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# Poisson Statistics

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## Contents

# 1 Abstract

In this experiment, it is aimed to observe Poisson Gaussian distributions and their differences by detecting if radioactive decays are applicable processes to Poisson distributions. The experiment consists of two parts; one is completed by recording the data via Geiger counter's scaler, where the other is completed via a chart recorder which records the peaks derived from gamma rays. At the end of the analysis it is understood that radioactive decay is an applicable process for Poisson distribution. Furthermore, Poisson distribution is a better way to describe distributions with small means where Gaussian distribution describes better the ones with large means. Finally, to observe those distributions described perfectly, the data collected should be increased as possible.


# 2 Theory

In probability theory and statistics, the Poisson distribution, named after French mathematician Siméon Denis Poisson, is a discrete probability distribution that explains the probability of a given number of events happening in a fixed interval of time or space as these events happen with a known constant rate and not depending on the time since the last event occurs [1]. Poisson distribution is an appropriate model if the following assumptions are true: k is the number of times an event occurs in an interval and k can take values of zero or positive integer. The probability of one event to occur is independent of the other events do occur. That means that events happen independently. The rate at which events occur is constant. The rate cannot be higher in some intervals and lower in other intervals. Two events cannot occur at exactly the same instant; instead, at each very small sub-interval exactly one event either occurs or does not occur. The probability of an event in a small sub-interval is proportional to the length of the sub-interval. If these conditions are true, then k is a Poisson random variable, and the distribution of k is a Poisson distribution [2].

Starting from Binomial Distribution, Poisson Distribution is derived below.

Equations used are given below, for detailed analysis please see : Advanced Physics Experiments. Erhan Gülmez, 1999.

- Poisson Distribution[3]:
$$P(\mu, n) = \frac{\mu^n e^{-n}}{n!} \tag{1}$$
where P is the probability of observing n counts, $\mu$ is the mean of the distribution and n is the number of occurrence of the events.

- The mean value of the Poisson Distribution is equal to its variance.
$$\sigma^2 = \mu \tag{2}$$

- The probability of observing n counts in a time interval of t is given by [5]:
$$P(\mu, n) = P(\alpha, t, n) = \frac{(\alpha t)^n e^{-\alpha t}}{n!} \tag{3}$$
where
$$\alpha = \mu/t \tag{4}$$

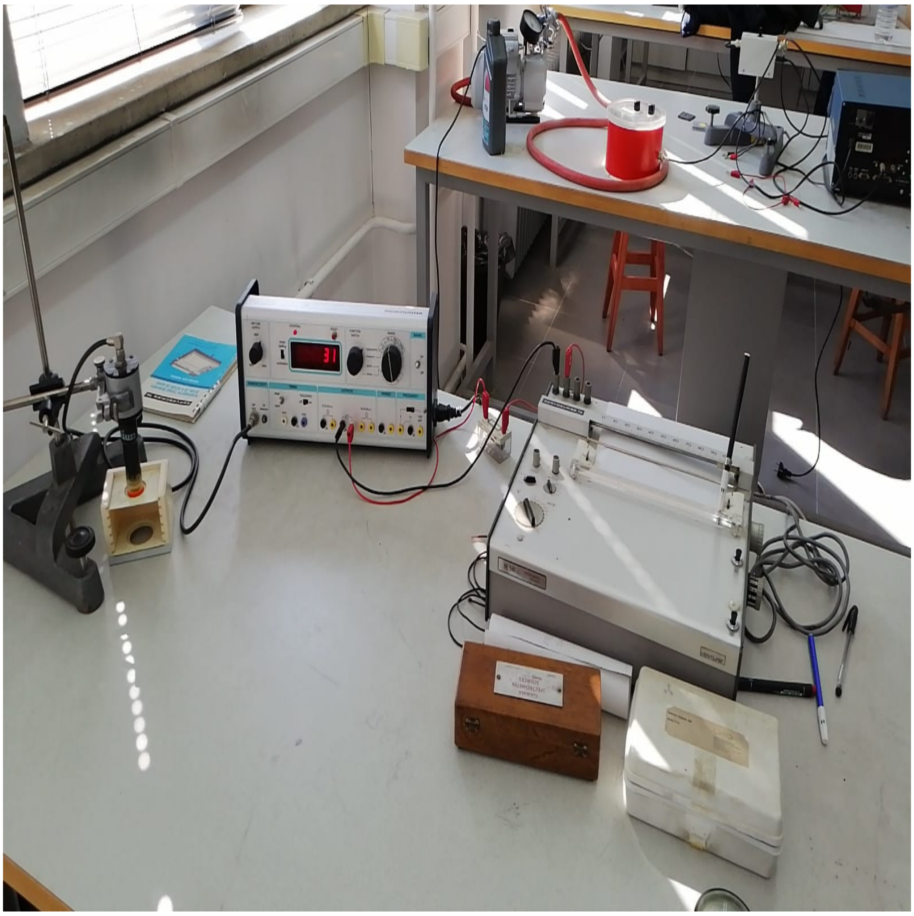
- As n value goes to larger values, Poisson distribution approaches the Gaussian distribution [6].

