Boğaziçi University
Physics Department

Scattering in Two Dimensions

Experiment 5

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Contents

1 Abstract

In this experiment, we scattered a circular object(which radius is 2.81 ± 0.01cm measured by vernier scale) by shot steel balls with 143 ± 12 shots/cm flux(I). Our aim was measuring radius of the circular object. Reflected balls leave trace on the paper and this trace gave us reflected angle. After completed the experiment, we use 2 different calculation method to measure radius;

By using first method, we found radius as 6.81 ± 1.05 cm and its χ^2 is equal to 14.6

By using second method, we found radius as 2.90 ± 0.53 cm and its χ^2 is equal to 0.066

2 Introduction

Improvement of microscope lead us to measure the dimension of the very small object. The most powerful one has a resolution of half the width of a hydrogen atom[1]. However, it is not possible to measure particles which has width about 1 femtometer or less and, the unstable particles with microscope. These particles width measured with scattering method.

Almost everything we know about nuclear and atomic physics has been discovered by scattering experiments.For example, Rutherford's discovery of the nucleus, the discovery of sub-atomic particles (such as quarks), etc. In low energy physics, scattering phenomena provide the standard tool to explore solid state systems. For example, neutron, electron, x-ray scattering, etc.[3]

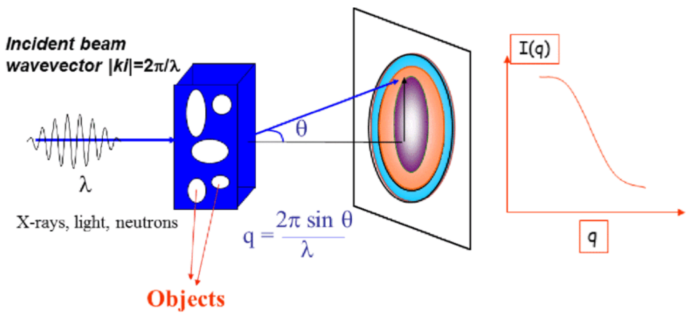

Figure 1: Schematic diagram of nuclear scattering[2]

2.1 Neutron Scattering

Neutron scattering is the technique of choice for condensed matter investigations in general because thermal/cold neutrons are a non-invasive probe; they do not change the investigated sample since they do not deposit energy into it.

Neutrons interact through nuclear interactions. X-rays interact with matter through electromagnetic interactions with the electron cloud of atoms. Electron beams interact through electrostatic interactions. Light interacts with matter through the polarizability and is sensitive to fluctuations in the index of refraction. For this, neutrons have high penetration (low absorption) for most elements making neutron scattering a bulk probe. Sample environments can be designed with high Z material windows (aluminum, quartz, sapphire, etc) with little loss.
In neutron scattering, scattering nuclei are point particles whereas in x-ray scattering, atoms have sizes comparable to the wavelength of the probing radiation. In the very wide angle (diffraction) range, x-ray scattering contains scattering from the electron cloud, whereas neutron scattering does not. In the Small Angle Neutron Scattering range, this is not the case.[4]

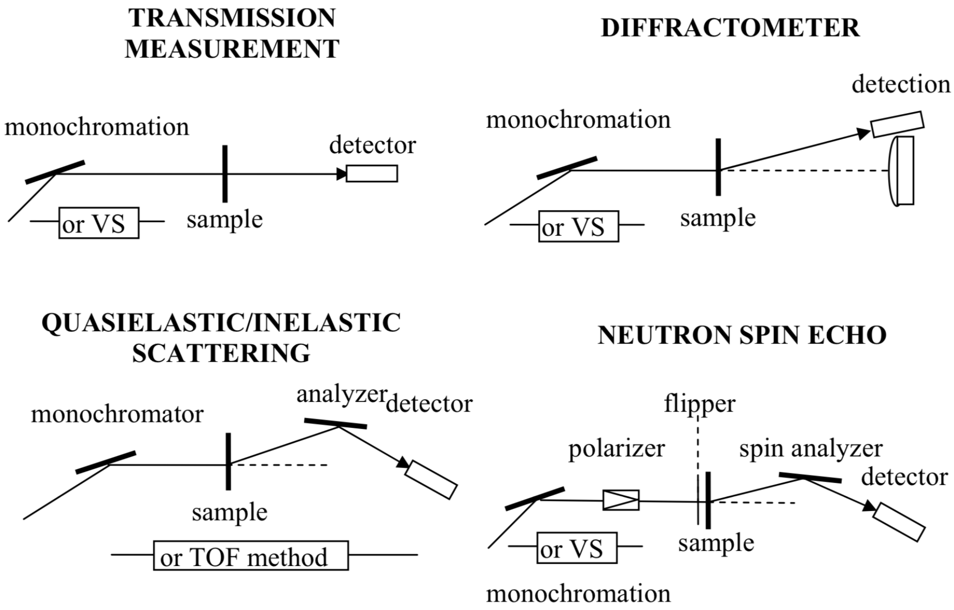

Figure 2: Schematic diagram of the four types of neutron scattering methods[4]

3 Theory

The differential cross section is defined as simply the ratio of the number scattered particles observed in a region normalized by corresponding solid angle to the total number of particles shot towards the target.

$$\frac{d\sigma}{d\Omega} = \frac{Y}{Id\Omega} \tag{1}$$

where Y is the yield dΩ,dΩ is the solid angle, and I is the incident flux. Some correction may have to be added.

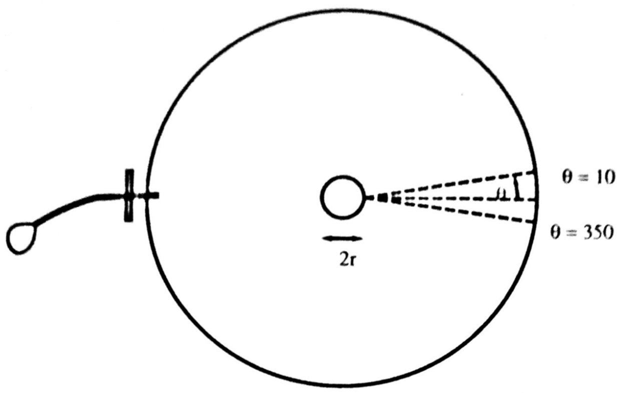

Figure 3: The scattering Tray[5]

The experiment here is a simple two dimensional scattering from a vertically placed cylindrical target (Figure 3). A steel ball shot towards the target with an impact parameter b will scatter at an angle θ (Figure 4). Assuming that the distance between the target and the detector is large, the scattering angle θ is given by;

$$\frac{b}{r} = cos(\frac{\theta}{2}) \tag{2}$$

The number of particles scattered at a given angle is

$$dN = -Idb = \frac{Ir}{2} \times sin(\frac{\theta}{2})d\theta \tag{3}$$

where I is the incident flux (shots/cm).Hence, the differential cross section is simply

$$\frac{\sigma}{d\theta} = \frac{dN}{Id\theta} = \frac{r}{2} \times sin(\frac{\theta}{2}) \tag{4}$$

Integrating the differential cross section over the full andular range from 0 to 2 π, would yield the total cross section.

$$\sigma = \int_0^{2\pi} \frac{r}{2} sin(\frac{\theta}{2})d\theta \tag{5}$$

$$\sigma = 2r \tag{6}$$

which is the expected result

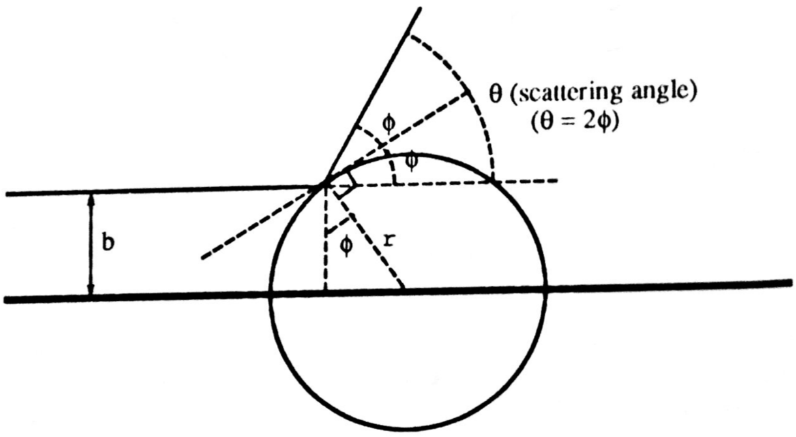

Figure 4: Geometry of the scattering in two dimensions[5]

4 Experiment

4.1 Apparatus

- Scattering Tray
- Pressure Sensitive Paper Tape
- Air Gun
- 20 Steel Balls
- Plexiglass Circular Target
- Vernier Scale
- Ruler

4.2 Setup

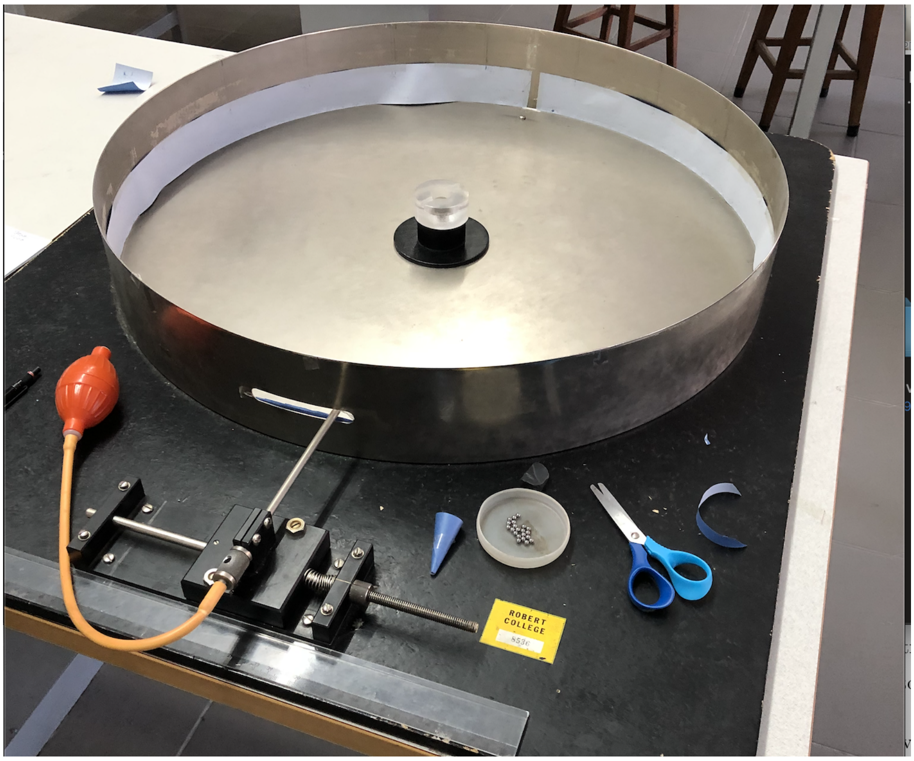

Figure 5: Image of Setup

4.3 Procedure

- First we line the rim with pressure sensitive paper and mark relevant angle region. We set $0°$ degree in the opposite side of the shooting area.
- Set minimum position of the gun where 1 turn after when the shot ball don't hit the target.
- Shot 20 balls and moved gun 1 turn.
- Measure the distance traveled in 1 turn
- repeat previous step again and again to the when the gun did not hit target again
- Count the hit trace on the paper in relevant angle. This number give us the dN
- Histogram dN as a function of angle(at which angle we place each value)
- Make a plot of dN versus $\sin(theta/2)$. Make line fit the data on the graph with considering the error.
- Estimate the uncertainty on the slope. Determine the radius of the target from the slope value.
- We compare the result with that we obtain by measuring the radius with vernier scale.

5 Raw Data

5.1 Raw Data Tables

θ (angle)	dN	Error of dN
10°-30°	28	5
30°-50°	26	5
50°-70°	36	6
70°-90°	38	6
90°-110°	64	8
110°-130°	64	8
130°-150°	68	8
150°-170°	101	10
170°-190°	86	9
190°-210°	61	8
210°-230°	69	8
230°-250°	47	7
250°-270°	43	6
270°-290°	38	6
290°-310°	27	5
310°-330°	22	5
330°-350°	13	4

Table 1: Number of hits in the 20° interval of given angles

Shoots in each turn	20±1
Total number screw turned	42 ±1
Total distance after 30 turn	4.32 cm
Distance per turn (ΔL)	0.14 cm
$\sigma_{distance}$	0.01 cm
Scattered Shots	831 ±29
radius of object(measured with vernier scale)	2.81±0.01

Table 2: Measurements and numbers

5.2 Raw Data Plots

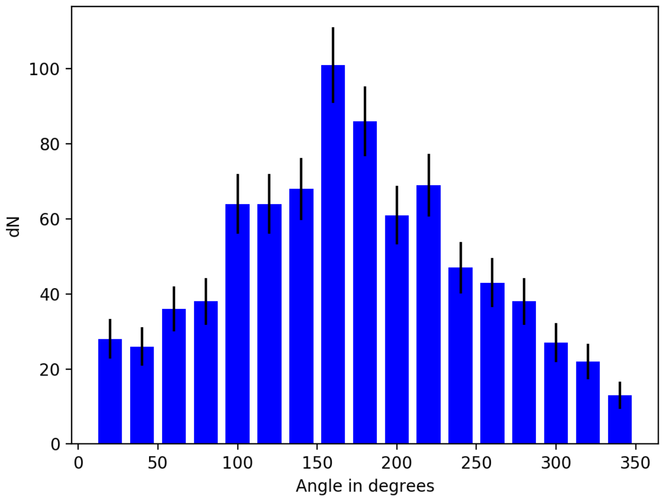

Figure 6: Histogram of the scattered data in given angles with Error bars

6 Data Analysis

We combine θ and its counterpart $360 - \theta$. Then we get this table and histogram;

θ (angle)	dN	Error of dN
20°	42	6
40°	48	7
60°	63	8
80°	76	9
100°	107	10
120°	111	10
140°	137	12
160°	162	13
180°	172	13

Table 3: Number of hits in the 20° interval of given angles

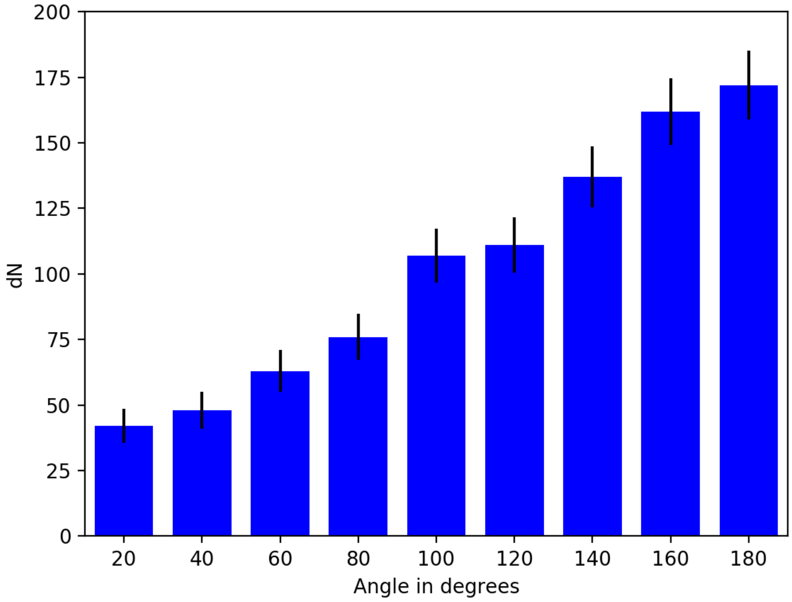

Figure 7: Histogram of the scattered data in given angles with Error bars

6.1 Meausrement of raidus (Method 1)

This equation will give us radius of the object where derive from Equation 3;

$$r = \frac{2dN}{Id\theta sin(\theta/2)} \tag{7}$$

where I is flux, dN is number of shots

We plotted $sin(\theta/2)$ versus dN graph with considering error via python;

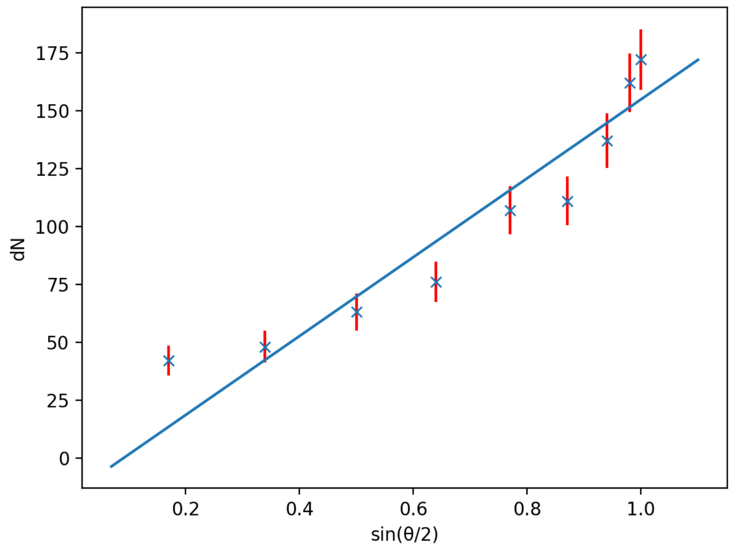

Figure 8: sin($\theta/2$) versus dN

We calculated the slope of the fitted line and its error as;

$$slope = 170. \pm 22 \tag{8}$$

Then r and its uncertainty become;

$$r = \frac{2 \times (slope)}{Id\theta} \tag{9}$$

$$\sigma_r = 2 \times \sqrt{\frac{(\sigma_{slope})^2}{I^2(d\theta)^2} + \frac{(\sigma_{d\theta})^2(slope)^2}{I^2(d\theta)^4} + \frac{(\sigma_I)^2(slope)^2}{I^4(d\theta)^2}} \tag{10}$$

