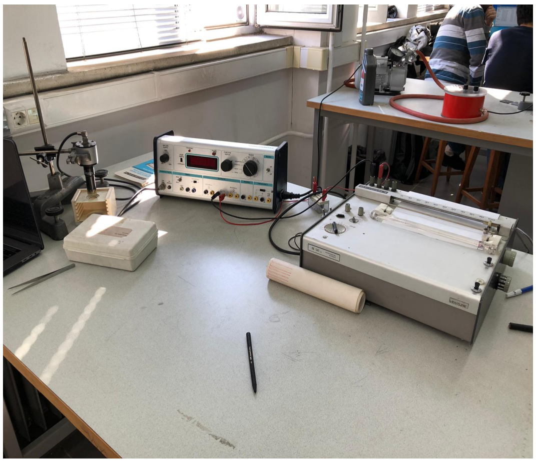


Figure 1: Caption

Figure 2: Caption

4 Experiment

4.1 Setup

The setup used for the experiment is given above in Figure [1] and [2]

4.2 Apparatus

- Geiger Counter with a Scaler
- Sample Holder(Isolate radiation caused from elements9
- Various Gamma-Ray Sources (Emits gamma particles when it goes radioactive decay)
- Lead Absorbers
- Chart Recorder

  What is Geiger Counter and how it works ?

  A Geiger counter is a metal cylinder filled with low-pressure gas sealed in by a plastic or ceramic window at one end. Going down the center of the tube there's a thin metal wire made of tungsten. The wire is designed to connect to a high, positive voltage so there's a strong electric field between it and the outside tube.

  If radiation enters the tube, it results ionization, scattering gas molecules into ions and electrons. The electrons, being negatively charged, are instantly attracted by the high-voltage positive wire and as they zoom through the tube collide with more gas molecules and produce further ionization. The result is that lots of electrons instantly reach at the wire, creating a pulse of electricity that can be measured on a meter and (if the counter is connected to an amplifier and loudspeaker) heard as a "click." The ions and electrons are quickly absorbed among the billions of gas molecules in the tube so the counter effectively resets itself in a fraction of a second, ready to detect more radiation. Geiger counters can detect alpha, beta, and gamma radiation[5].

4.3 Procedure

- First of all,we need to find the operating voltage of the Geiger Tube . To do this, select a Gamma-ray source and place it on the sample holder, beneath the Geiger Tube assembly. We set the counter to 100 s, radioactivity and single mode. We choose a source and tray position that gives about 10 counts/s.
- Plot the number of counts as in term of HV. At some point in our plot, the counts became approximately constant, we select a voltage value from this region.Our voltage value is 480 Volt.We set the Geiger Counter to this voltage value.

- We set the counter to 10 s interval and continuous recording.
- Place one of the sources in the holder and record the number of counts for 100 times in 10 seconds intervals.
- Set the counter to 1 s interval and repeat the above.
- Second Part
- We select a source and tray position that gives a count rate of 1 per second.
- Set the chart recorder speed to 12 inches per minute and turn it on.
- Connect the chart recorder to Geiger counter and run it.
- Turn the chart recorder off and remove the portion of the paper that was used for drawing.

0	22,5	48,9	74,1	95,2
3,50	22,9	49,5	75,3	96
3,8	23,6	50	75,5	97,2
4,5	24,6	50,4	75,8	97,4
6	25,9	51,5	77,3	99,1
6	27,8	51,7	79,2	99,5
6,5	28,3	52,5	80,2	100,1
7,4	29,7	53,1	83	100,2
8,1	30,3	54,5	83,6	100,8
9,2	31,2	55,4	83,7	101,2
10,1	32,2	56,6	83,8	102,1
10,6	32,6	57	84,4	103
11,2	34,4	59,1	85,6	103,2
11,9	35,6	62,2	86,4	104
12,7	35,7	62,9	87	107,3
14	36,1	63,6	88	108,3
14,2	36,3	63,8	88,4	108,9
15,4	36,5	64,1	88,8	109,2
16	37,4	66,2	88,5	110,1
17,7	39	67,1	90	110,5
17,9	42	70,4	90,9	110,7
19,1	43,2	70,8	92	111,2
20,4	44,5	71,4	92,2	111,6
21,5	46	72,5	93,9	112,2
21,9	47,3	72,9		114,6

