POISSON STATISTICS

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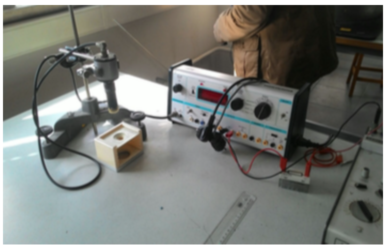
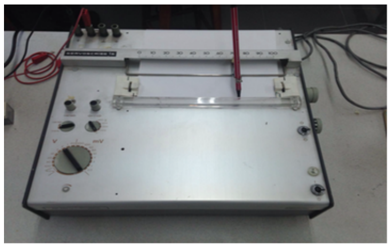
INTRODUCTION

Radioactive decay is a random process in which the emission of radiation depends on the number of atoms that can decay and a probability function that is characteristic their natural lifetimes. The detection of particles is random and any two measurements of the particles over equal periods of time will most likely be different. For large numbers, the difference between the measurements will be a small percentage. The probability of detecting a specific number of events for a given measurement is given by the standard normal or Gaussian distribution as was studied in the experiment on statistical analysis. However, for the measurement of a small number of events, the probability distribution for detecting a specific number of events is different and is given by a Poisson distribution. For rare events, the average number detected might also be much less than one, The Poisson distribution applies to these measurements and is useful for determining the probability of detecting a single event or more than one event in the same period. The Poisson distribution is a special case of the binomial distribution, similar to the Gaussian distribution being a special case.

APPARATUS





-Geiger Counter with a Scaler
-Chart Recorder
-Sample Holder
-Lead Absorbers
-Various Gamma-ray Sources

THEORY

Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occuring in a fixed interval of time and, or space if these events occur with a known avarage rate and independently of the time since the last event.

$$P_p(\mu,n) = \frac{\mu^n e^{-\mu}}{n!}, \sigma^2 = \mu$$

where μ is the mean of the distribution. $P_p(\mu,n)$ is the probability of observing n counts in a distribution with a mean μ

$$\sum_n P_p(\mu,n) = 1$$

Poisson distribution approaches the Gaussian distribution as μ becomes large.

$$P_G(\mu,\sigma,x) = \frac{1}{\sigma\sqrt{2\pi}} exp[-\frac{(x-\mu)^2}{2\sigma^2}]$$

where $P_G(\mu,\sigma,x)$ is Gaussian distribution.

Calculation of χ^2 : To reach the best fit of Gaussian and Poisson distribution ;

$$\chi^2_{poisson} = \sum_{i=n} \frac{[y(n) - L\frac{\lambda^n e^{-\lambda}}{n!}]^2}{\sigma^2_{y_i}}$$

$$\chi^2_{gaussian} = \sum_{i=n} \frac{[y(n) - L\frac{1}{\sigma\sqrt{2\pi}} exp[-\frac{(n-\lambda)^2}{2\sigma^2}]^2}{\sigma^2_{y_i}}$$

where L is width of bin and also λ is equal to mean value μ and variance σ^2 .

Probability of observing n counts during a time interval t :

$$P(\mu,n) = P(\alpha,t,n) = \frac{(\alpha t)^n e^{-\alpha t}}{n!}$$

where $\alpha = \mu/t$ and/or $\alpha = \lambda/t$.
Probability of having one event in a time interval dt is

$$P(\alpha,dt,1) = \frac{(\alpha dt)e^{-\alpha dt}}{1!}$$

Probability of n events in a t interval followed by by another event within a dt time is

$$P_q(n+1,t)dt = P(\alpha,t,n)P(\alpha,dt,1) = \frac{(\alpha t)^n e^{-\alpha t}}{n!}\frac{(\alpha dt)e^{-\alpha dt}}{1!} \simeq \frac{(\alpha t)^n e^{-\alpha t}\alpha dt}{n!}$$

$$P_q(n+1,t) = \frac{(\alpha t)^n e^{-\alpha t}\alpha}{n!}$$

DATA-ANALYSIS

In the first step we chose a random radioactive source 'Cs^{137}' (this selection is independent from the source , it is about the ionization effect)and then collected data at $20V$ intervals to plateau the counter. We started from $300V$ to $500V$. Then since the number of counts started to appear in a narrow range , we chose our V_{eff} as 440 V.

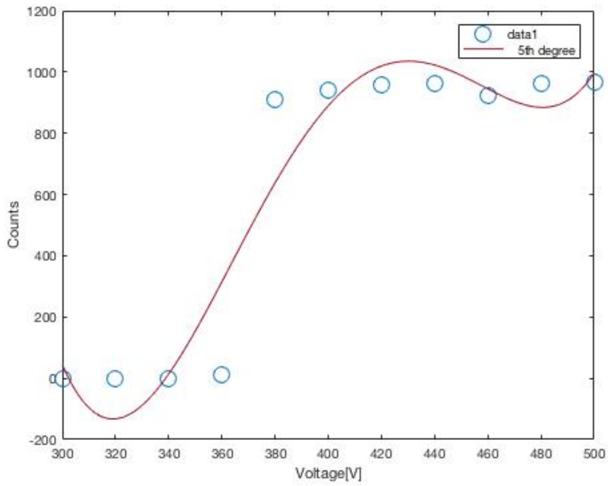


Figure 1: plot with spline interpolant

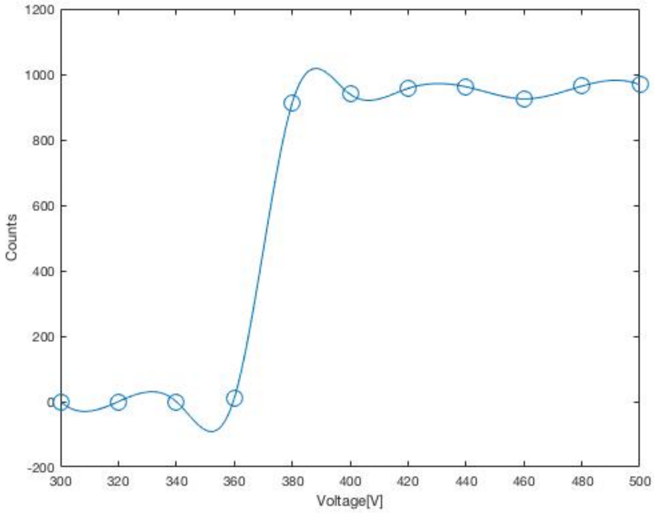
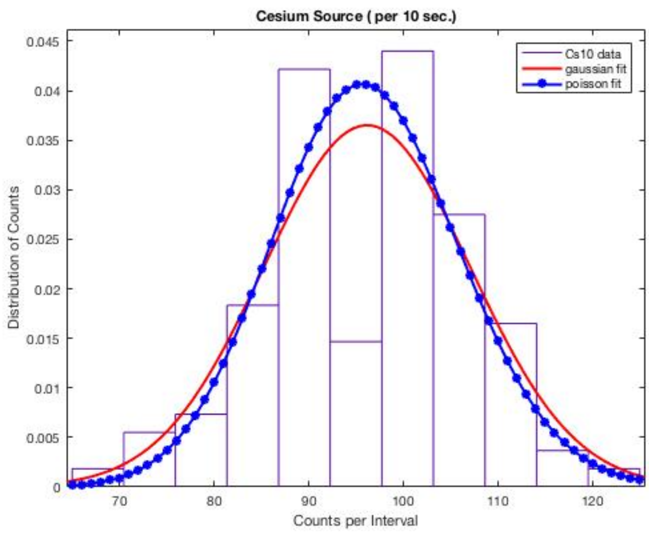


Figure 2: plot of polynomial fit of 5th degree

Secondly, we used Cs^{137} as the first source. And we got 100 counts with 10 sec. intervals. and then we got 100 counts with 1 sec. intervals. Next we used another radioactive source Ba^{133} and repeated same counting steps . With these data , we had four histograms with the poisson fitting and gaussian fitting.

Analysis of Cs^{137} data for 10 *sec.* :

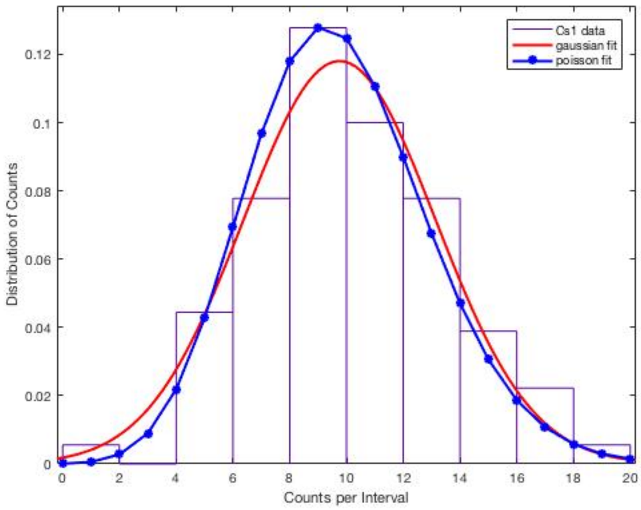


μ	σ
96.17	10.92

Bin Center	Actual Values of Bin Centers
71	1
76	3
81	4
86	10
91	23
96	8
101	24
106	15
111	9
116	2
121	1

Values of Poisson Fit	Values of Gaussian Fit	$\chi^2_{poisson}$	$\chi^2_{gaussian}$
0.8400	1.7133	0.0010	0.0046
3.1133	4.4333	0.0140	0.0305
8.3333	9.2867	0.1103	0.1382
16.4000	15.7867	0.4166	0.3843
24.1800	21.7600	0.8472	0.6717
27.1200	24.3333	1.2094	0.9680
23.4667	22.0733	0.7866	0.6869
15.8733	16.2400	0.3655	0.3842
8.4933	9.6933	0.1023	0.1367
3.6267	4.6933	0.0206	0.0353
1.2533	1.8400	0.0023	0.0053

Analysis of Cs^{137} data for 1 *sec.* :

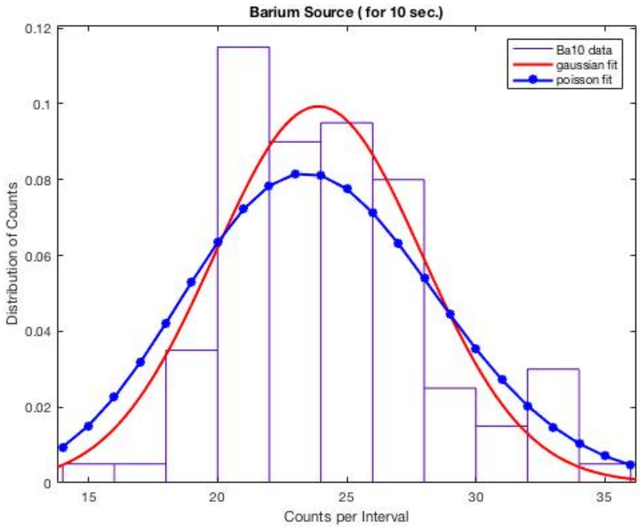


μ	σ
9.76	3.38

Bin Center	Actual Values of Bin Centers
0	1
4	9
6	16
8	26
10	20
12	16
14	7
16	4
18	1

Values of Poisson Fit	Values of Gaussian Fit	$\chi^2_{poisson}$	$\chi^2_{gaussian}$
0.0116	0.3660	0.0103	0.0674
4.3760	5.5380	11.8189	21.3551
13.8820	12.7340	151.9309	124.4784
23.5900	20.6280	444.3852	326.3142
24.9460	23.5460	535.4678	471.5205
17.9860	18.9360	272.4225	305.2930
9.4060	10.7300	77.1996	102.7099
3.7300	4.2840	11.1941	15.2874
1.1600	1.2040	1.1409	1.2386

Analysis of Ba^{133} data for 10 *sec.* :

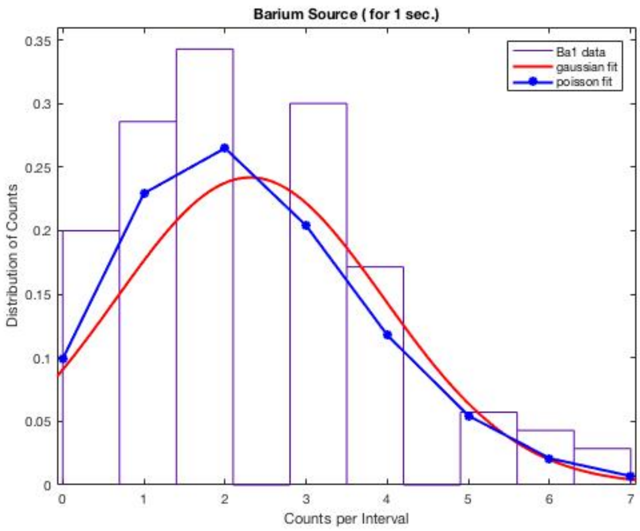


μ	σ
23.9	4.02

Bin Center	Actual Values of Bin Centers
10	1
13	1
16	7
19	23
22	18
25	17
28	16
31	5
34	3
37	6
40	1

Values of Poisson Fit	Values of Gaussian Fit	$\chi^2_{poisson}$	$\chi^2_{gaussian}$
0.1400	0.0500	0.0010	0.0047
1.1120	0.5000	1.2543	0.1894
4.5200	2.8700	17.5348	5.4810
10.6140	9.4380	81.3101	59.3099
15.6820	17.7620	234.3910	311.0745
15.5120	19.1340	232.3777	372.3693
10.7740	11.7980	101.3356	125.7177
5.4540	4.1640	30.0440	16.3035
2.0740	0.8420	3.7938	0.3240
0.6080	0.0980	0.0043	0.4049
0.1400	0.0064	0.0010	0.0137

Analysis of Ba^{133} data for 1 *sec.* :



μ	σ
2.31	1.65

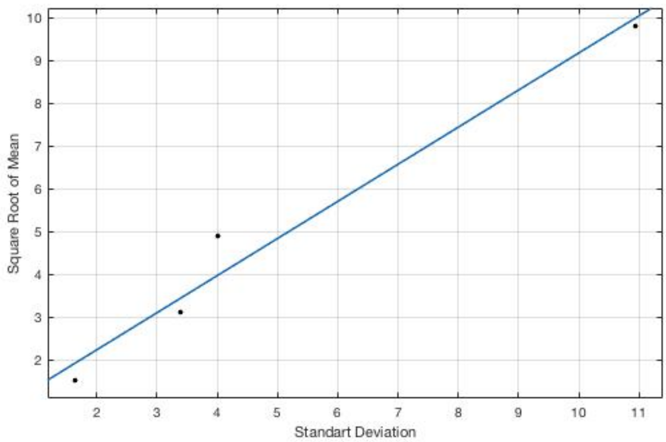
Bin Center	Actual Values of Bin Centers
0	13
1	19
2	23
4	20
5	11
7	4
8	3
9	2

Values of Poisson Fit	Values of Gaussian Fit	$\chi^2_{poisson}$	$\chi^2_{gaussian}$
6.6173	6.0480	23.2192	18.2255
15.2860	11.7627	155.2980	82.1040
17.6553	15.8407	203.5225	156.6167
7.8513	9.5393	26.6787	46.1642
3.6273	4.2266	4.7261	7.5757
0.4607	0.2833	0.0014	0.0437
0.1333	0.0420	0.0544	0.1033
0.0340	0.0043	0.0435	0.0562

As mentioned Theory section , χ^2 values per degrees of freedom are

	Cs 10s	**Cs 1s**	**Ba 10s**	**Ba 1s**
Poisson χ^2_ν	39.1499	26.5738	5.3662	4.1772
Gaussian χ^2_ν	35.1532	24.4376	6.9284	3.1723

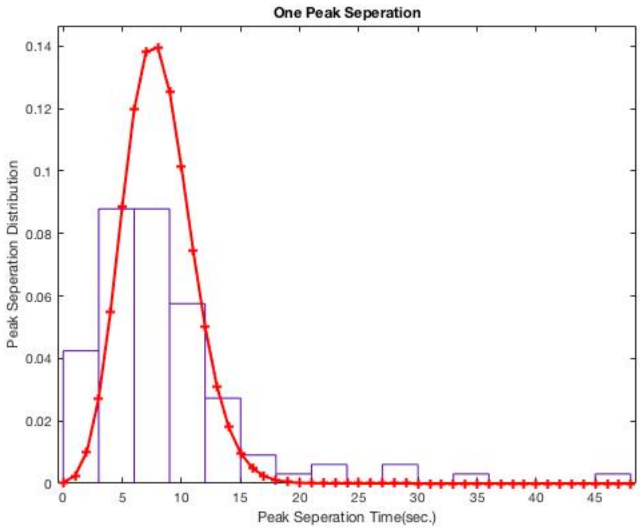
To calculate the relation between $\sqrt{\mu}$ and standart deviation σ we plotted a fit below ; —

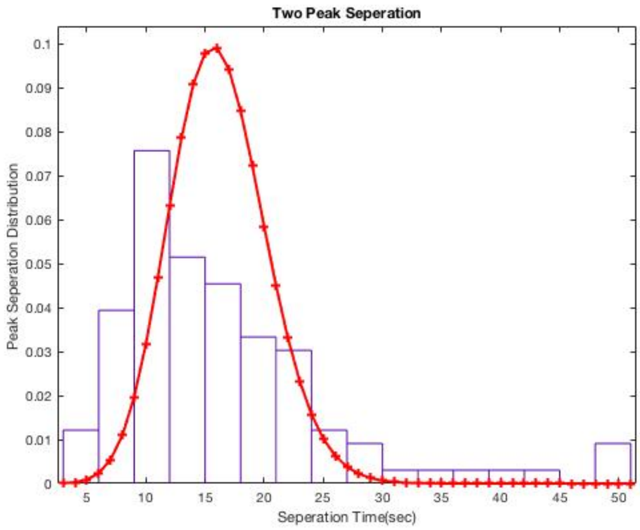


Linear model Poly1:
f(x) = p1*x + p2
Coefficients (with 95p1 = 0.8667 (0.4127, 1.321)
p2 = 0.5069 (-2.27, 3.284)

Goodness of fit:
SSE: 1.112
R-square: 0.9712
Adjusted R-square: 0.9568
RMSE: 0.7458

In the second part of the experiment , we found probabilities of observing n=0 and n=1 counts during a time intervals . And we had histograms of data from Chart Recorder ;





	σ	μ	$\alpha = \frac{\mu}{t}$
n=0[peak sep.]	6.1256	4.0410	0.037
n=1[two peak sep.	8.6128	16.1819	0.147

CONCLUSION

In this experiment, with the radioactive decay of the different sources we tried to find which distribution is valid for this event. We collected sets of data to create histograms and then we fitted these to poisson and gaussian distributions. In the second step of the experiment it was shown in time interval dt there is no event and then one event occurs.
When we controlled the χ^2 for each data set, except the data of Barium in every 10 seconds, all other data sets showed a larger $\chi^2_{poisson}$ than $\chi^2_{poisson}$. Hence , poisson distribution is better to fit this physical event.

REFERENCES

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http://web.mit.edu/8.13/www/experiments.shtml
http://www.umass.edu/wsp/resources/poisson/in dex.html