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Photoelectric Effect & The Planck's Constant

Experiment 1

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1 Abstract

In this experiment, we send light beams of different frequency separately to the photocell and observe the current generated by the liberated electrons moving from cathode to anode. Then increase the stopping potential to balance electron's Kinetic energy. We found the corresponding stopping potential of the each light ray's frequency, after analyze all data we can find the Planck's constant by finding the electron charge divided by slope of the stopping potential vs frequency graph.
In conclusion we find the Planck's constant as $5.88x10^{-34}Js \pm 3.02x10^{-35}$ and its χ^2 is equal to 6.

2 Theory

The emission of electron from metal surface by incident light beam to the metal surface called photoelectric effect. The energies of electrons are liberated by light depend on the frequency of the light.

Photoelectric effect is firstly discovered by Heinrich Hertz in 1887. During his experiments on em waves, Hertz noticed that sparks occurred more readily in the air gap of his transmitter when ultraviolet light was directed at one of the metal balls. He did not follow up this observation, but others did[1].Moreover, Philip Lenard is the man who performed clear researches in order to photoelectric effect. Lenard was assistant of Hertz, he used metal surfaces held under a vacuum. Then, he observe the effect merely on the metal without any surface contaminants or oxidation[2]. A successful theory of the photoelectric effect was published in 1905 by Albert Einstein(his achievement was awarded with the Nobel Prize in 1921[3]). Five years earlier, in 1900, Max Planck had developed a theory of thermal radiation. His theory explain the wavelength distribution of light emitted by heated objects. Based partly on Planck's ideas, Einstein proposed that the energy of electromagnetic radiation is not continuously distributed over the wave front, but instead is concentrated in localized bundles or quanta (also known as photons). The energy of a photon associated with an electromagnetic wave of frequency ν is

$$E = h\nu \tag{1}$$

Einstein said that photoelectron is released as a result of an encounter with a single photon. The whole energy of the photon is delivered instantaneously to a single photoelectron. If the photon energy hν is greater than the work function W of the material, the photoelectron will be released. If the photon energy is smaller than the work function, the photoelectric effect will not occur. This explanation thus accounts for two of the failures of the wave theory: the existence of the cutoff frequency and the lack of any measurable time delay[4].

If the photon energy hν exceeds the work function, this exceeded energy will be equal to the kinetic energy of the electron:

$$K_{max} = h\nu - W \tag{2}$$

Robert A. Millikan found Einstein's hypothesis reckless. He thought that if experiment made properly, we could not photoelectric effect. Ironically, in 1916 Millikan has proved experimentally Einstein's studies and he measured Planck's constant with great accuracy.[5]According to Millikan's observation;

- There is a linear relation between stopping potential(V) and frequency(ν).
- $\frac{dV}{d\nu}$ is equal to h/e.
- At the critical frequency ν_0 at which V = 0, W = hν_0, in other words ; the intercept of the V vs ν_0 line on the ν_0 axis is the lowest frequency to liberate electron from metal.[6]

Today's experimental value of Planck constant is $6.626070040x10^{34}J \cdot s$ with $0.000000081x10^{34}J \cdot s$ standart uncertainty. [8]

To calculate Planck's constant via Millikan's method, these formula will be used.

$$h\nu = KE_{electron} + W \tag{3}$$

$$KE_{electron} = q_e V_s \tag{4}$$

$$V_s = \frac{1}{q_e}(h\nu - W) \tag{5}$$

or

$$h = \frac{q_e V_s + W}{\nu} \tag{6}$$

where W is work function which depend on metal, h is Planck's constant, V_s is stopping Potential(V),ν is frequency of light,$KE_{electron}$ is maximum kinetic energy of electron and q_e is charge of an electron.

$$q_e = 1.6021766208x10^{-19}C \pm 0.0000000098x10^{-19}C[8] \tag{7}$$

Thanks to all of these scientists, now we know light is both particle and wave. We will try to find Planck's constant with same setup which Millikan used

3 Experiment

3.1 Apparatus

- High pressure mercury lamp with power supply
- Spectrography with a transmission grating
- Photocell with housing
- Current Amplifier
- Ampermeter (0-300 μA)
- Power Supply (0-3 V)
- DC Voltmeter (0-3 V) (Thurlby 1503 digital multimeter)
- Moving coil DC Voltmeter for the current amplifier (0-3 V)
- Connecting Leads

3.2 Setup

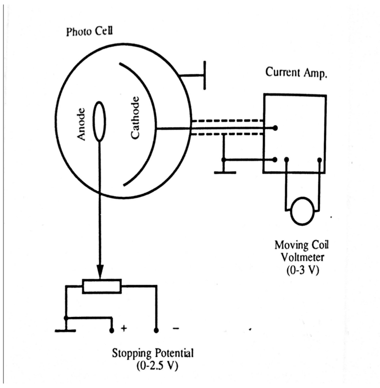

Figure 1: Schematic diagram of the setup [7]