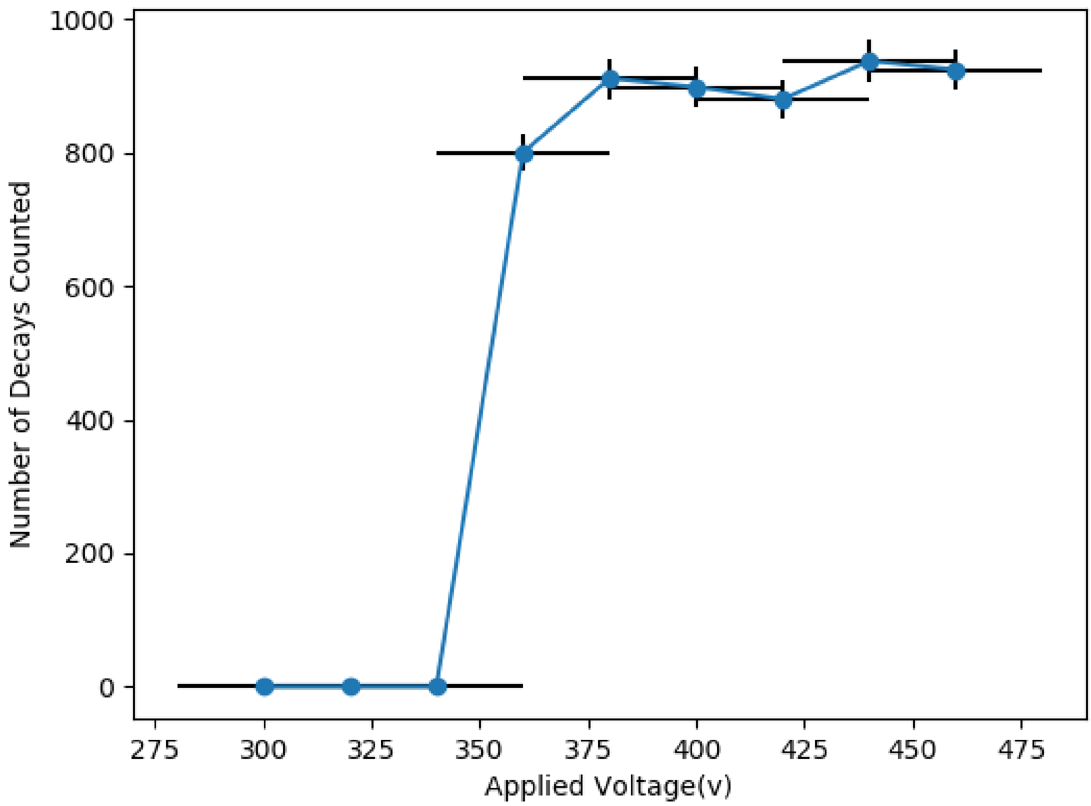
5 Raw Data

Measured data of length intervals which represent time difference of each decay is given in the table above.Also, it is given another measured data which represents the number of decays between the fixing time intervals such as 10 seconds and 1 second for different radioactive elements Cesium and Barium, in the table below.

Cs for 10 seconds	Cs for 1 second	Ba for 10 seconds	Ba for 1 second
114	15	49	3
113	16	77	6
102	11	49	8
105	9	59	5
104	9	50	5
100	15	66	6
97	12	58	8
105	14	64	6
110	18	68	4
113	11	62	5
112	10	54	3
127	9	56	2
124	13	70	13
115	9	63	5
96	15	61	8
95	11	49	5
118	11	63	6
97	9	51	3
122	8	67	8
100	9	65	6
104	10	56	9
95	10	62	5
121	7	61	10
93	13	70	6
86	13	60	6
131	8	59	2
102	12	67	7
112	7	47	9
119	8	76	6
96	10	56	8
95	8	64	2
107	16	54	7
93	8	59	5
119	10	60	4
137	16	67	10
100	11	60	5
101	6	59	9
102	8	51	3
108	16	53	2
112	10	55	8
104	11	69	5
120	14	58	4
106	9	48	5
130	12	69	9
94	13	75	5
103	10	66	9
104	7	68	10
109	11	51	2
88	13	62	4
109	13	69	11
79	9	59	6
122	6	71	5
120	9	51	9
105	15	62	9
110	10	75	10
104	11	70	5
99	9	70	3
106	14	72	4
94	9	61	5
87	7	69	4
103	13	77	1
92	5	56	6
108	8	55	1