$$G(n, \sigma, \mu) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{n-\mu}{\sigma})^2}$$

# 3 Experiment

## 3.1 Setup

Figure 1: Photo of the setup



## 3.2 Apparatus

1. Geiger Counter with a Scaler : The Geiger counter is mainly based on its detector. This detector captures, detects and then gives signals that a radioactive particle, known as a radioactive isotope, has reached the detector.At its simplest level, the detector which is in a shape of metallic hollow tube that contains gas inside it and a conductive wire running straight through its centre. There is a battery giving this wire positive charge.When a radioactive atom breaks down, the particles and its energy shoots off knock electrons off billions of nearby atoms.Even if these free electrons may have a little amount of charge, as combined have enough power to result in a short electrical pulse if given the opportunity.The opportunity, in this case, is through the conductive wire which, being positively charged, attracts the electrons.Every time a radioactive atom breaks down, knocking off nearby electrons, this builds up enough charge to create a pulse and therefore a "click" on the detector. Each click demonstrate a single atom has decayed, and so more clicking means more atoms and therefore, more radiation.[7]

2. Sample Holder
3. Cs137 Sample
4. Ba133 Sample
5. Lead Absorbers
6. Chart Recorder

## 3.3 Procedure

### 3.3.1 Part 1

1. A Cs137 sample is placed on the holder which is under the Geiger tube in order to determine the operating voltage of the Geiger tube.
2. The counter is set to 100 seconds in radioactive - single mode
3. The potential is applied from 300 V to 500V with 20V intervals.
4. The operating potential is chosen.The counter is changed to continuous mode with 10 seconds interval. 100 counts are recorded.
5. The counter is set to 1 sec interval and the previous process is repeated
6. A Ba133 sample is placed on the holder and its position is arranged to obtain approximately 40 counts in a 10 seconds interval. Previous steps are repeated.

### 3.3.2 Part 2

(a) A Ba133 sample is placed on the holder and its position is arranged to obtain approximately 1 count / second .

(b) The recorder is run about two minutes, then it is turned off and the portion of paper is removed.

(c) The time interval between adjacent pulses is measured.

# 4 Data and Analysis

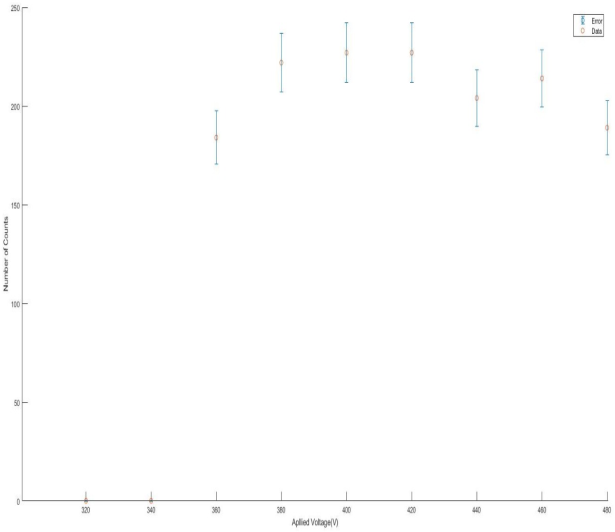
## 4.1 Raw Data Tables

The potential applied and the counts obtained to determine the operating voltage are given in table below. 400 V is chosen to be the working potential for remaining parts since the counts are at their closest position at that voltage.

| Potential(V) | Number of counts |
|---|---|
| 300 | 0 |
| 320 | 0 |
| 340 | 0 |
| 360 | 184 |
| 380 | 222 |
| 400 | 227 |
| 420 | 227 |
| 440 | 204 |
| 460 | 214 |
| 480 | 189 |

Graph of the table above is given below with corresponding error given in equation [2]:

Figure 2




As told in the procedure part, number of radioactive decays are counted for Cs137 and Ba133 elements for 10 and 1 seconds intervals respectively. This table consisting of number of decays are given in the appendix part.

