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X-Ray Scattering and Duane-Hunt Law
Phys442 Project

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Contents

1 Abstract

In this experiment we try to find Planck Constant by examining X-ray diffraction of NaCl crystal. To do so we use Bragg's Law and we calculate frequency of X-ray beams by converting angles we get from experiment to frequency. Then,by using Duane-Hunt Law, we compare maximum frequency and energy ratios, and we get Planck constant experimentally. While comparing these values we do line fit and get some uncertainties. Also as converting angle data which we get from experiment to frequency, we perform line fit and get some other corresponding uncertainties.By taking into consideration of all uncertainties we end up with an experimental Planck constant. .

2 Theory

In an x ray tube, electrons are emitted from a cathode which is heated and accelerated in a beam by a voltage supplied between anode and cathode then these emitted electrons strike the anode [1]. While electrons travel through the atoms of the anode, a beam electron interacts the target atom's electrons [2]. These electrons are in an inner subshell. Energetic beam electron interacts with the atomic electron via coulomb interaction. If the energetic electron has enough energy to excite bound electron, the latter will be ejected from the atom and the atom will be ionized. [3] This causes the atom to be in a highly excited state. Then the outer shell electrons of atom goes into the ground state and this leaves an exceed high energy photons which belongs its x-ray line spectrum. The Total spectrum x radiation by an x ray tube includes the discrete line spectrum combined on a continuum. When the beam electrons are accelerating due to the target nuclei of the atoms in the anode, the continuum spectrum occurs because of the Bremsstrahlung processes. The shape of the Bremsstrahlung continuum is related to energy of electron beam, on the other hand, the shape of x-ray line spectrum depends on particular atoms which anode consists. The qualitative explanation of this process follows. When an electron hits the anode, some of electron's energy is converted to heat the anode and remaining of its energy goes to create a photon, so energy of photon depends on how much energy carries of incoming electron. The maximum photon energy is obtained if the total kinetic energy of incoming electron is transferred to a photon. [4]

$$E_{max} = e.U_{acc} \qquad (1)$$

As it can be seen, the maximum energy of photon is proportional to the voltage applied. According to Duane-Hunt Law, if the maximum energy of the photon is known, the maximum frequency and the corresponding minimum wavelength can also be determined and vice-versa.

$$E = h\nu = hc/\lambda \qquad (2)$$

In the Equation 2 above, c is the speed of ligth and equals to 3*10e+8 m/s and h is the Planck constant and equals to 4.14e-15 eV s.

Secondly,English physicists Sir W.H. Bragg and his son Sir W.L. Bragg developed a relationship in 1913 to explain why the cleavage faces of crystals appear to reflect X-ray beams at certain angles of incidence (theta).This relationship can be written in the form of equation below, and this equation is called Bragg's Law.[5]

$$2d\sin(\theta) = n\lambda \tag{3}$$

The variable d is the distance between atomic layers in a crystal, and the variable λ is the wavelength of the incident X-ray beam; n is an integer. In this experiment, NaCl crystal was used and distance between atomic layers of NaCl crystal is 0.28 10e-9 m.