6 Data Analysis

First Part of Experiment



Second Part of Experiment

The histograms are created by using our recorded row data and bin centers and corresponding histogram values are calculated. In addition, we calculated the mean and standard deviation of mean for both Gaussian and Poisson distributions, with the formulas given below;

$$\overline{X} = \sum \frac{X}{n} \tag{26}$$

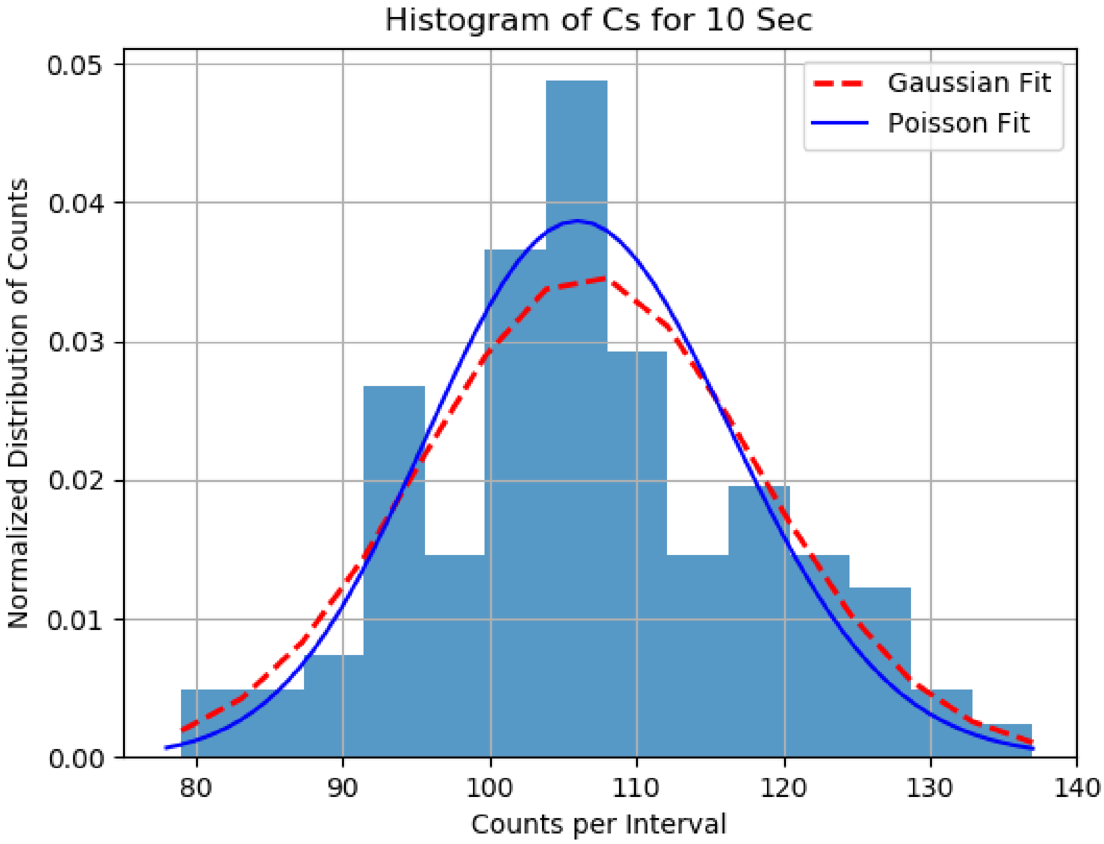
$$\nu_\sigma = \frac{\sigma}{\sqrt{n}} \tag{27}$$