3.3 Procedure

1. Connect the circuit given in Figure 1.
2. Turn on the power supplies. Wait until the mercury lamp gets warm. Then,turn off the room light because photocell should get photon only from mercury lamp via Spectrograph.
3. Check the alignment of the Spectrograph, grating, mercury lamp and the photocell. Make sure that the opening in front of the photocell casing will allow the light to fall only on the cathode and not on any part of the anode. Light falling on the anode may cause a reverse current to flow.
4. Adjust the orientation of the grating so that a desired light beam falls on the photocell. Adjust the scale of the current amplifier and the Voltmeter connected to the output of the amplifier to read the current comfortably. Give voltage to the photocell from power supply. Then, begin from zero retarding potential if it is possible.
5. Slowly increase the retarding potential; then, it cause decrease in the current. Read voltages from the Thurlby 1503 digital multimeter. Do not increase the retarding potential over 2.5 V. It is adequate that measure 2-3 negative current
6. Repeat these steps for other colors in mercury spectrum. Do not take data for red. Take data from Colors which are yellow, green, turquoise, blue, and violet.
7. Plot photocell current vs retarding potential graph for every color with error bars. Calculate retarding potential for every color with uncertainty.
8. Plot retarding potential vs frequency graph for every color that you take data; then measure the Planck's constant.

4 Raw Data

4.1 Raw Data Tables

YELLOW			
Voltage(V)	$Sigma_{Voltage}(V)$	Current(A)	$Sigma_{Current}(A)$
0,050	0,001	1,6 e-13	0,2 e-13
0,100	0,001	1,2 e-13	0,2 e-13
0,150	0,001	0,8 e-13	0,2 e-13
0,200	0,001	0,6 e-13	0,2 e-13
0,250	0,001	0,4 e-13	0,2 e-13
0,300	0,001	2,6 e-14	0,2 e-14
0,350	0,001	1,4 e-14	0,2 e-14
0,400	0,001	0,2 e-14	0,2 e-14
0,450	0,001	-0,6 e-14	0,2 e-14
0,500	0,001	-0,2 e-13	0,2 e-13
0,550	0,001	-0,2 e-13	0,2 e-13

Table 1: Yellow Table

GREEN			
Voltage(V)	$Sigma_{Voltage}(V)$	Current(A)	$Sigma_{Current}(A)$
0,050	0,001	0,6 e-12	0,2 e-12
0,100	0,001	0,4 e-12	0,2 e-12
0,150	0,001	0,2 e-12	0,2 e-12
0,200	0,001	2,4 e-13	0,2 e-13
0,250	0,001	1,8 e-13	0,2 e-13
0,300	0,001	1,2 e-13	0,2 e-13
0,350	0,001	0,6 e-13	0,2 e-13
0,400	0,001	0,4 e-13	0,2 e-13
0,450	0,001	1,6 e-14	0,2 e-14
0,500	0,001	-0,4 e-14	0,2 e-14
0,550	0,001	-0,2 e-13	0,2 e-13
0,600	0,001	-0,4 e-13	0,2 e-13

Table 2: Green Table

TURQUOISE			
Voltage(V)	$Sigma_{Voltage}(V)$	Current(A)	$Sigma_{Current}(A)$
0,050	0,001	0,8 e-13	0,2 e-13
0,100	0,001	0,8 e-13	0,2 e-13
0,150	0,001	0,6 e-13	0,2 e-13
0,200	0,001	0,6 e-13	0,2 e-13
0,250	0,001	0,4 e-13	0,2 e-13
0,300	0,001	0,4 e-13	0,2 e-13
0,350	0,001	3,0 e-14	0,2 e-14
0,400	0,001	2,2 e-14	0,2 e-14
0,450	0,001	1,6 e-14	0,2 e-14
0,500	0,001	1,2 e-14	0,2 e-14
0,550	0,001	0,8 e-14	0,2 e-14
0,600	0,001	0,6 e-14	0,2 e-14
0,650	0,001	3,0 e-15	0,2 e-15
0,700	0,001	1,0 e-15	0,2 e-15
0,750	0,001	-0,6 e-15	0,2 e-15
0,800	0,001	-0,2 e-14	0,2 e-14

Table 3: Turquoise Table

BLUE			
Voltage(V)	$Sigma_{Voltage}(V)$	Current(A)	$Sigma_{Current}(A)$
0,050	0,001	2,8 e-12	0,2 e-12
0,100	0,001	2,6 e-12	0,2 e-12
0,150	0,001	2,4 e-12	0,2 e-12
0,200	0,001	2,0 e-12	0,2 e-12
0,250	0,001	1,8 e-12	0,2 e-12
0,300	0,001	1,6 e-12	0,2 e-12
0,350	0,001	1,4 e-12	0,2 e-12
0,400	0,001	1,2 e-12	0,2 e-12
0,450	0,001	1,0 e-12	0,2 e-12
0,500	0,001	0,8 e-12	0,2 e-12
0,550	0,001	0,6 e-12	0,2 e-12
0,600	0,001	0,6 e-12	0,2 e-12
0,650	0,001	0,4 e-12	0,2 e-12
0,700	0,001	0,4 e-12	0,2 e-12
0,750	0,001	2,6 e-13	0,2 e-13
0,800	0,001	1,8 e-13	0,2 e-13
0.850	0,001	1,2 e-13	0,2 e-13
0,900	0,001	0,8 e-13	0,2 e-13
0,950	0,001	0,4 e-13	0,2 e-13
1,000	0,001	1,0 e-14	0,2 e-14
1,050	0,001	-0,6 e-14	0,2 e-14
1,100	0,001	-0,2 e-13	0,2 e-13
1,150	0,001	-0,4 e-13	0,2 e-13