Calculation of the flux and its uncertainty by error propagation formula;

$$I = \frac{number\ of\ shots\ in\ each\ turn\ (N)}{distance\ per\ turn(\Delta L)} \tag{11}$$

$$\sigma_I = \sqrt{\frac{N^2\sigma_{\Delta L}^2}{\Delta L^4} + \frac{\sigma_N^2}{\Delta L^2}} \tag{12}$$

We can express I as a;

$$I = 143 \pm 12 \quad shots/cm \tag{13}$$

dθ is equal to 20,0°, the value in radian;

$$d\theta = 0.349 \pm 0.001 \tag{14}$$

Then we calculated r with equation [9];

$$r_1 = 6.82\ cm\ \pm 1.05\ cm \tag{15}$$

6.2 Meausrement of raidus (Method 2)

The result of the Equation 6, we can find radius r_2 with another method;

$$r = \frac{\sigma}{2} = \frac{dN}{2 \times I} \tag{16}$$

$$\sigma_r = \frac{1}{2} \times \sqrt{\frac{(\sigma_{dN})^2}{I^2} + \frac{(\sigma_I)^2(dN)^2}{I^4}} \tag{17}$$

$$r_2 = 2.90\ cm\ \pm 0.53\ cm \tag{18}$$

6.3 Calculation of χ^2

χ^2 of r_1 found by method 1 and we take nominal value of r as we measured by vernier scale;

$$\chi^2 = \left(\frac{6.82\ cm\ -\ 2.81\ cm}{1.05\ cm}\right)^2 = 14,6 \tag{19}$$

χ^2 of r_2 found by method 2 and we take nominal value of r as we measured by vernier scale;

$$\chi^2 = \left(\frac{2.90\ cm\ -\ 2.81\ cm}{0.35\ cm}\right)^2 = 0.066 \tag{20}$$

7 Conclusion

The setup was a classical model of a nuclear scattering. We realize that we can measure dimension of a object without saw it. Neutron scattering or electron scattering are made in 3 dimensions. Hence result is more reliable and less uncertain than 2 dimension.

Both χ^2 values of our results are not good and the evidence of some mistake happened in experiment.
In first calculation method; even we have a very high uncertainty, our χ^2 values is very high. And some possible reasons are;

- We realize that sometimes we failed to shot balls, we cannot exactly shot 20 balls. Then we fire 21 balls and make calculation with 20±1 balls. This correct our accuracy but decrease our precision.
- Some high speed balls reflected again from paper and make another trace in opposite side. This situation cause miscalculation.
- We assume gun shot all balls directly, however it is possible that balls fire some variation of angle. If its uncertainty is very high, it is also considerably effect our result.
- We turned the screw 42 times. However we might have forget to turn screw or turn twice without shot balls. Because of this reason we set ±1 uncertainty to total number of screw. We didn't use this value in calculation but if it is happened, it affect dN values in given angles.

In second calculation method;our χ^2 values is very low because of high uncertainty. And some possible reasons are;

- The good accuracy is because of that we shot 21 balls and took the number of shot 20±1. In reality we try to shot 21 × 42 = 882 balls but we saw a only 831 trace. However, when we take it 20× 42 = 840 is very close and only 0.3 σ away from scattered shots.
- Bad precision is also because of same reason. We add extra uncertainty to the flux while making shot number in each turn 20±1 shot. But the main reason is the high uncertainty of of total number of shot.

- If we shot 200 balls instead of 20 balls in each turn. Then our uncertainty will be lower than this one.

References

[1] World's Most Powerful Microscope [accessed date: 03/04/2019]
$https://www.azonano.com/nanotechnology-video-details.aspx?VidID=354$

[2] Principle of the Small Angle Neutron Scattering (SANS) experiment [accessed date: 03/04/2019]
$https://www.researchgate.net/figure/Principle-of-the-Small-Angle-Neutron-Scattering-SANS-experiment_fig1_282976864$

[3] Scattering Theory [accessed date: 03/04/2019]
$http://www.tcm.phy.cam.ac.uk/\ bds10/aqp/lec20-21_compressed.pdf$

[4] Introduction to Neutron Scattering [accessed date: 03/04/2019]
$https://www.ncnr.nist.gov/staff/hammouda/distance_learning/chapter_6.pdf$

[5] Advanced Physics Experiments, Erhan Gulmez, Bogazici University Publications, 1999,ISBN 975-518-129-6

[6] Constants [accessed date: 03/04/2019]
$https://physics.nist.gov/cuu/Constants/$

Appendices

Python code used in data analysis;
$https://github.com/sinaaktas/2D-scattering/blob/master/2dSinaAktas.ipynb$