For the obtained Poisson and Gaussian distributions at the values corresponding to the position of each bin, χ^2 and χ^2 per degrees of freedom calculated by;

$$\chi^2 = \sum_{i=n} \frac{[y(n) - L\frac{\lambda^n e^{-\lambda}}{n!}]^2}{\sigma_{y_i}^2} \tag{28}$$

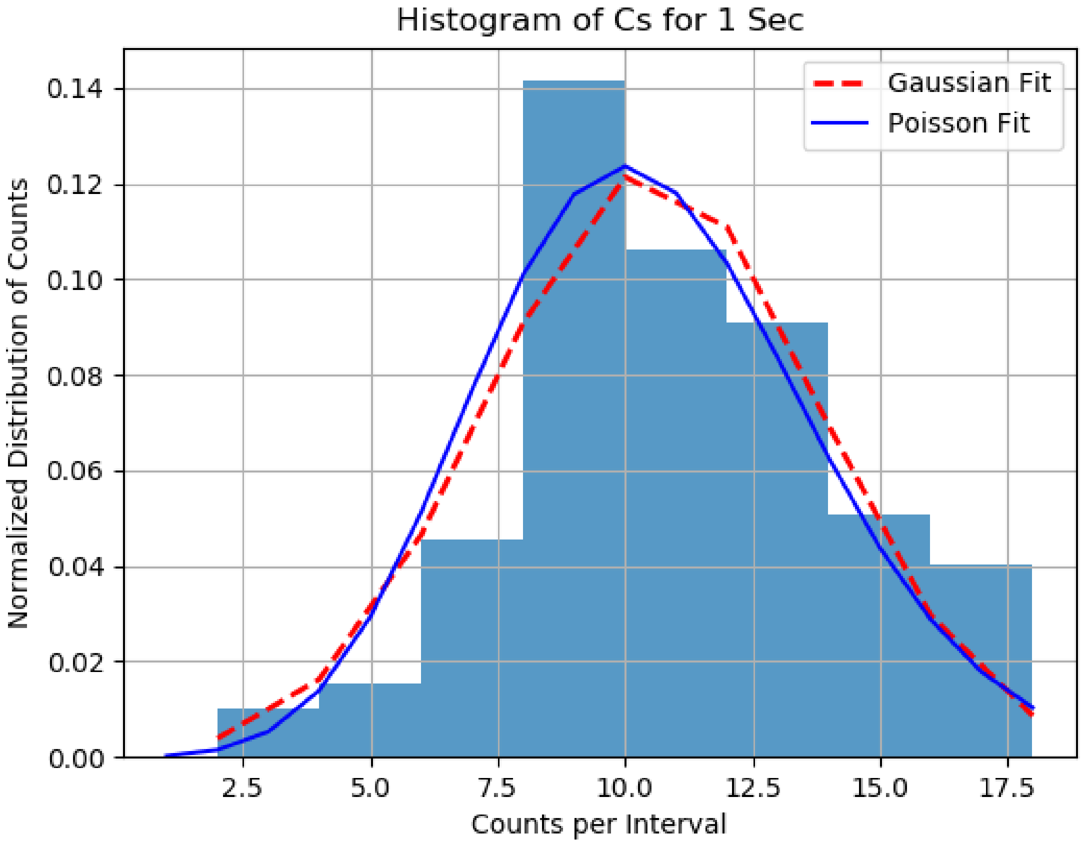
$$\chi^2 = \sum_{i=n} \frac{[y(n) - L\frac{1}{\sigma\sqrt{2\pi}} exp[-\frac{(n-\lambda)^2}{2\sigma^2}]^2}{\sigma_{y_i}^2} \tag{29}$$

$$\chi^2 = \frac{1}{\nu} \sum_{i=n} \frac{[y(n) - L\frac{1}{\sigma\sqrt{2\pi}} exp[-\frac{(n-\lambda)^2}{2\sigma^2}]^2}{\sigma_{y_i}^2} \tag{30}$$