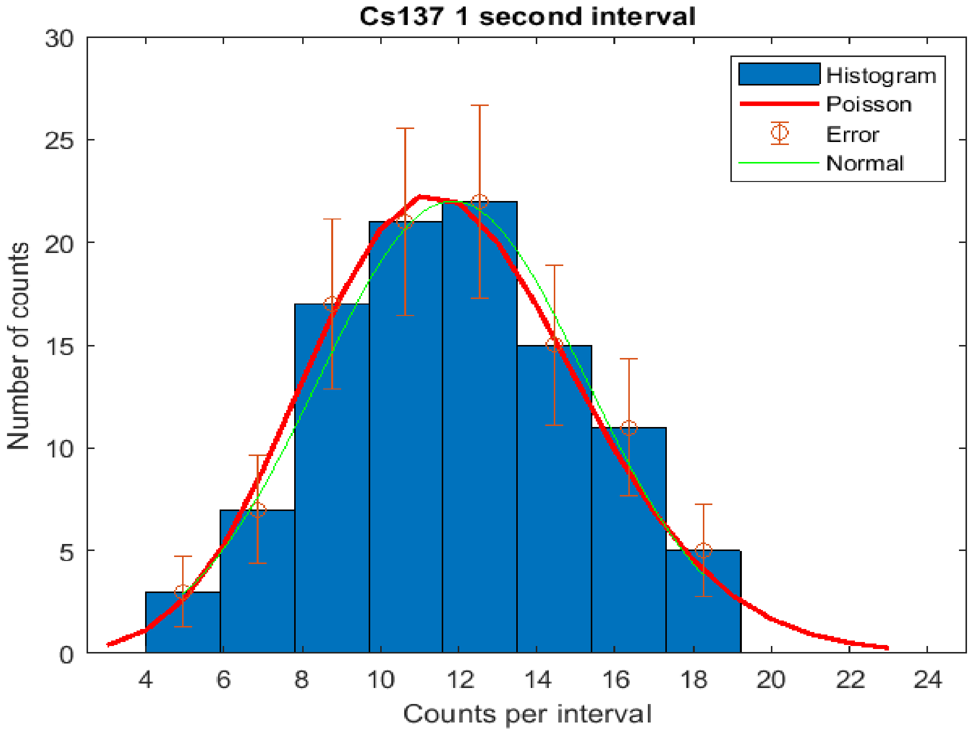
## 4.2 Analysis

All the calculation in this experiment are performed by using MATLAB and in the Appendix part used codes and data are given as online.Also, for normal distribution function mean ($\mu$) and standard deviation of the normal distribution function ($\sigma$) with corresponding uncertainty values are given as a table in Appendix part.

### 4.2.1 Part 1

In this part histograms are made and Poisson and Normal distribution fits are performed on each histogram.