Table 4: Blue Table

VIOLET			
Voltage(V)	$Sigma_{Voltage}(V)$	Current(C)	$Sigma_{Current}(A)$
0,050	0,001	0,8 e-12	0,2 e-12
0,100	0,001	0,6 e-12	0,2 e-12
0,150	0,001	0,6 e-12	0,2 e-12
0,200	0,001	0,6 e-12	0,2 e-12
0,250	0,001	0,6 e-12	0,2 e-12
0,300	0,001	0,6 e-12	0,2 e-12
0,350	0,001	0,4 e-12	0,2 e-12
0,400	0,001	0,4 e-12	0,2 e-12
0,450	0,001	0,4 e-12	0,2 e-12
0,500	0,001	0,2 e-12	0,2 e-12
0,550	0,001	2,6 e-13	0,2 e-13
0,600	0,001	2,4 e-13	0,2 e-13
0,650	0,001	2,0 e-13	0,2 e-13
0,700	0,001	1,8 e-13	0,2 e-13
0,750	0,001	1,4 e-13	0,2 e-13
0,800	0,001	1,2 e-13	0,2 e-13
0,850	0,001	1,0 e-13	0,2 e-13
0,900	0,001	0,6 e-13	0,2 e-13
0,950	0,001	0,4 e-13	0,2 e-13
1,000	0,001	0,4 e-13	0,2 e-13
1,050	0,001	2,2 e-14	0,2 e-14
1,100	0,001	1,2 e-14	0,2 e-14
1,150	0,001	0,4 e-14	0,2 e-14
1,200	0,001	-0,2 e-14	0,2 e-14
1,250	0,001	-0,6 e-14	0,2 e-14

Table 5: Violet Table

5 Data Analysis

To analyze the data; first,we plotted photocell current vs retarding potential graph. Positive current and negative current fitted separately with considering their uncertainty. Fitting made by orthogonal distance regression Then intersection of two line is calculated and also uncertainty of intersection point calculated with error propagation. We implement Proper Error propagation formula to our code. All calculation made in Python.

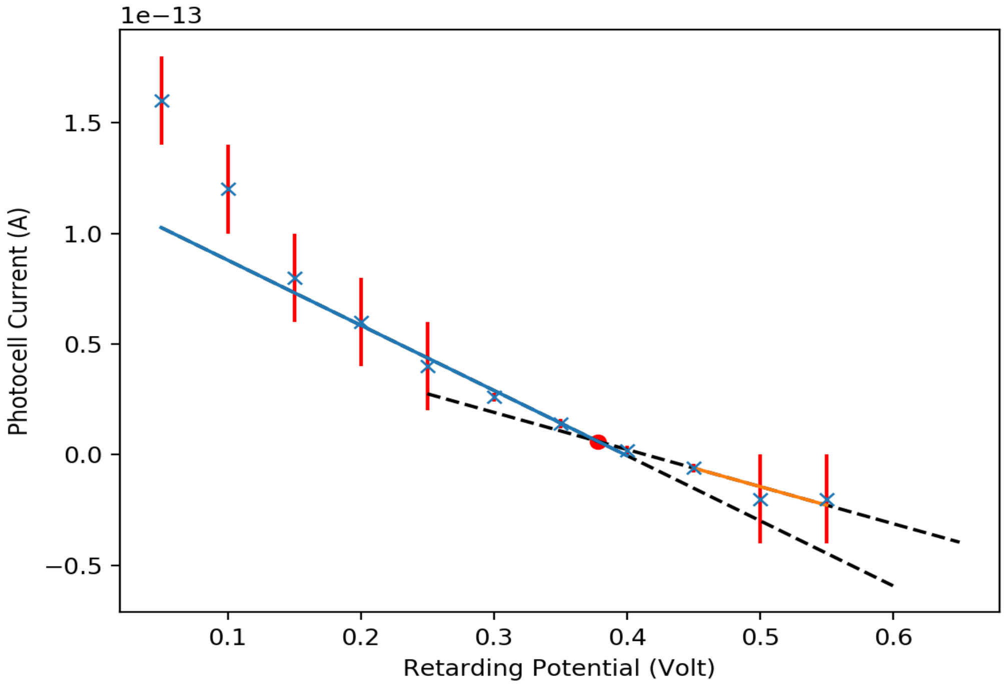

Figure 2: Yellow - Photocell Current vs Retarding Potential

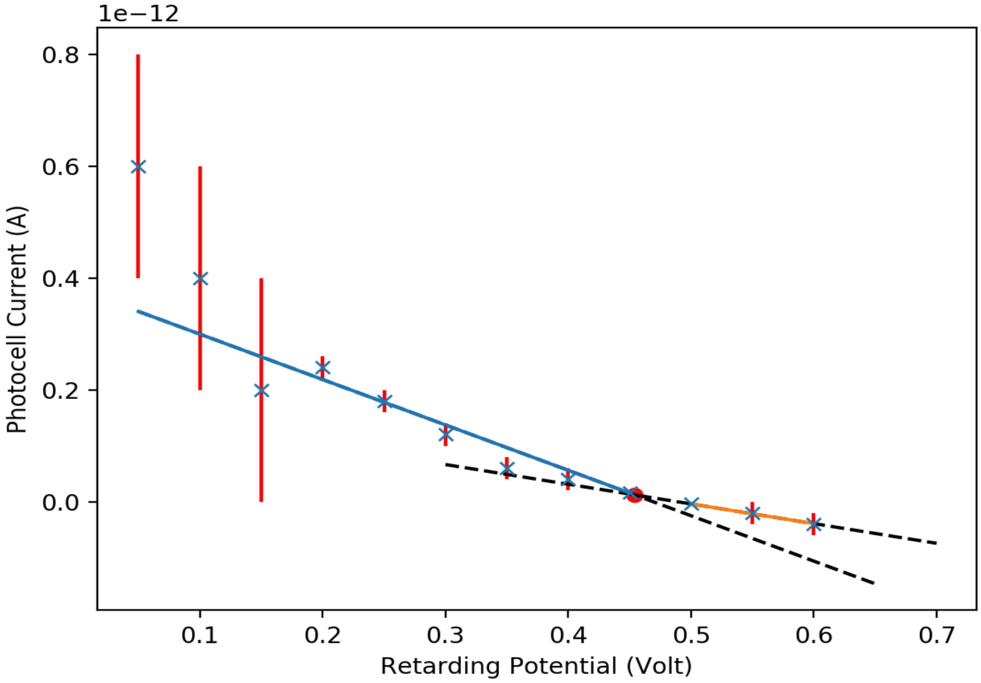

Figure 3: Green - Photocell Current vs Retarding Potential

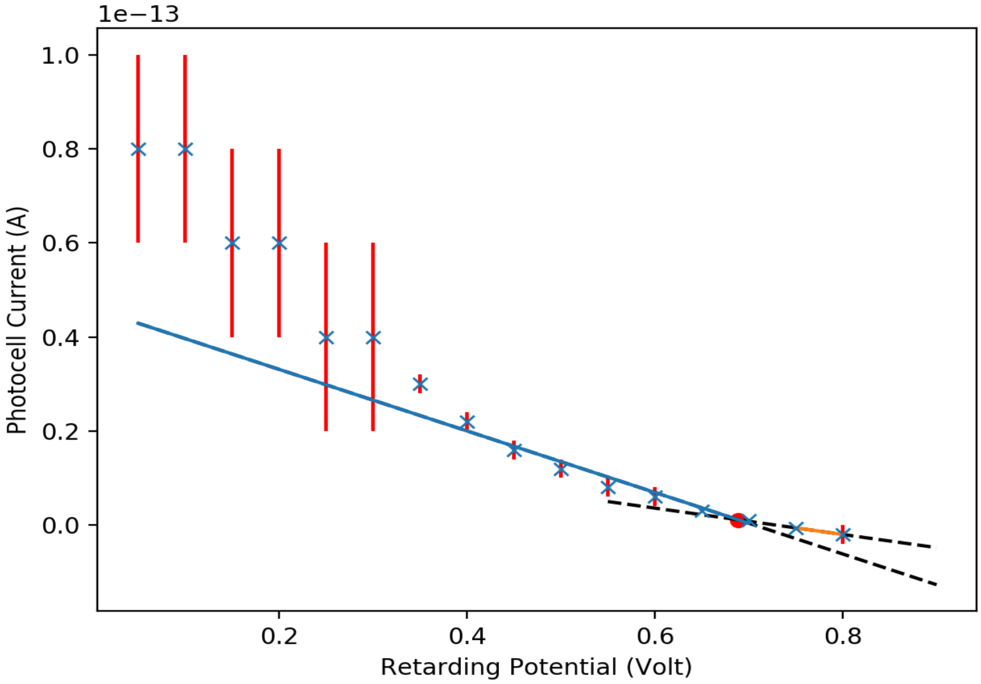

Figure 4: Turquoise - Photocell Current vs Retarding Potential

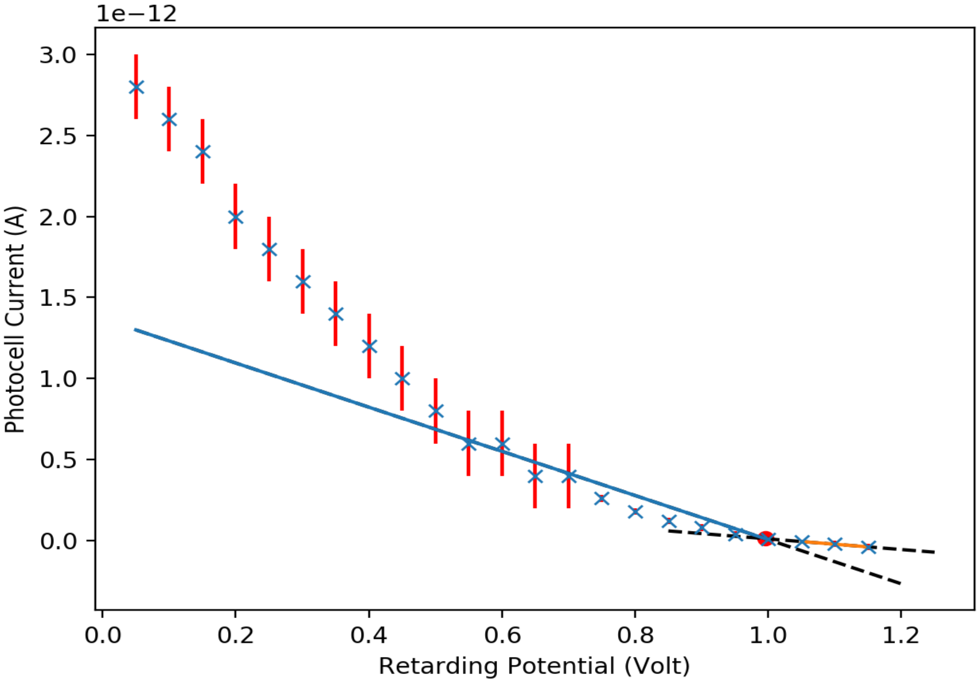

Figure 5: Blue - Photocell Current vs Retarding Potential

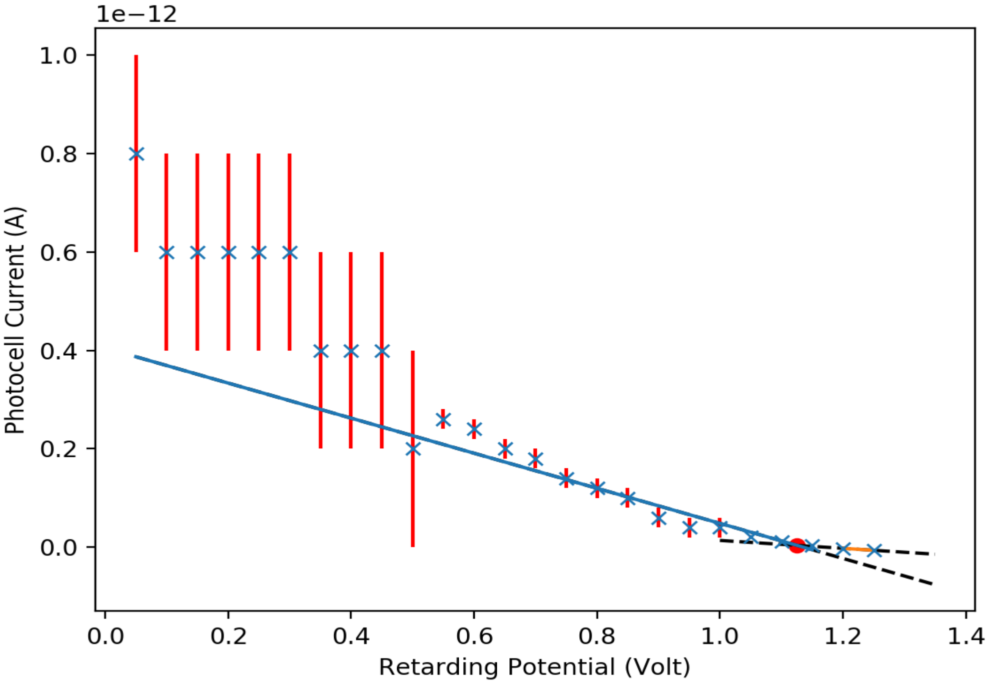

Figure 6: Violet - Photocell Current vs Retarding Potential

Reason of finding the intersection points is that point represents the balance between forward and reverse currents. The method employed for obtaining the maximum energy of emission in volts was not to attempt to locate experimentally just that retarding potential which would prevent the escape of electrons from metal but rather to plot the photocurrent-potential curve in the neighborhood of the balance point and then to find interception point's voltage.[6]
Calculation of Retarding potential(V) and its uncertainty formulas from the graph;

$$V_{retardingPotential} = \frac{c_n - c_p}{m_p - m_n} \tag{8}$$

$$\sigma_V = \sqrt{\left(\frac{\sigma_{c_p}}{m_p - m_n}\right)^2 + \left(\frac{\sigma_{c_n}}{m_p - m_n}\right)^2 + \left(\frac{(c_n - c_p)\sigma_{m_p}}{(m_p - m_n)^2}\right)^2 + \left(\frac{(c_n - c_p)\sigma_{m_n}}{(m_p - m_n)^2}\right)^2} \tag{9}$$

Where m_p is slope of positive current line, m_n is slope of negative current line, c_p is intersection point of y-axis of positive current line, and c_n is intersection point of y-axis of negative current line.

NOTE 1: There is negative correlation between uncertainty of Photocell current and uncertainty of Retarding potential(V). That will decrease the uncertainty. However, we don't know correlation between them and we couldn't imply it to error propagation formula.

NOTE 2: Error propagation formula work well when the uncertainty is much less then the mean value. In our data, uncertainty of current very close to mean value, even it is same in some value. But we don't have any chance to use other methods. we keep continue using error propagation formula.

Color	$V_s(V)$	$\sigma_V(V)$
Yellow	0.378	0.303
Green	0.454	0.084
Turquoise	0.689	0.156
Blue	0.995	0.249
Violet	1.126	0.158

Table 6: Calculated Stopping potential with uncertainties for each color in proper significant figure

Color of The Light Beam	**Frequency (Hz)**
Yellow	$5.19x10^{14}$
Green	$5.56x10^{14}$
Turquoise	$6.08x10^{14}$
Blue	$6.88x10^{14}$
Violet	$7.41x10^{14}$

Table 7: Photon's Frequency of specific colors[4]. Since their uncertainty are not given. Their uncertainty taken as $0.01x10^{14}$ [7]

The graph in Figure 7 is plotted frequency vs retarding potential and their errors. The line fitted by considering their uncertainties. When the charge of an electron divided by slope of the graph, that will give us Planck's constant.

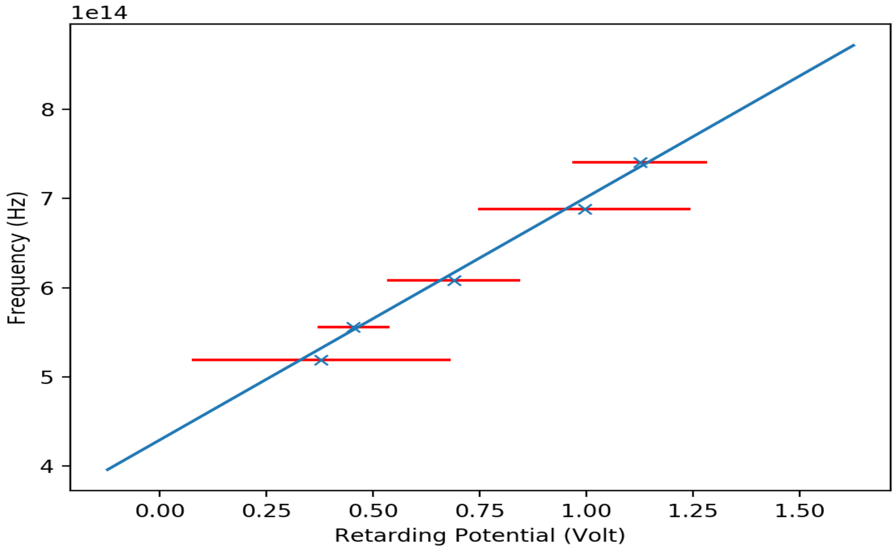

Figure 7: Frequency vs Retarding Potential

$$h = \frac{q_e}{m} \tag{10}$$

Uncertainty of Planck's constant by error propogation formula;

$$\sigma_h = \sqrt{\left(\frac{\sigma_{q_e}}{m}\right)^2 + \left(\frac{q_e \sigma_m}{m^2}\right)^2} \tag{11}$$

Where m is slope of graph in Figure 7, σ_m is uncertainty of slope, q_e is electron charge in Coulomb, and σ_{q_e} is its uncertainty

$$q_e = 1.6021766208x10^{-19}C \quad \& \quad \sigma_{q_e} = 0.0000000098x10^{-19}C[8] \tag{12}$$

σ_{q_e} is very small when we compare it to other uncertainty value. So it has not too much effect to our uncertainty of Planck's constant, but it used in anyway. After whole analyze, we found;

$$h_{measured} = 5.88x10^{-34}Js \tag{13}$$

$$\sigma_h = 3.02x10^{-35}Js \tag{14}$$

In Figure 7, the line intersect the positive y-axis(Frequency) and negative x-axis(Retarding potential). If we multiply absolute value of intersection point of x-axis with charge of electron we will get the work function(W).

$$\nu(V) = mV + c \quad (Equation\ of\ the\ line\ in\ Figure\ 7) \tag{15}$$

where m is slope and c is minus of intersection point of x-axis
In x-axis intersection point $\nu = 0$.Then,

$$V_0 = \frac{-c}{m} \tag{16}$$

We will combine equation 5 and 16. Then leave Work function alone in the left hand side of the equation;

$$W = -q_e V_0 \quad => \quad W = \frac{q_e c}{m} \tag{17}$$

Uncertainty of work function by error propagation;

$$\sigma_W = \sqrt{(\frac{c\sigma_{q_e}}{m})^2 + (\frac{q_e\sigma_c}{m})^2 + (\frac{cq_e\sigma_m}{m^2})^2} \tag{18}$$

$$W_{measured} = 2.52x10^{-19}J \tag{19}$$

$$\sigma_W = 9.98x10^{-21}J \tag{20}$$

All uncertainty calculation via error propagation is made with Python code.

6 Conclusion

This is the χ^2 of Planck's constant measurement which we found

$$\chi^2 = \left(\frac{5.88x10^{-34} - 6.62x10^{-34}}{3.02x10^{-35}}\right)^2 = 6 \tag{21}$$

And this χ^2 of Planck's constant measurement was found by Millikan in 1916.he title that he thought his measured uncertainty of Planck's constant is no more than $\frac{1}{3}$ percent[6]. So we can took it roughly $2x10^{-34}$

$$\chi^2 = \left(\frac{6.57x10^{-34} - 6.62x10^{-34}}{2x10^{-34}}\right)^2 = 6x10^{-4} \tag{22}$$

- If we found χ^2 between 1 and 2. I can say this is very good measurement. the number of 6 say us, we don't have good accuracy. Our uncertainty is 5% percent, quite good value. The reason Millikan found χ^2 very small is that in this era devices are not so precise(high uncertainty).He found Planck's constant in good accuracy but not good uncertainty.
- While uncertainty data measured by Voltmeter are between 0.1% and 1%, however; our uncertainty of Planck's constant is 5% since uncertainty of data measured by ampermeter is various between 100% and 6, 7%. Using better ampermeter will decrease uncertainty.It might increase our accuracy as well. We took data in negative current just 2 or 3. This cause missing the accurate value of Planck's constant. More negative current data may have taken.
- Other possible source of error is environmental factor. We didn't made our experiment in absolutely dark room because we also need a light to read data. Leaked light to photocell house can affect significantly our data we took.
- Even we split the mercury lamp's light with Spectrography, it is impossible to separate it absolutely. It is possible that other colours leak to the desired colour. Then, this affect photocell current.
- When we look the fitted retarding potential vs photocell current graph all data look like in proper position. We didn't delete any data while fitting the line.
- We used orthogonal distance regression in fitting the graph. since our graph is linear, orthogonal distance regression give exactly same function of the best line with least square method. In other word orthogonal distance regression is least square calculation of the co-secant of the errors.

References

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[9] line fitting with orthogonal distance regression in data analysis (scipy.odr)
https://docs.scipy.org/doc/scipy/reference/odr.html