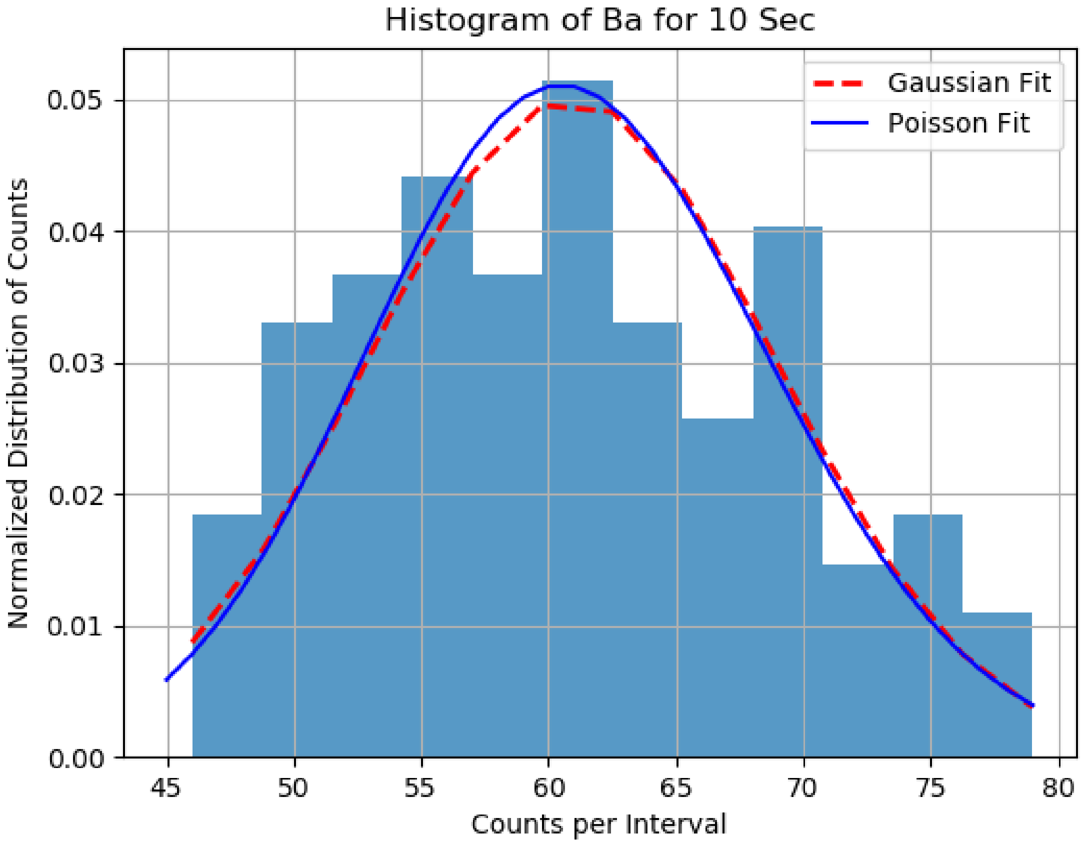
Mean Gaussian fit	σ Mean Gaussian	χ^2/DOF	Mean Poisson fit	σ Mean Poisson	χ/DOF
0.016028	0.003105	0.009668	0.016136	0.003541	0.012489

Table 1: Means,standard deviations and chi square/degree of freedom values of Gaussian and Poisson fit of Cs for 10 sec.