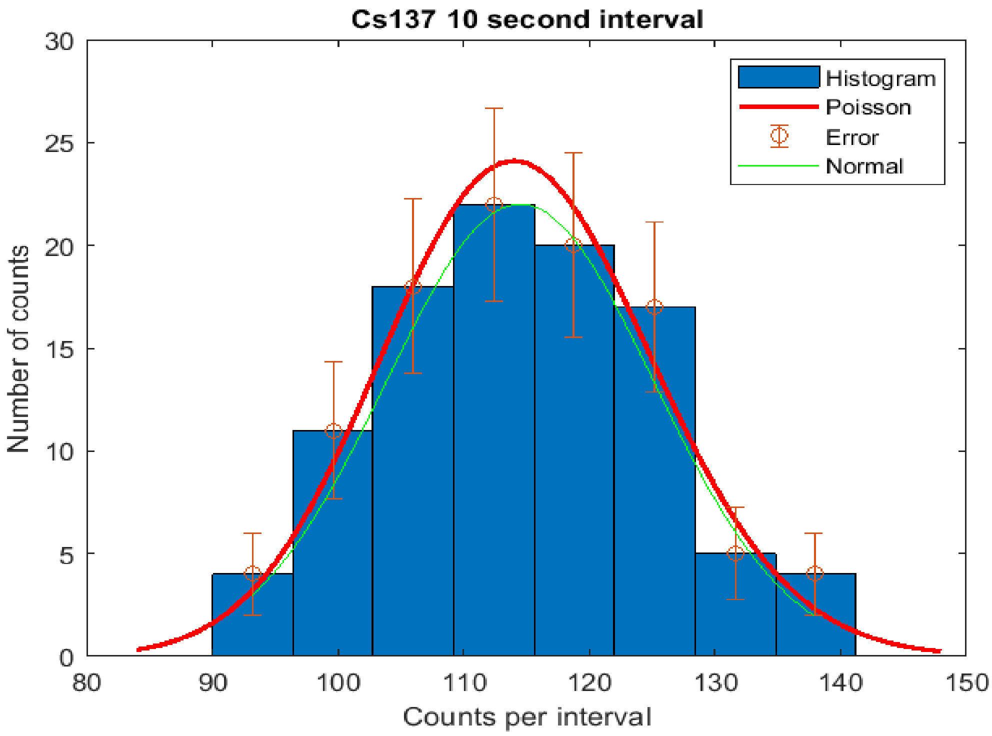
Figure 3




$\mu$ values are the same for both Normal and Poisson distribution and equals to 11.84 with $\sigma_\mu$=0.34.And, $\chi^2$ values are given below.

| $\chi^2_{Normal}$ | $\chi^2_{Poisson}$ |
|---|---|
| 1.1136 | 1.0357 |

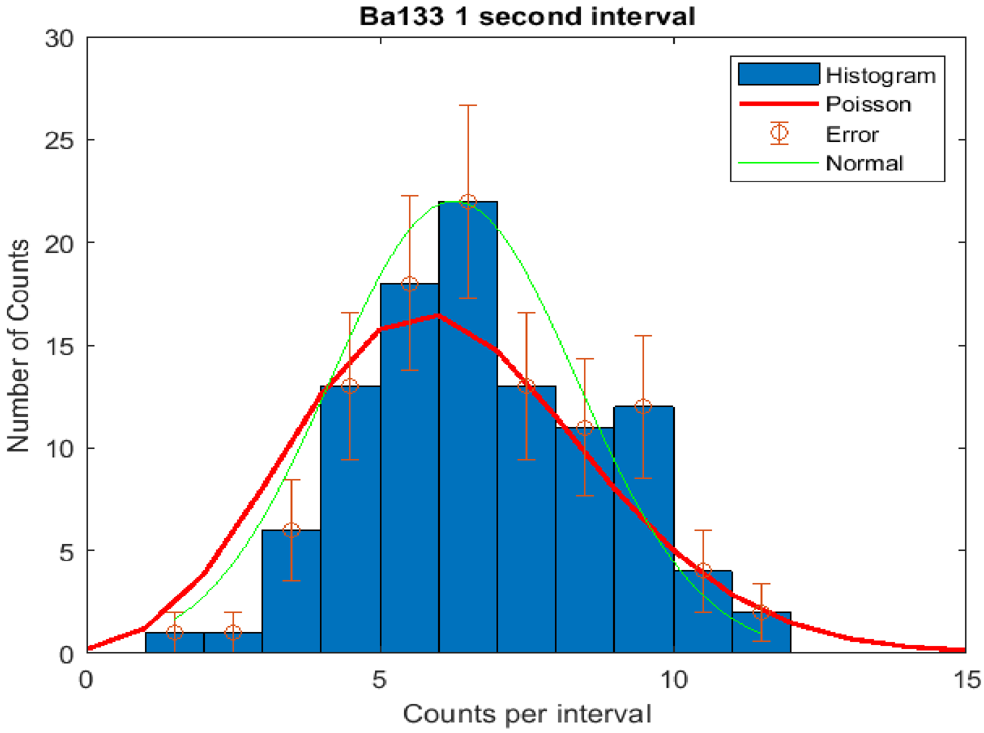
Figure 4




$\mu$ values are the same for both Normal and Poisson distribution and equals to 114.50 with $\sigma_\mu$=1.06.And, $\chi^2$ values are given below.