Appendices

```
##CODE FOR THE CURRENT vs RETARDING POTENTIAL GRAPH PLOTTING AND FITTING#
####INTERSECITON POINT####
###ERROR PROPAGATION####

import numpy as np
import matplotlib.pyplot as plt
from scipy.odr import *
from scipy.stats import linregress
from scipy.optimize import fsolve
import random

#Violet
v=[0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45,0.5,0.55,0.6,0.65,
    0.7,0.75,0.8,0.85,0.9,0.95,1.0,1.05,1.1,1.15,1.2,1.25]
i=[0.8e-12,0.6e-12,0.6e-12,0.6e-12,0.6e-12,0.6e-12,0.4e-12,0.4e-12,
    0.4e-12,0.2e-12,2.6e-13,2.4e-13,2.0e-13,1.8e-13,1.4e-13,
    1.2e-13,1.0e-13,0.6e-13,0.4e-13,0.4e-13,2.2e-14,1.2e-14,
    0.4e-14,-0.2e-14,-0.6e-14]
ve=[0.001, 0.001, 0.001, 0.001, 0.001, 0.001, 0.001, 0.001, 0.001,
    0.001,0.001, 0.001, 0.001, 0.001,0.001, 0.001, 0.001,0.001, 0.001,
    0.001, 0.001,0.001, 0.001,0.001, 0.001]
ie=[0.2e-12,0.2e-12,0.2e-12,0.2e-12,0.2e-12,0.2e-12,0.2e-12,0.2e-12,
    0.2e-12,0.2e-12,0.2e-13,0.2e-13,0.2e-13,0.2e-13,0.2e-13,0.2e-13,
    0.2e-13,0.2e-13,0.2e-13,0.2e-13,0.2e-14,0.2e-14,0.2e-14,0.2e-14,0.2e-14]
```

```
mid=23 #it divide positive and negatvie current for incoming codes


# Define the function you want to fit against.:
def fit_func(p, x):
    m, c = p
    return m*x + c

# Create a Model.:
linear = Model(fit_func)

# Create a RealData object using our initiated data from above.
data1 = RealData(v[:mid], i[:mid], sx=ve[:mid], sy=ie[:mid])
data2 = RealData(v[mid:], i[mid:], sx=ve[mid:], sy=ie[mid:])

# Set up ODR with the model and data.
# ODR: Orthogonal distance regression
#make least square of orthogonal distance to best line

odr1 = ODR(data1, linear, beta0=[0., 1.])
odr2 = ODR(data2, linear, beta0=[0., 1.])

# Run the regression.
out1 = odr1.run()
out2 = odr2.run()

# Use the in-built pprint method to give us results.
out1.pprint()
out2.pprint()

xp_fit = np.linspace(v[0], v[mid-1], 1000)
xp_fitd = np.linspace(v[0], v[mid-1]+0.2, 1000)

xn_fit = np.linspace(v[mid], v[-1], 1000)
xn_fitd = np.linspace(v[mid]-0.2, v[-1]+0.1, 1000)

yp_fit = fit_func(out1.beta, xp_fit)
yp_fitd = fit_func(out1.beta, xp_fitd)

yn_fit = fit_func(out2.beta, xn_fit)
yn_fitd = fit_func(out2.beta, xn_fitd)

intr = (out2.beta[1]-out1.beta[1])/(out1.beta[0]-out2.beta[0])

plt.errorbar(v, i, xerr=ve, yerr=ie, linestyle='None', marker='x',ecolor='r')

plt.xlabel('Retarding Potential (Volt)')
plt.ylabel('Photocell Current (A)')
plt.plot(xp_fitd, yp_fitd,'r--',color='black',)
plt.plot(xn_fitd, yn_fitd,'r--',color='black')
plt.plot(xp_fit, yp_fit)
```

```
plt.plot(xn_fit, yn_fit)
plt.scatter(intr, intr*out1.beta[0]+out1.beta[1],color='red')
plt.show()

#interception of two line
print('V:',(out2.beta[1]-out1.beta[1])/(out1.beta[0]-out2.beta[0]))
#sigma of interception
print('sigma V:',np.sqrt((out2.sd_beta[1]/(out1.beta[0]-out2.beta[0]))**2+
    (out1.sd_beta[1]/(out1.beta[0]-out2.beta[0]))**2+(((out2.beta[1]-
    out1.beta[1])*out1.sd_beta[0])/((out1.beta[0]-out2.beta[0])**2))**2+
    (((out2.beta[1]-out1.beta[1])*out2.sd_beta[0])/
    (out1.beta[0]-out2.beta[0])**2)**2))

########################OUTPUT######################
Beta: [-3.56036495e-13  4.04697477e-13]
Beta Std Error: [2.78180312e-14 3.03675841e-14]
Beta Covariance: [[ 2.26538797e-28 -2.46691667e-28]
 [-2.46691667e-28  2.69966761e-28]]
Residual Variance: 3.4159396412567995
Inverse Condition #: 0.00695567331870224
Reason(s) for Halting:
  Sum of squares convergence
Beta: [-8.0e-14  9.4e-14]
Beta Std Error: [4.33572182e-28 5.31236523e-28]
Beta Covariance: [[ 3.20509269e-27 -3.92623859e-27]
 [-3.92623859e-27  4.81164552e-27]]
Residual Variance: 5.865191899384127e-29
Inverse Condition #: 0.002232130240262949
Reason(s) for Halting:
  Parameter convergence

#####GRAPH#####

V: 1.1255666680230234
sigma V: 0.1580170824052551

#yel = [0.3782899257855594,0.3030606347947359]
#green =[0.45410066856005177,0.08391985168630223]
#turq = [0.6890779794066331,0.15647378577495694]
#blue=[0.995455284322239,0.24945242343145158]
#violet =[1.1255666680230234,0.1580170824052551]

###################################################################

#CODE FOR THE FREQUENCY vs RETARDING POTENTIAL GRAPH PLOTTING AND FITTING#
####PLANCK CONSTANT AND WORK FUNCTION CALCULATION####
###ERROR PROPAGATION####

import numpy as np
import matplotlib.pyplot as plt
from scipy.odr import *
from scipy.stats import linregress
```

```
from scipy.optimize import fsolve
import random

v=[0.3782899257855594,0.45410066856005177,0.6890779794066331,
    0.995455284322239,1.1255666680230234]
ve=[0.3030606347947359,0.08391985168630223,0.15647378577495694,
    0.24945242343145158,0.1580170824052551]
f=[5.19e14,5.56e14,6.08e14,6.88e14,7.41e14]
fe=[0.01e14,0.01e14,0.01e14,0.01e14,0.01e14]

def fit_func(p, x):
    m ,c= p
    return m*x+c

linear = Model(fit_func)
data = RealData(v, f, sx=ve, sy=fe)
odr= ODR(data, linear, beta0=[0., 1.])
out = odr.run()
out.pprint()

x_fit = np.linspace(v[0]-0.5, v[-1]+0.5, 1000)
y_fit = fit_func(out.beta, x_fit)

plt.errorbar(v, f, xerr=ve, yerr=fe, linestyle='None', marker='x',ecolor='r')
plt.xlabel('Retarding Potential (Volt)')
plt.ylabel('Frequency (Hz)')
plt.plot(x_fit, y_fit)
plt.show()


print("h:",1.6021766208e-19/out.beta[0])
print("sigma h:",np.sqrt((0.0000000098e-19/out.beta[0])**2+
    ((1.60217662e-19*out.sd_beta[0])/out.beta[0]**2)**2))
print('W:',(1.6021766208e-19*out.beta[1])/out.beta[0])
print("sigma W:",np.sqrt(((out.beta[1]*0.0000000098e-19)/out.beta[0])**2+
    ((out.sd_beta[1]*1.6021766208e-19)/out.beta[0])**2+
    ((1.60217662e-19*out.beta[0]*out.sd_beta[0])/out.beta[0]**2)**2))

######################OUTPUT######################
Beta: [2.72684689e+14 4.28785339e+14]
Beta Std Error: [1.40160965e+13 9.58362484e+12]
Beta Covariance: [[ 4.35901921e+27 -2.75399968e+27]
 [-2.75399968e+27  2.03795333e+27]]
Residual Variance: 0.045067698014414966
Inverse Condition #: 0.025182089632486245
Reason(s) for Halting:
  Sum of squares convergence

#####GRAPH#####

h: 5.8755650178089235e-34
sigma h: 3.020062716228129e-35
```

```
W:  2.519356139755606e-19
 sigma W:  9.976301534670817e-21
```