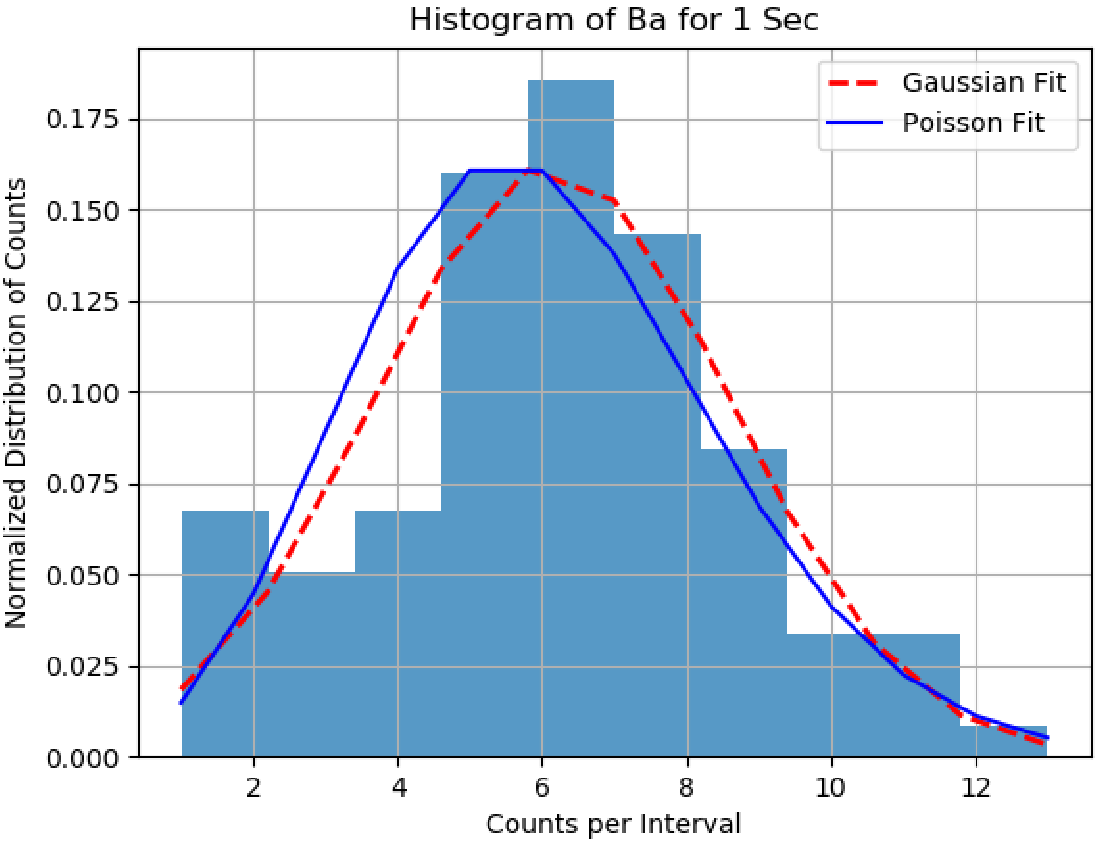
Mean Gaussian fit	σ Mean Gaussian	χ^2/DOF	Mean Poisson fit	σ Mean Poisson	χ/DOF
0.055254	0.042138	0.036153	0.055120	0.042740	0.037284

Table 2: Means,standard deviations and chi square/degree of freedom values of Gaussian and Poisson fit of Cs for 1 sec.



Mean Gaussian fit	σ Mean Gaussian	χ^2/DOF	Mean Poisson fit	σ Mean Poisson	χ/DOF
0.027320	0.004398	0.009973	0.027478	0.004525	0.010497

Table 3: Means,standard deviations and chi square/degree of freedom values of Gaussian and Poisson fit of Ba for 10 sec.



Mean Gaussian fit	σ Mean Gaussian	χ^2/DOF	Mean Poisson fit	σ Mean Poisson	χ/DOF
0.075418	0.017579	0.049582	0.074431	0.017213	0.048169

Table 4: Means,standard deviations and chi square/degree of freedom values of Gaussian and Poisson fit of Ba for 1 sec.

Root of mean for Ba for 1s	Root of mean Cs for 10 s	Root of mean Ba for 10 s	Root of mean Cs for10 s
2.4761488507989844	3.2442645584558183	7.803780487622002	10.328444591358744
Sigma of Ba for 1s	Sigma of Cs for 10 s	Sigma of Ba for 10 s	Sigma of Cs for10 s
2.4603788188512374	3.24533366432431	7.965195692137099	11.472257519005993

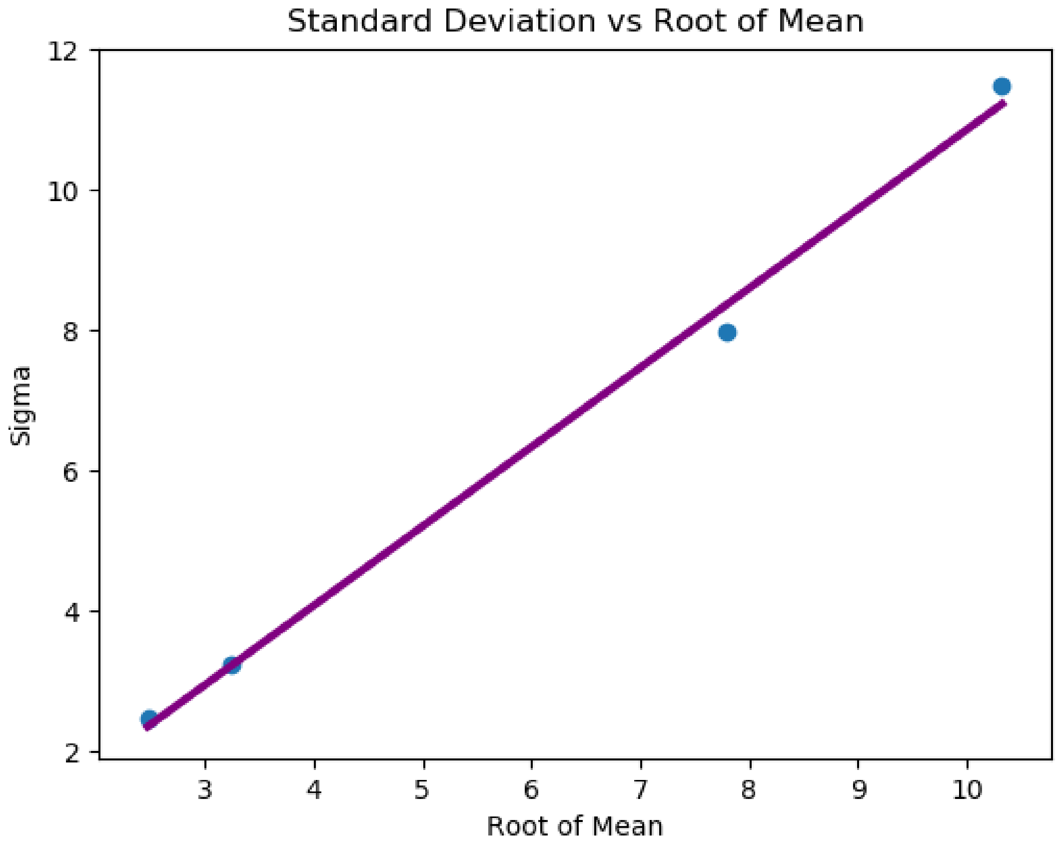


Figure 3: Linear Fit with 1.13003535 x -0.4527898

Third Part of Experiment

In the third part of experiment,we measured the length difference,which represents time interval for each decay in the chart recorder.Obtained data is tested by ;

$$P_P(0+1,t) = \frac{(\alpha t)^0 e^{-\alpha t}\alpha}{0!} = \alpha e^{-\alpha t} \tag{31}$$

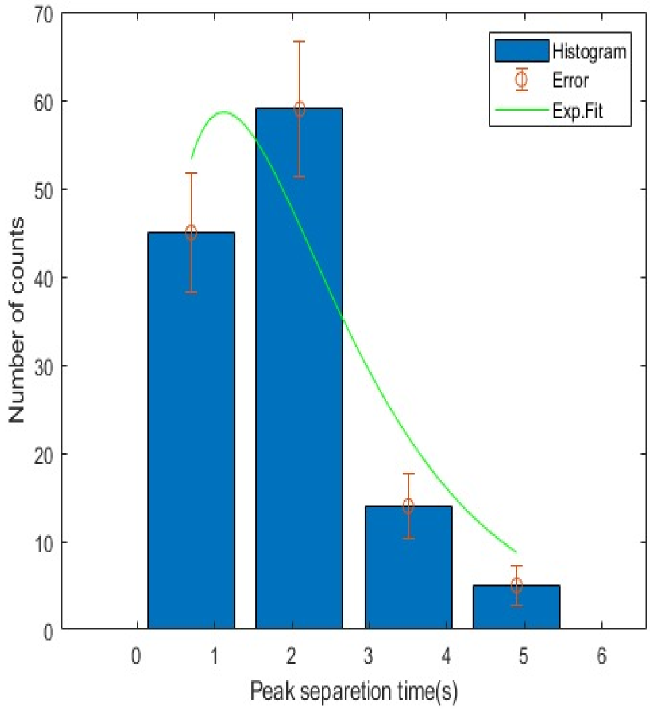


Figure 4: Histogram,and fitting curve plotted according to equation[31]

α_0	σ_{α_0}	$\chi^2/Degree\,of\,freedom$
0.8545	0.1465	3.3695

$$P_P(1+1,t) = \frac{(\alpha t)^1 e^{-\alpha t}\alpha}{1!} = \alpha^2 t e^{-\alpha t} \tag{32}$$

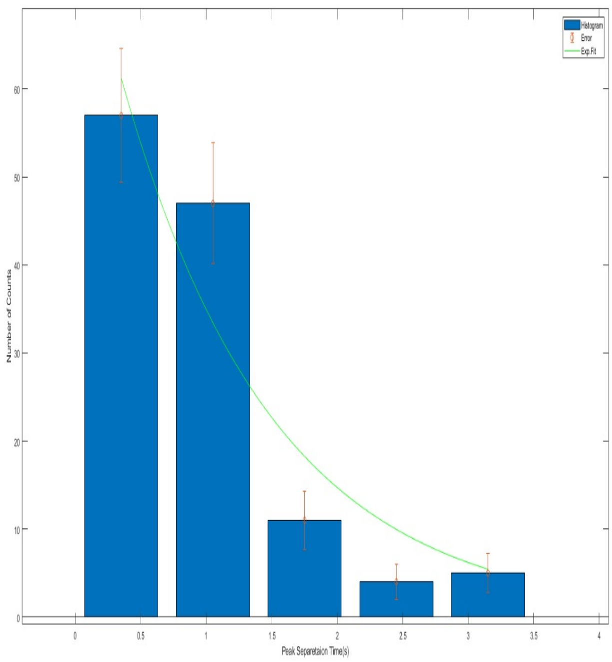


Figure 5: Histogram,and fitting curve plotted according to equation[32]

α_0	σ_{α_0}	$\chi^2/Degree\,of\,freedom$
0.8643	0.3525	3.5938

7 Conclusion

We seek to obtain minimum χ^2 value and we compare the Poisson and Gaussian fits.Generally, we obtain that Poisson distribution fits better than Gaussian but normal and Poisson have similar fits when mean gets larger.Also we plotted a linear fit between the standard deviation and root of mean for each element's counted number of radioactive decay distribution to observe whether their deviations could increase linearly together or not. In the second part of experiment α and nominal values are nearby values.We can deduce that Poisson distribution can also valid for radioactive decay of huge number of particles. To conclude, when we try to examine the distribution of the number of radioactive decay for a fixed time interval, because of the events behave as independent considering the nucleus of the atom can be thought that it has no memory caused by the previous decay situation when the number of nuclei is large,thus we can use Poisson Distribution in this experiment.

References

[1] Poisson Distribution `https://www.britannica.com/topic/Poisson-distribution`

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[3] Advanced Physics Experiments, Erhan Gulmez, Bogazici University Publications, 1999, ISBN 975-518-129-6

[4] Radioactive Decay `http://serc.carleton.edu/files/nagt/jge/abstracts/Huestis`$_v 50n5p524.p$

[5] Radioactive Decay `https://www.explainthatstuff.com/how-geiger-counters-work.html`

8 Appendix

https://github.com/EgemenYuzbasi/Poisson-442.git