| $\chi^2_{Normal}$ | $\chi^2_{Poisson}$ |
|---|---|
| 3.2675 | 2.4603 |

Figure 5

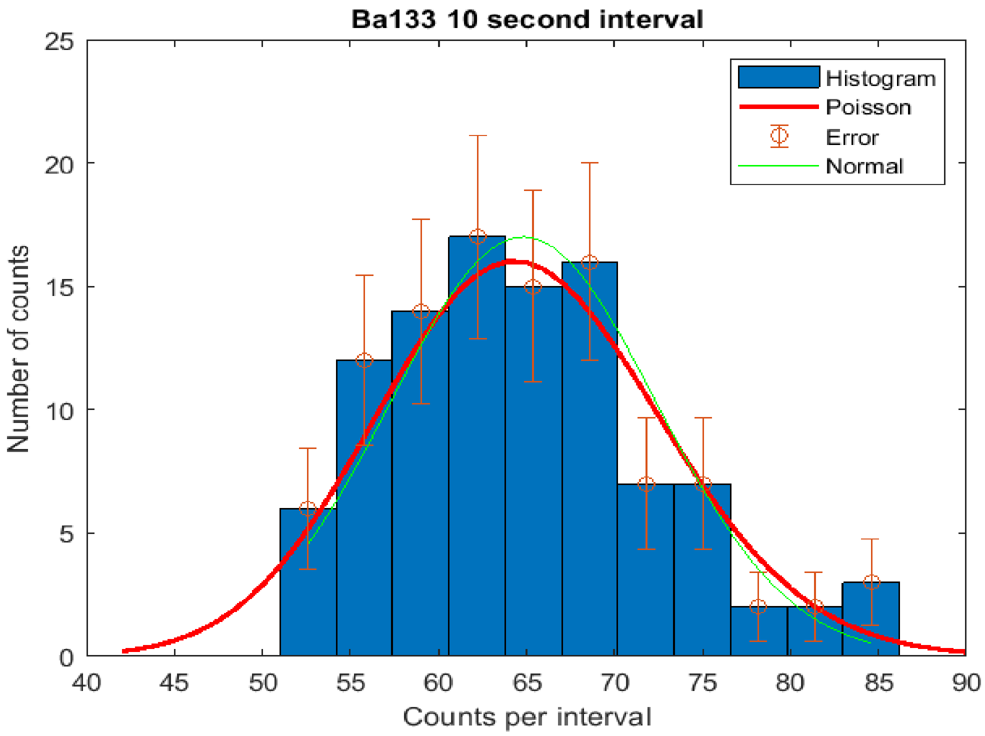



$\mu$ values are the same for both Normal and Poisson distribution and equals to 6.25 with $\sigma_{\mu}$=0.25.And, $\chi^2$ values are given below.

| $\chi^2_{Normal}$ | $\chi^2_{Poisson}$ |
|---|---|
| 20.2146 | 34.7443 |

| - | Cs137 10 s | Cs137 1 s | Ba133 10 s | Ba133 1s |
|---|---|---|---|---|
| $\frac{\chi^2}{dof}$ Poisson | 2,4603/6 | 1,0357/6 | 6,7570/8 | 34,7443/8 |
| $\frac{\chi^2}{dof}$ Normal | 3,2675/6 | 1,1136/6 | 7,5766/8 | 20,2146/8 |

Figure 6




$\mu$ values are the same for both Normal and Poisson distribution and equals to 64.82 with $\sigma_\mu$=0.80.And, $\chi^2$ values are given below.

| $\chi^2_{Normal}$ | $\chi^2_{Poisson}$ |
|---|---|
| 7.5766 | 6.7570 |

As seen in the plots above red line indicates Poisson distribution function which is stated in the equation [1]. And, green line indicates Normal distribution which is in the form of:

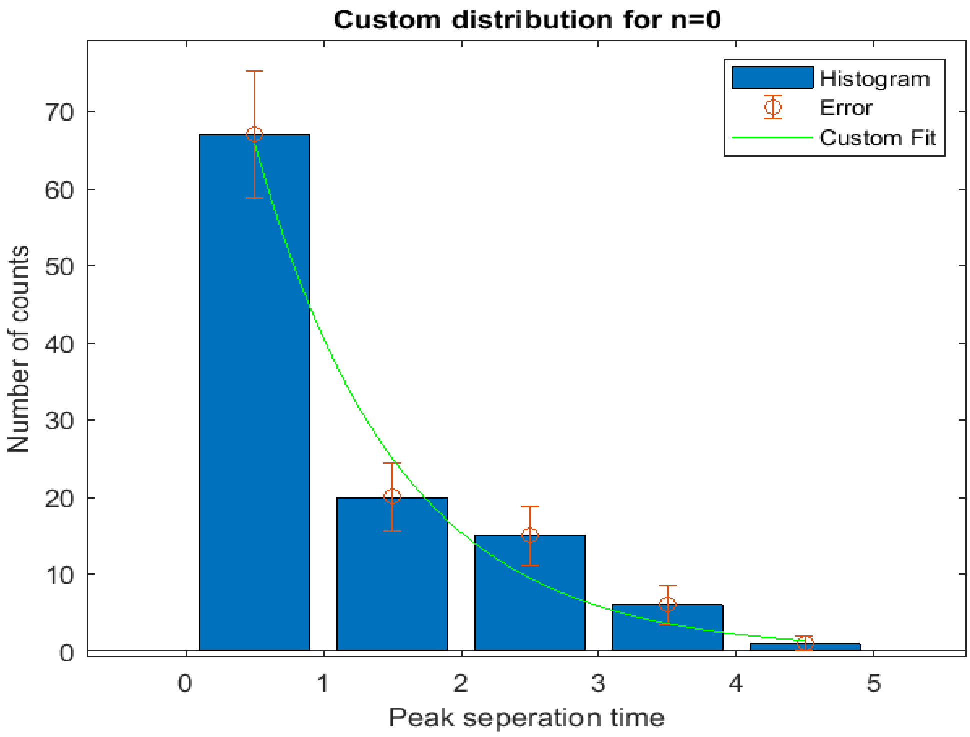
$$G(n, \sigma, \mu) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{n-\mu}{\sigma})^2}$$

Then,by dividing $\chi^2$ to degree of freedoms we get the table just above the Figure 8

#### 4.2.2 Part 2

In the second part, the length differences between successive peaks of records are analyzed. Two intervals are taken into account; one is that there is no gamma ray observed (n = 0) and the other is that only one gamma ray is observed (n = 1). The graphs are plotted and $\alpha$ values are calculated by using equation [3].