3 Experiment

3.1 Apparatus

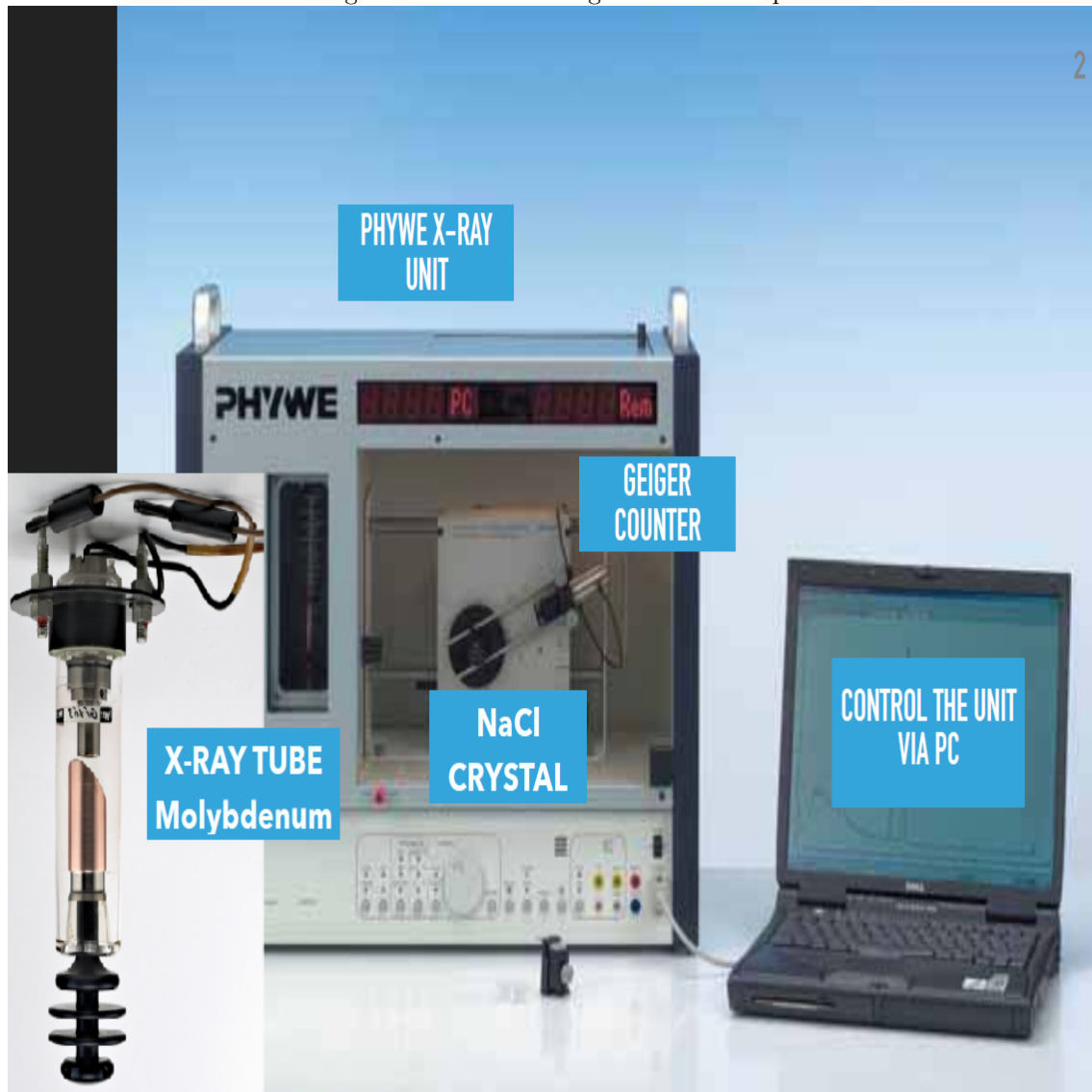X-Ray Tube Molybdenum

PHYWE x-Ray Unit

Geiger Counter

NaCl Crystal

Control The Unit via PC

3.2 Setup

Figure 1: Schematic diagram of the setup

3.3 Procedure

1. First set the voltage to the highest possible value i.e 35kV to take full spectrum for calibration. : from 5 to 35 degree.

2. Then set voltage applied to 15kV and : from 5 to 20 degree. You may stop taking data when you see the plateau.

3. Repeat the same procedure by incrementing voltage 3kV each time up to 30kV.

4 Data and Analysis

4.1 Calibration

This is the first part of the experiment. Main goal of this part is to understand how accurate the device is taking the data. In order to test this accuracy we take correspond-

ing x-axis values of peak values in the plot. These intersecting x-values of peak values are characteristic emission line angles of Molybdenum which are K_{α_1}, K_{β_1}, K_{α_2}, K_{β_2}, K_{α_3}, K_{β_3}, K_{α_4} respectively.By comparing these experimental values with the theoretical values, we get the calibration function. It can be seen in the graph below.

Figure 2:
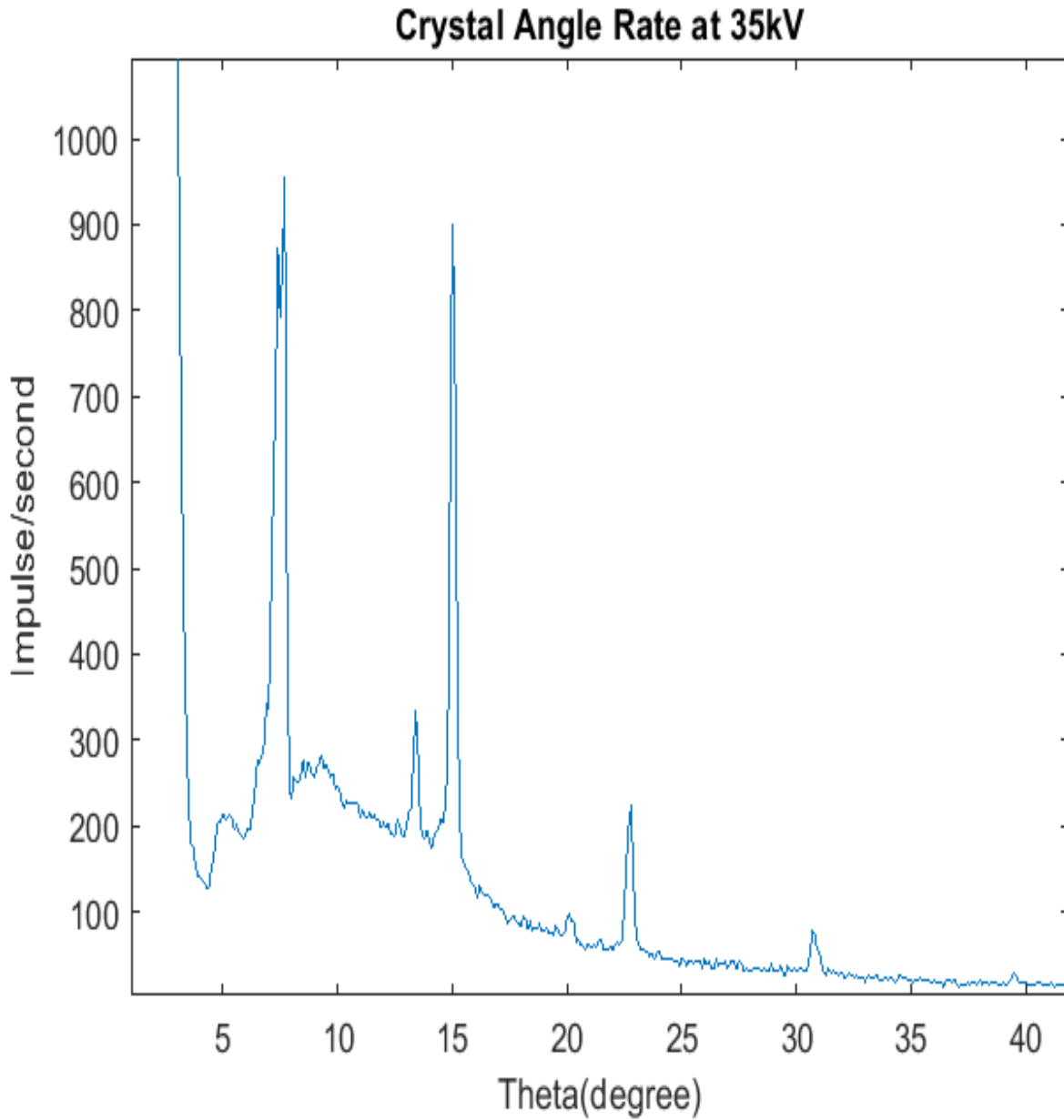
There is a big jump in the intensity at around 5 degree or lower since at the angles smaller than 5 X-Rays are arriving to the detector without being scattered from the crystal.

Theta Theoretical(degree)	Theta Experimental(degree)
7.28	7.70
14.70	15.00
22.37	22.80
30.49	30.70
6.47	7.40
13.02	13.40
19.7634	20.10

Table 1: Theta Theoretical vs Experimental

By using points in the Table 1 above, we make a plot and by using MATLAB, we do line fitting.

Figure 3:
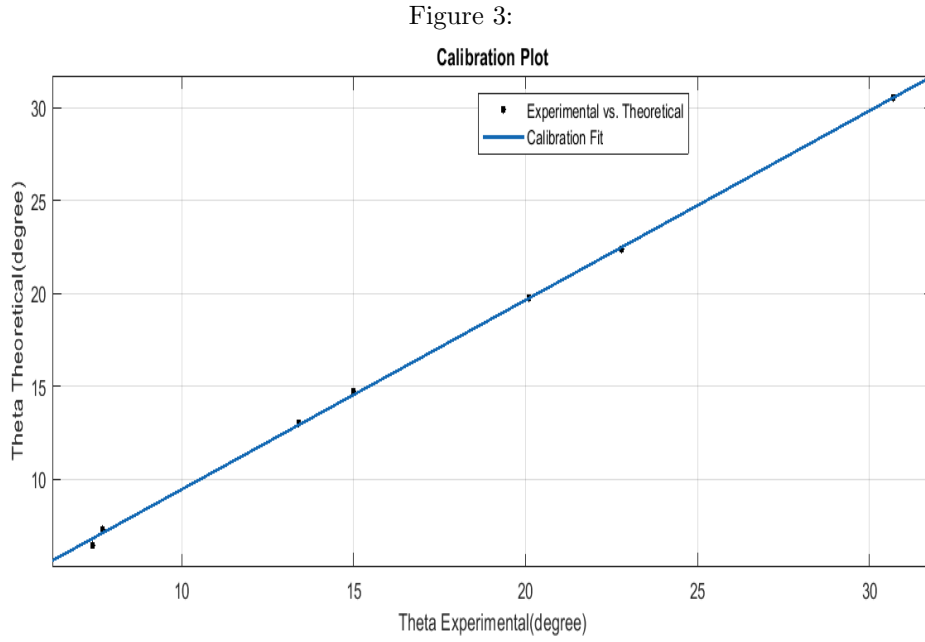
Then, our calibration function is:

$$\theta_{Calibrated} = p_1.\theta_{experimental} + p_2 \tag{4}$$

where

$$p_1 = 1.02, \sigma_{p_1} = 0.03$$

and

$$p_2 = -0.73, \sigma_{p_2} = 0.45$$

By using calibration function above we will be able to make more accurate calculations.

4.2 Data for Different Voltages

In this section of the experiment, starting from 15kV and increasing 3kV each time until reaching 30kV, we take K_{α_1}, K_{β_1}, K_{α_2}, K_{β_2}, K_{α_3}, K_{β_3}, K_{α_4} values from the plots.Then we make line fitting and get θ values intersecting the x-axis.By making line fitting, we can get more accurate θ values with uncertainties.
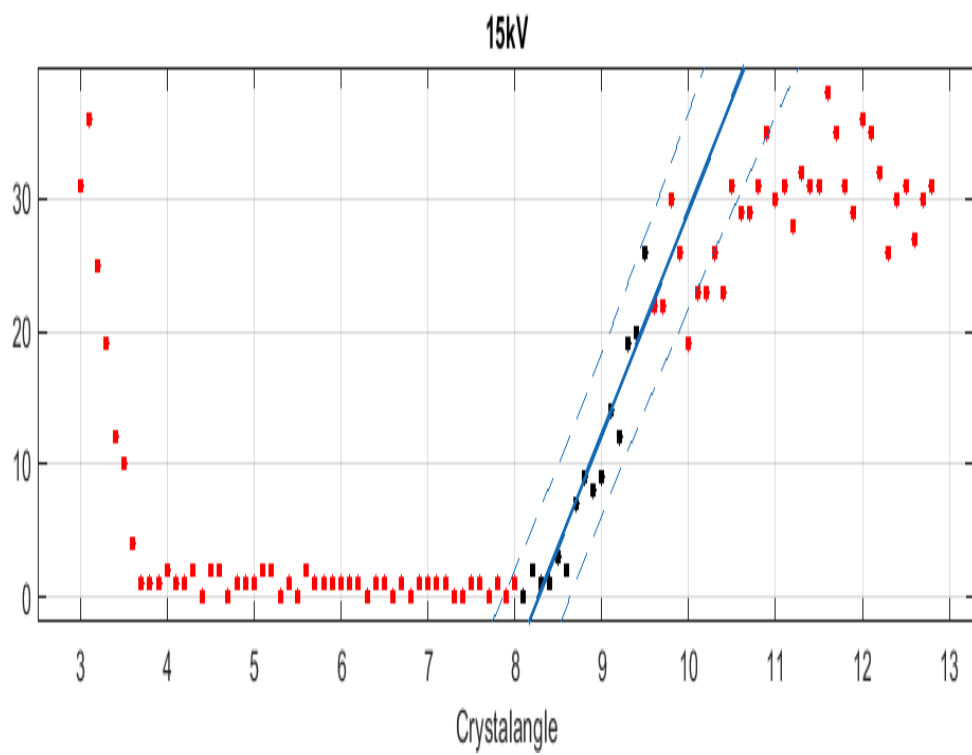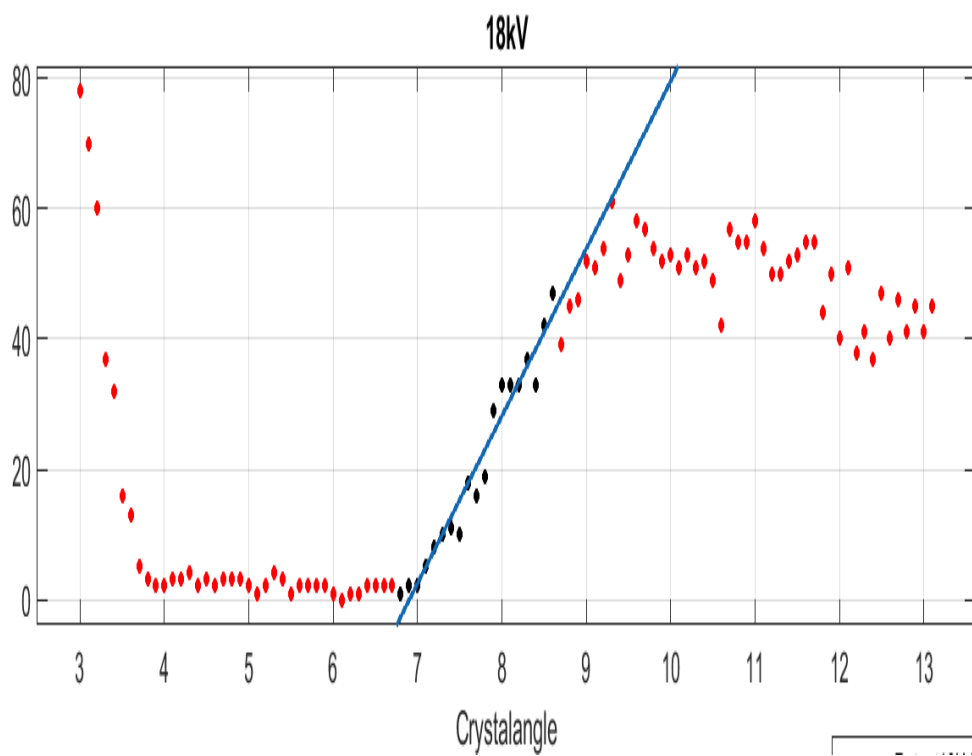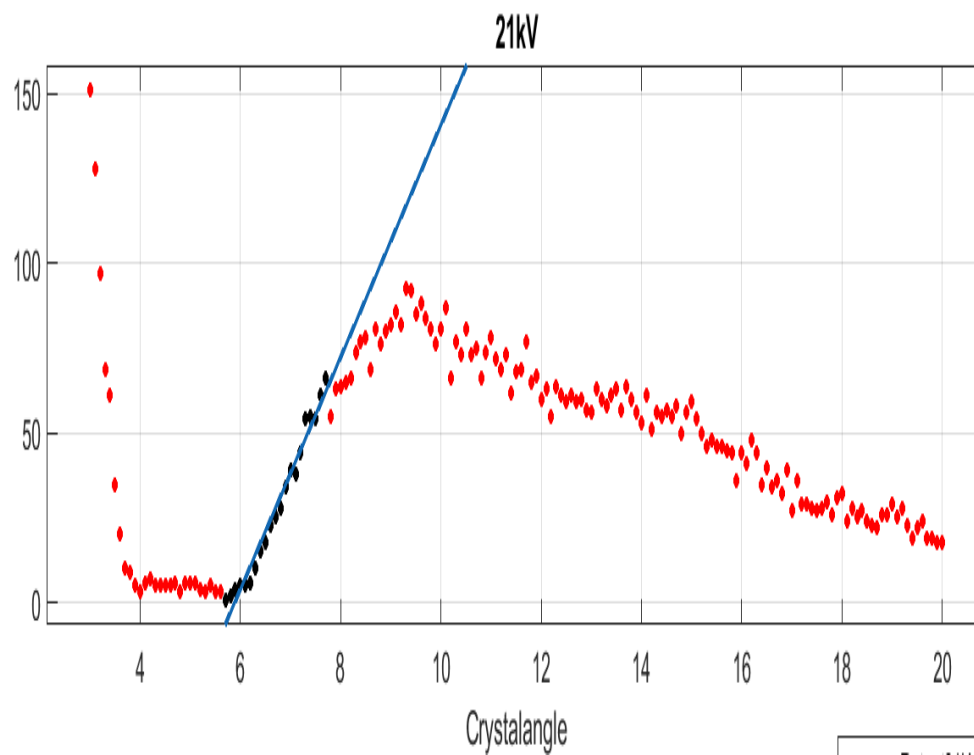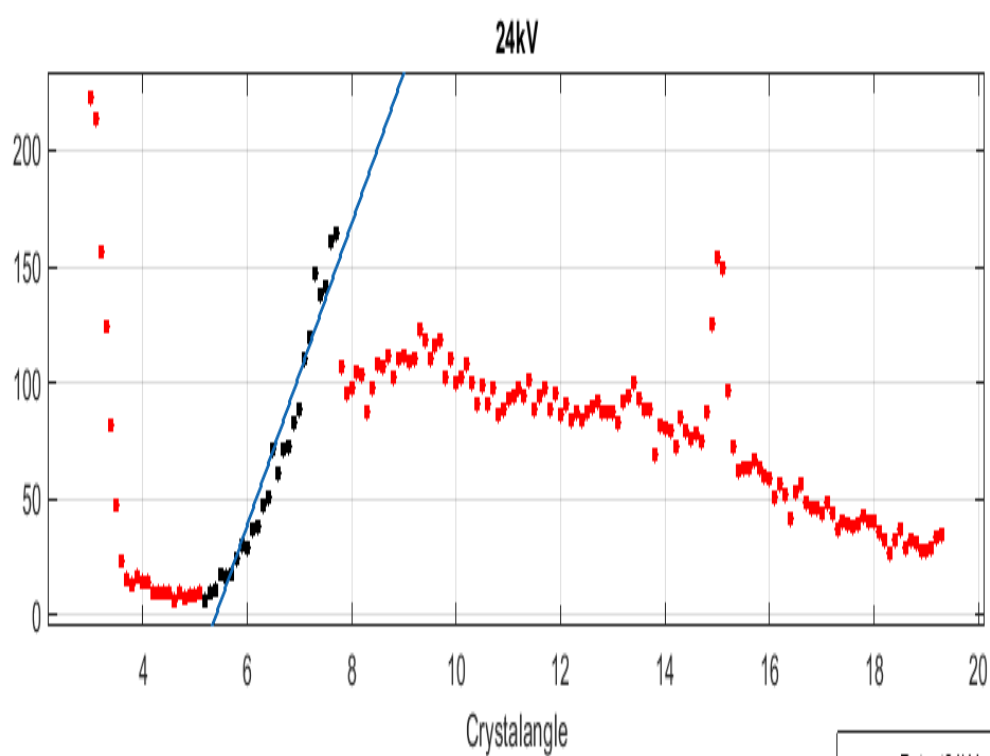
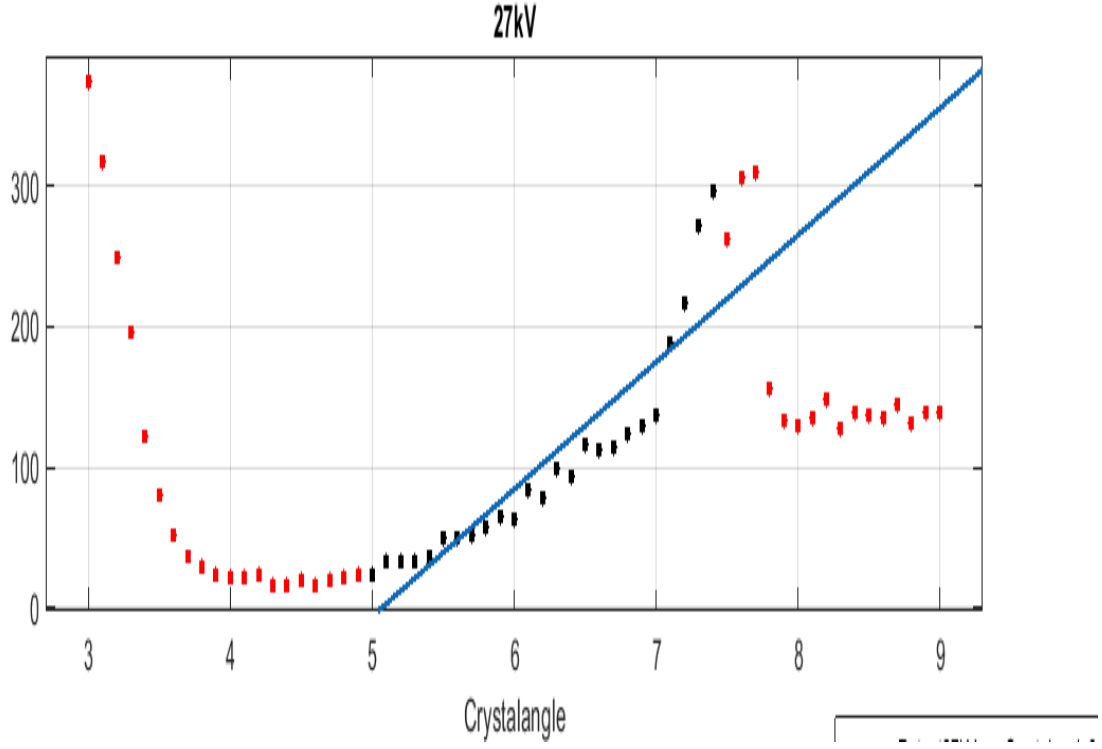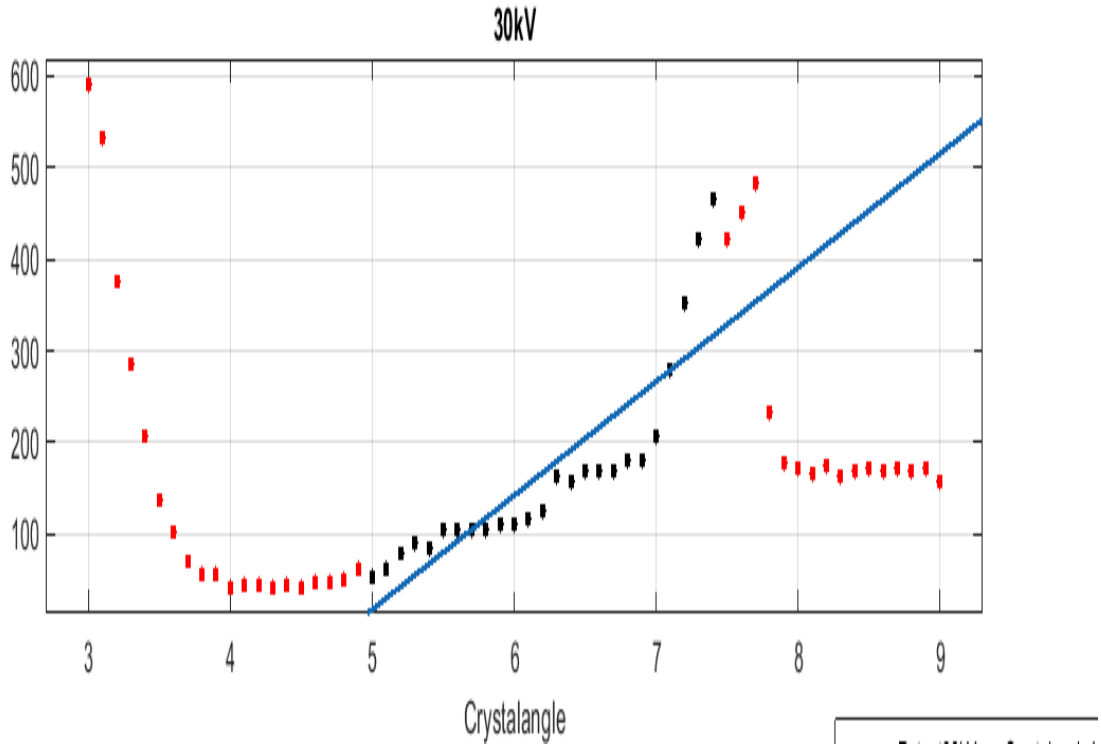Figure 4:

Figure 5:

Figure 6:

Figure 7:

27kV

30kV

In the Table 2 given below, we can see θ values with corresponding uncertainties taken from MATLAB.

Theta(degree)	$\sigma_{Theta}(rad)$
8.20	6.11E-04
6.90	5.23E-04
5.88	4.18E-04
5.26	5.41E-04
4.70	4.36E-04
4.65	4.53E-04

Table 2: Theta Experimental(degree) vs σ_{Theta}

Theta(degree)	$\sigma_{Theta}(rad)$
7.67	1.51E-04
6.28	1.48E-04
5.25	1.45E-04
4.62	1.43E-04
4.00	1.43E-04
4.00	1.43E-04

Table 3: Theta Calibrated(degree) vs $\sigma_{ThetaCalibrated}(rad)$

Then,putting values in the Table 2 into the calibration function and performing error propagation which is in the form of equation below

$$\sigma^2_{final} = \sum_{i=1}^{N} (\frac{\partial f}{\partial x_i})^2 \sigma^2_{x_i} \tag{5}$$

we can make θ values more accurate with corresponding uncertainties.As seen in the Table 3.

4.3 Converting Theta to Frequency

By combining Bragg's scattering law

$$2dsin(\theta) = m\lambda \tag{6}$$

and Duane-Hunt Displacement Law

$$E = h\nu = hc/\lambda \tag{7}$$

and

$$c = \nu\lambda \tag{8}$$

we get

$$\nu = cm/2dsin(\theta)Hz \tag{9}$$

where where n=1 and d=2.8*E-10 m(Atomic size of NaCl crystal.)

By using data given in the Table 3 and performing error propagation, we get the Table 4.

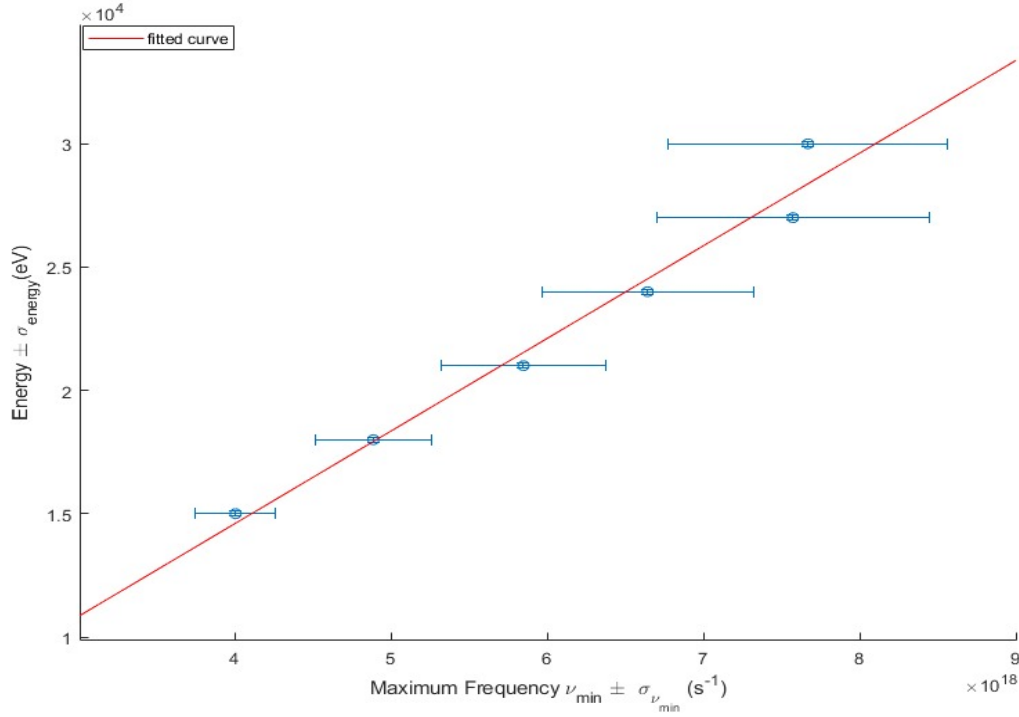| Frequency(Hz)	$\sigma_{Frequency}(Hz)$
4.01E+18	2.58E+17
4.89E+18	3.75E+17
5.85E+18	5.29E+17
6.64E+18	6.77E+17
7.57E+18	8.74E+17
7.67E+18	8.96E+17

Table 4: Frequency(Hz) vs $\sigma_{Frequency}(Hz)$

4.4 Planck Constant

$$E = h\nu \qquad (10)$$

As seen in the Equation 8 above, Planck Constant is the ratio of Energy and Frequency.

Figure 10:

Slope of the Figure 10 above gives the Planck Constant with uncertainty.Actual value of the Planck Constant is 4.14E-15 eV s.As seen in the Table 5, by performing this experiment we can get

$$h = 3.74E - 15eVs \qquad (11)$$

with

$$\sigma_{PlanckConstant} = 5.8E - 16eVs \qquad (12)$$

Planck Constant(10E-15 eV s)	$\sigma_{PlanckConstant}(10E-15eVs)$
3.76	0.58

Table 5: Planck Constant vs $\sigma_{PlanckConstant}$

5   Conclusion

In this experiment we calculate Planck constant as 3.76E-15 eV s with uncertainty of 0.58E-15 eV s. Actual value of the Planck constant is 4.14E-15 eV s. By using equation below, we calculate error of the experiment as %9.Also, we are 0.66σ away from the actual value.

$$\frac{\mid h_{actual} - h_{calculated} \mid}{\mid h_{actual} \mid} \cdot 100 = \%error \tag{13}$$

$$\frac{\mid 4.14 - 3.76 \mid}{\mid 4.14 \mid} \cdot 100 = \%9 \tag{14}$$

Also,the reason of that we do only see K-lines is absorption process.Additionally, analysis of the data needs line fitting and this fitting process needs human intervene. While taking line fitting which Matlab is used to do so, it is likely to perform some mistakes. This processes may cause to calculation to have some extra errors. .

6 References

[1],[2],[3],[4]:R. Eisberg and R. Resnick. Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles. John Wiley and Sons, Inc., 1985.

[5]: University of Sydney, "X-Rays" . https://myscope.training/legacy/xrd/ background/concepts/xrays.

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