Figure 7



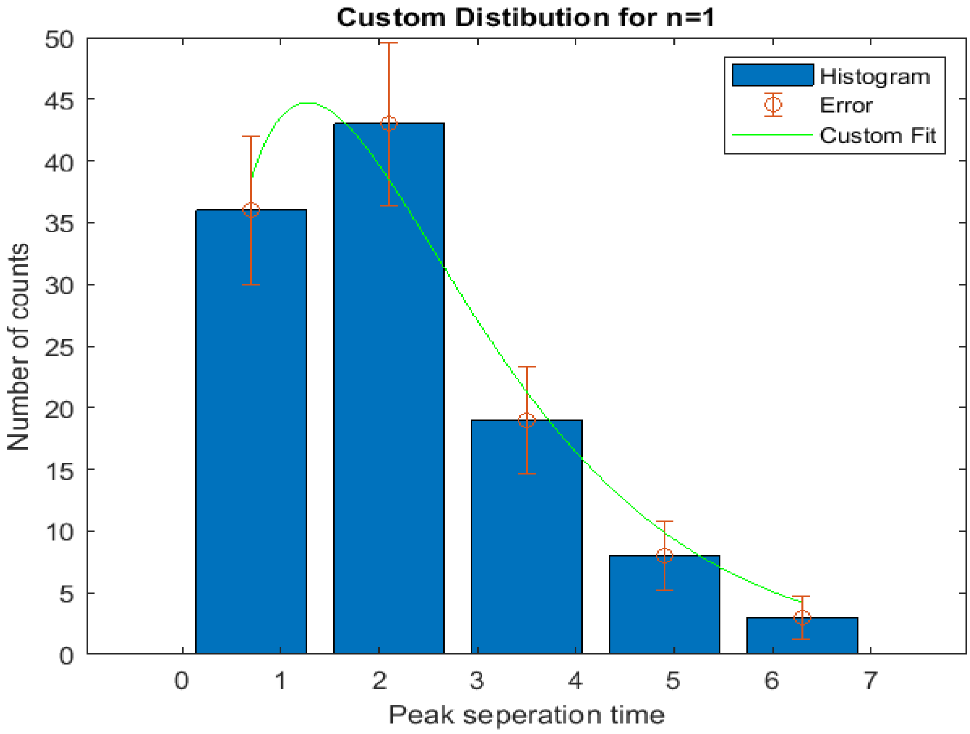

In the plot above for n=0, custom fit function is in the form of:

$$A\alpha e^{-\alpha t} \tag{5}$$

where $A = 110.3$, $\alpha = 0.9677$ with $\sigma_\alpha = 0.244$.And, $\chi^2$ value is 4.37967.

| - | $\alpha$ | $\sigma_\alpha$ | $\frac{\chi^2}{dof}$ |
|---|---|---|---|
| n=0 | 0,9677 | 0.224 | 4.3796/3 |
| n=1 | 0.7877 | 0,167 | 1.8897/3 |

Figure 8




In the plot above for n=1, custom fit function is in the form of:

$$A\alpha^2 te^{-\alpha t} \tag{6}$$

where $A = 154.4$, $\alpha = 0.7877$ with $\sigma_\alpha = 0.167$.And, $\chi^2$ value is 1.8897.

Then,by dividing $\chi^2$ to respective degree of freedom we get:

## 5 Conclusion

In the first part, considering $\chi^2$ values it is possible to compare Gaussian and Poisson distributions. Taking those values into account, Poisson distribution is an appropriate distribution for radioactive decays.$\chi^2$/dof values for Poisson fits are more close to 0 than Gaussian fits do. This means that for this experiment,radioactive decays of Cs137 and Ba133 elements performs Poisson distribution as seen tables given Data Analysis part..Also, it can clearly be said that as the mean value ($\mu$) increases it becomes hard to distinguish two distributions. Furthermore, for small $\mu$ values Poisson distribution describes it better than Gaussian does. In the second part, fits describe the distribution properly. In this case, instead of refusing the Poisson distribution, different fit functions are used and these functions are derived above for using equation [3] with inserting n=0 and n=1 respectively.And, their $\chi^2$ and $\chi^2$ per dof values given in the Data Analysis part above.

| - | $Cesium^{137}$ 10 s | $Cesium^{137}$ 1 s | $Barrium^{133}$ 10 s | $Barrium^{133}$ 1s |
|---|---|---|---|---|
| $\frac{\chi^2}{dof}$ poisson | 2,4603/6 | 1,0357/6 | 6,7570/8 | 34,7443/8 |
| $\frac{\chi^2}{dof}$ normal | 3,2675/6 | 1,1136/6 | 7,5766/8 | 20,2146/8 |
| $\mu$ | 114,50 | 11,84 | 64,82 | 6,25 |
| $\sigma_\mu$ | 1,06 | 0,34 | 0,80 | 0,25 |
| Variance | 10,6552 | 3,4197 | 2,1090 | 7,5382 |
| $\sigma_{variance}$ | 0,75516 | 0,2424 | 0,53440,1495 | 0,5344 |

# 6 References


[1]: Frank A. Haight. Handbook of the Poisson Distribution. New York: John Wiley Sons, 1967.

[2]: Frank A. Haight. Handbook of the Poisson Distribution. New York: John Wiley Sons, 1967.

[3]: Advanced Physics Experiments. Erhan Gülmez, 1999.

[4]: Advanced Physics Experiments. Erhan Gülmez, 1999.

[5]: Advanced Physics Experiments. Erhan Gülmez, 1999.

[6] :Advanced Physics Experiments. Erhan Gülmez, 1999.

[7] :Cosmos The Science of Everything,"How does a Geiger counter work?". https://cosmosmagazine.con does-geiger-counter-work,19.03.2019


# 7 Appendix

All the codes used in analyzing this experiment are taken from website given below:
"https://github.com/emreelibollar/phys442poisson.git"

Also, $\sigma$ values of Normal distribution function and corresponding uncertainty values are given in the table end of the Conclusion part: