Boğaziçi University
Physics Department

The Cavendish Experiment

Experiment 4

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Contents

1 Abstract

In this experiment, we measured the angular displacement of a torsional pendulum caused by two big masses near to it which exerts gravitational force. Then, we measured frequency of oscillation by analyzing data graph and found torsion constant. By this constant, we found the torque caused by the two big masses disturbing the system, then it gave us force exert on pendulum. From this force we measured the gravitational constant as $3.5 \times 10^{-10} N.m^2.kg^{-2}$ with $0.4 \times 10^{-10} N.m^2.kg^{-2}$ and its χ^2 is equal to 50.

2 Introduction

Newton had published his law of gravitation in 1687. However, he didn't make any attempt to determine the constant G or the mass of Earth. By the 1700s, astronomers wanted to know the density of Earth, as it would make it possible to determine density of the other planets.[1] Hence, the calculation of the mass of the earth and G constant have a important role in science history.

2.1 History

The Cavendish Experiment, was one of his most notable experiments. Cavendish performed the experiment in 1797-1798. The Cavendish Experiment was the first experiment to measure the force between masses in the laboratory. Moreover, the first experiment to produce definitive values for the gravitational constant and the mass density of the Earth. The experiment was originally conceived by John Michell before 1783, however in 1793 Michell died before completing his work. Soon thereafter, Cavendish was given Michell's apparatus for the experiment, which he then re-constructed his own model, while keeping major components similar to Michell's original plan. The results of the Cavendish Experiment was the mass density of the earth, yet others were able to derive the actual value of the gravitational constant from the experiments results. The Cavendish Experiment's purpose is frequently misunderstood to think its goal was to determine the gravitational constant(G). When in fact, Cavendish's only goal was to measure the mass density of the Earth. The gravitational constant does not appear in Cavendish's published paper on the topic, nor is there any indication that he regarded it as a goal of this experiment. Nearly 100 years later when G was first measure in a laboratory, they realized that Cavendish had obtained a value of G that was accurate to 1%.
Cavendish found that the Earth's density was 5.448 0.033 times that of water (due to a simple arithmetic error, found in 1821 by F. Baily, the erroneous value 5.48 0.038 appears in his paper).

Cavendish's equipment was remarkably sensitive for its time. The force involved in twisting the torsion balance was very small, 1.47 x 10 –7 N, about 1/50,000,000 of the weight of the small balls or roughly the weight of a large grain of sand. To prevent air currents and temperature changes from interfering with the measurements, Cavendish placed the entire apparatus in a wooden box about thick, tall, and wide, all in a closed shed on his estate. Through two holes in the walls of the shed, Cavendish used telescopes to observe the movement of the torsion balance's horizontal rod. The motion of the rod was only about 0.16 inch. Cavendish was able to measure this small deflection to an accuracy of better than one hundredth of an inch using vernier scales on the ends of the rod.[2]

3 Theory[7]

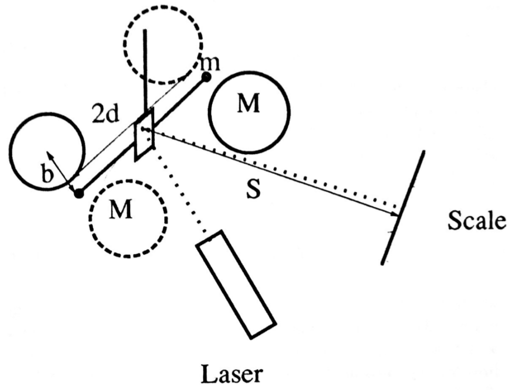

Figure 1: [3]

In Cavendish experiment, two small masses hanging on torsion pendulum in opposite side. The force exerted by heavier mass on the small mass create torque on pendulum. And the torque is equal to;

$$\tau = 2Fd = \kappa\theta_{eq} \tag{1}$$

where d is distance between center of mass and small mass.κ is torsion constant. θ_{eq} is change in the angle. F is equal to;

$$F = G\frac{Mm}{r^2} \tag{2}$$

$$2G\frac{Mm}{r^2}d = \kappa\theta_{eq} \tag{3}$$

$$G = \frac{\kappa\theta_{eq}r^2}{Mm2d} \tag{4}$$

to find κ;

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} = -\kappa\theta \tag{5}$$

Assume that it is an ideal system which has no friction this motion;

$$I\frac{d^2\theta}{dt^2} = -\kappa\theta \tag{6}$$

$$I\frac{d^2\theta}{dt^2} + \kappa\theta = 0 \tag{7}$$

Then the equation for θ is;

$$\theta = A\sin(w_0 t) \tag{8}$$

and

$$w_0 = \sqrt{\frac{\kappa}{I}} \tag{9}$$

and κ can be calculated as

$$\kappa = \omega_0^2 I \tag{10}$$

to find θ in the equation (5), we will use following method;

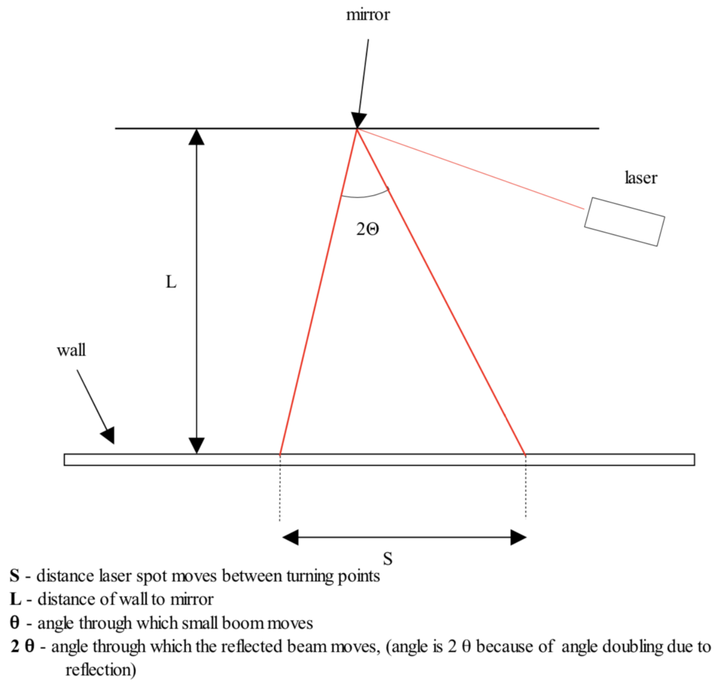

Figure 2: [5]

$$\Delta\theta = \alpha\Delta \tag{11}$$

$$dl = \frac{\Delta S_{weighted}}{2} \tag{12}$$

$$\Delta\theta = tan^{-1}(\frac{dl}{L}) \tag{13}$$

Where L is distance between light source and torsion pendulum. Conversion factor to convert Voltage to angle is

$$\frac{\Delta\theta}{\Delta V} \tag{14}$$

$$\theta_{eq} = V_{eq}\frac{\Delta\theta}{\Delta V} \tag{15}$$

$$I\frac{d^2\theta}{dt^2} + \kappa\theta + b\frac{d\theta}{dt} = 0 \tag{16}$$

and solution for $theta$ will be

$$\theta = Ae^{-\beta t}\sin(\omega_d t + \phi) + C_1 \tag{17}$$

where

$$\beta = \frac{b}{2l} \tag{18}$$

In the end of experiment we try to observe damping case by stopping the position of the masses.Result of this stop energy and amplitude started to decrease.

$$\omega_d = \sqrt{\omega_0^2 - \beta^2} \tag{19}$$

$$\omega_0 = \sqrt{\omega_d^2 + \beta^2} \tag{20}$$

Now, we can calculate G constant from;

$$G = \frac{\kappa \theta_{eq} r^2}{MmL} \tag{21}$$

where $L = 2d$
Once G has been found, the attraction of an object at the Earth's surface to the Earth itself can be used to calculate the Earth's mass and density;

$$mg = \frac{GmM_{Earth}}{R_{Earth}^2} \tag{22}$$

$$M_{Earth} = \frac{gR_{Earth}^2}{G} \tag{23}$$

$$\rho_{Earth} = \frac{M_{Earth}}{\frac{4}{3}\pi R_{Earth}^3} = \frac{3g}{4\pi R_{Earth} G} \tag{24}$$

4 Experiment

4.1 Apparatus[3]

1. **Low Power Laser**: To calculate $\Delta\theta$ rod of small masses turn.
2. **Scale** : Measuring the distance laser point travel.
3. **Cavendish Torsion Balance**: It is the most important part of the experiment. it include small and heavy masses, torsion pendulum etc.
4. **Ruler**: 2 meter ruler with 0.01 m uncertainties.
5. **Computer**: Collect and save the data from Cavendish Torsion Balance.

4.2 Setup

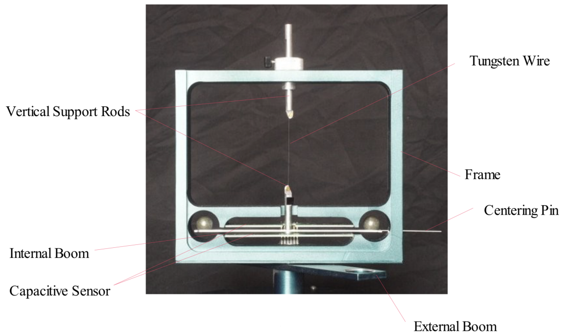

Figure 3: Cavendish Torsion Balance [5]

Figure 4: Image of setup

4.3 Procedure[3]

- Measure the distance between the laser pointer and the mirror on the torsional balance.
- Open the computer and check the system is in the balance or not. Wait until the system is balanced.
- Change the position of the big masses to disturb system to begin oscillation.
- Repeat the above step when speed of the pendulum is equal to zero.
- Take at least 5 peak at maximum amplitude.
- Stop changing the position of the masses and keep taking data to the oscillation end.

5 Raw Data

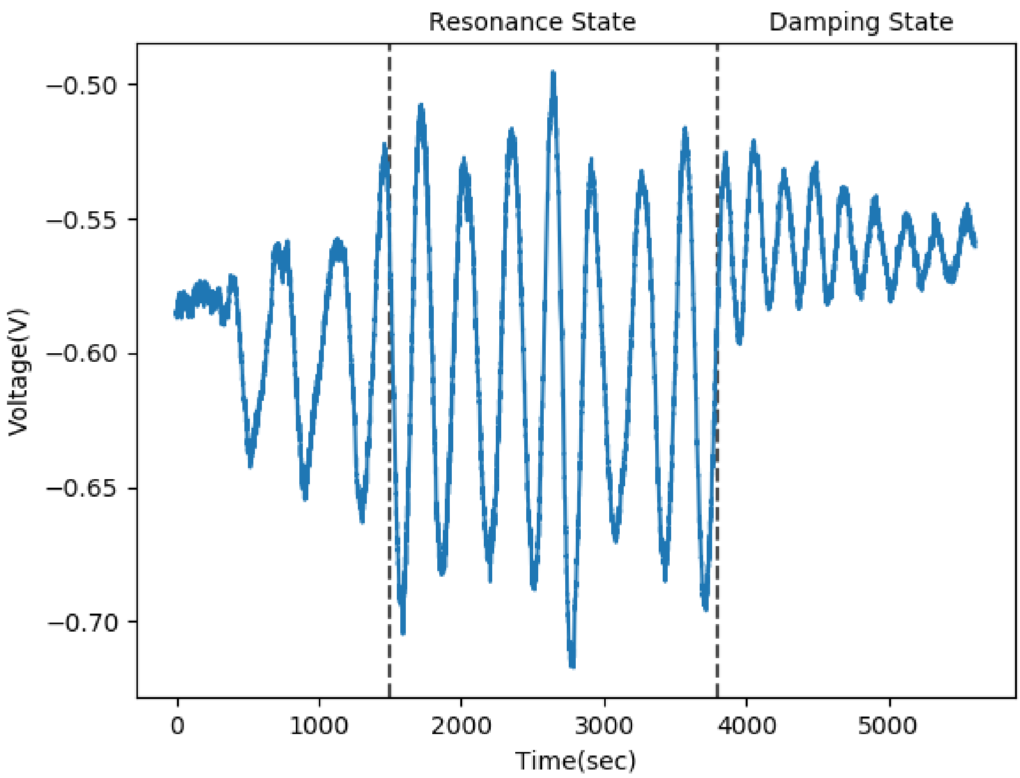

Figure 5: Time(sec) vs Voltage(V) graph of whole data

5.1 Raw Data Tables

Min(cm)	Max.(cm)	Δ S(cm)	σ S(cm)
7.8	9.7	1.9	0.1
8.2	9.6	1.4	0.1
8.0	9.7	1.7	0.1
7.8	9.7	1.9	0.1

Table 1: Max. and Min points for Resonance

$$L(distance\ between\ Cavendish\ balance\ and\ laser) = 161.3 \pm 0.1cm \quad (25)$$

5.2 Table of Constants

Variables	Value	Error	Unit
M_1(Heavy mass 1)	1.0385	0.001	kg
M_2(Heavy mass 2)	1.0386	0.001	kg
m_1(Small mass 1)	0.014573	0.000001	kg
m_2(Small mass 2)	0.014545	0.000001	kg
M_{avg}(Average of heavy mass)	1.0385	0.001	kg
m_{avg}(Average of small mass)	0.014559	0.000001	kg
DM(Distance between largest radius edges of small sphere)	0.146766	0.000066	m
ds_1(diameter of small sphere 1)	0.013452	0.000048	m
ds_2(diameter of small sphere 2)	0.013468	0.000048	m
d (distance between Center of masses of small spheres)	0.066653	$3.71x10^{-5}$	m
$L(2 \cdot d)$	0.13331	0.00007	m
DL_1(diameter of large sphere 1)	0.05612	0.00009	m
DL_2(diameter of large sphere 2)	0.05629	0.00017	m
W (Separation of glass surface)	0.0351	0.0001	m
G_1(gap between heavy sphere 1 and glass surface)	0.0007	0.0002	m
G_2(gap between heavy sphere 2 and glass surface)	0.0002	0.0002	m
r (Average distance between Center of masses of small spheres and large masses)	0.0461025	0.000158	m
I(Moment of inertia of two small masses respect to middle point)	0.000143	0.000001	kg$\cdot m^2$

Table 2: The experiment constants [5]

6 Data Analysis

6.1 Calculation of κ

First of all, we need to find damping frequency.In order to obtain damping frequency we fitted function;

$$A' \sin(\omega_d t + \phi')e^{-\beta t} + C' \tag{26}$$

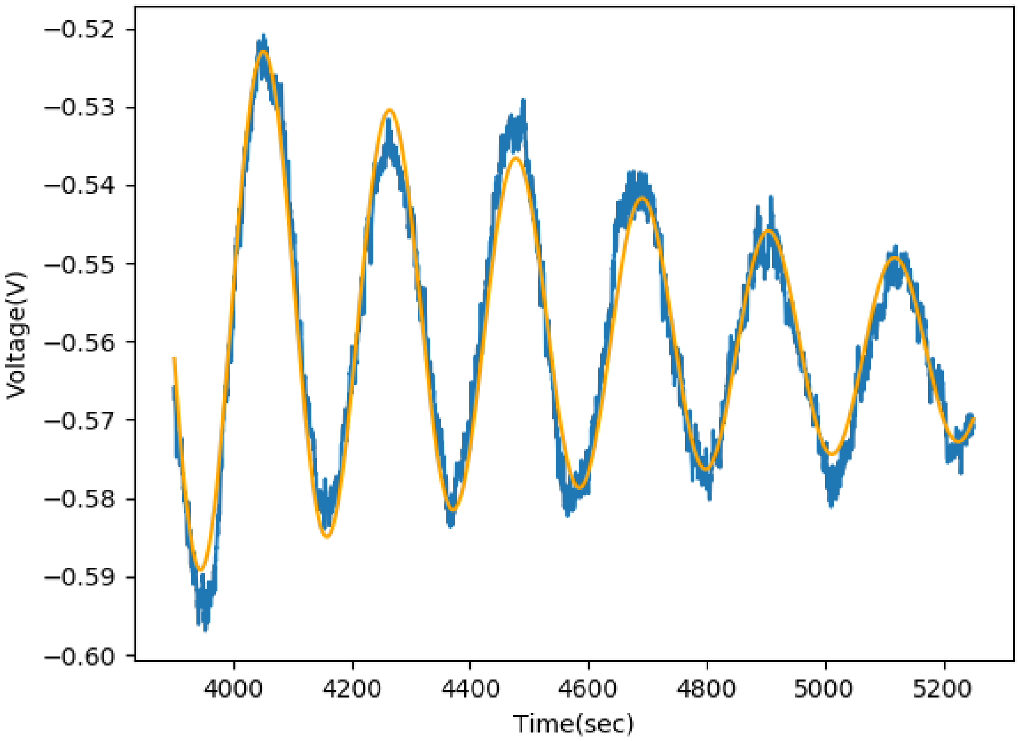

Figure 6: Time(sec) vs Voltage(Volt) fit for damping part

From the fitted graph by python;

$$\omega_d = 0.029460 \pm 0.96188 \times 10^{-6} rad/s \tag{27}$$

and damping parameter β is;

$$\beta = 9.0889 \times 10^{-4} \pm 0.12256 \times 10^{-4} \tag{28}$$

To calculate κ we need to find natural frequency ω_0, we can found with these formula;

$$w_0 = \sqrt{w_d^2 + \beta^2} \tag{29}$$

$$\sigma_{w_0} = \sqrt{(\frac{w_d}{(w_d+\beta)^2})^2 \cdot (\sigma_{w_d})^2 + (\frac{\beta}{(w_d+\beta)^2})^2 \cdot (\sigma\beta)^2} \tag{30}$$

$$\omega_0 = 2.9474 \times 10^{-2} \pm 0.00010330^{-2} rad/s \tag{31}$$

We will calculate κ and its uncertainty with these forumla

$$\kappa = w_0^2 I \tag{32}$$

$$\sigma_{\omega_0} = \sqrt{\Big(\frac{\omega_d.\sigma_{\omega_d}}{\sqrt{\omega_d^2+\beta^2}}\Big)^2 + \Big(\frac{\beta.\sigma_\beta}{\sqrt{\omega_d^2+\beta^2}}\Big)^2} \tag{33}$$

We found κ as;

$$\kappa = 1.2422 \times 10^{-7} \pm 0.0087 \times 10^{-7} Nm/rad \tag{34}$$

6.2 Calculation of θ_{eq}

We need to find mean value of voltage in the resonance part of oscillation.In order to find mean value of voltage we fitted a sinusoidal function in resonance part;

$$A \sin(\omega_d t + \phi) + C \tag{35}$$

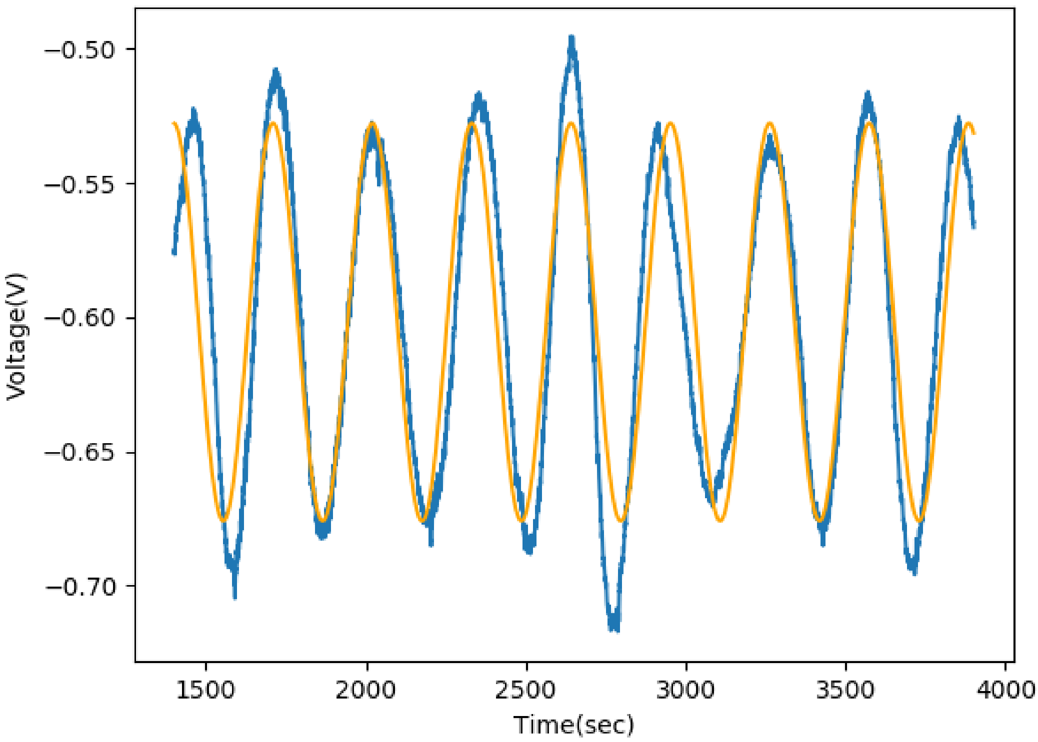

Figure 7: Time(sec) vs Voltage(Volt) fit on resonance part

The average ΔS value and its uncertainty from table 1;

$$\Delta S_{avg} = 1,7 \pm 0.2cm \tag{36}$$

$$\Delta\theta = tan^{-1}\Big(\frac{\Delta S_{avg}/2}{L}\Big) \tag{37}$$

$$\sigma_{\Delta\theta} = \sqrt{\Big(\frac{\sigma_{\Delta S}}{(1+(\frac{\Delta S}{2L})^2)2L}\Big)^2 + \Big(\frac{-\Delta S.\sigma_{2L}}{(1+(\frac{\Delta S}{2L})^2)2L^2}\Big)^2} \tag{38}$$

where

$$L = 161.3 \pm 0.1cm \tag{39}$$

Then we calculate $\Delta\theta$ as;

$$\Delta\theta = 0.0053rad \pm 0.0006 \tag{40}$$

Double of A value in our fit function will give us ΔV

$$\Delta V = 0.14823 \pm 0.00010Volt \tag{41}$$

$V_{mean_{res}}$ is equal to C in equation 1451345245

$$V_{mean_{res}} = -0.63965 \pm 0.00025Volt \tag{42}$$

$V_{mean_{damping}}$ is equal to C' in equation 23452345

$$V_{mean_{damping}} = -0.56538 \pm 0.00021Volt \tag{43}$$

These formula will give us V_{eq} and its uncertainty;

$$V_{eq} = |V_{mean_{damping}} - V_{mean_{res}}| \tag{44}$$

$$\sigma_{V_{eq}} = \sqrt{(\sigma_{V_{mean_{damping}}})^2 + (\sigma_{V_{mean_{res}}})^2} \tag{45}$$

Then we calculate V_{eq}

$$V_{eq} = 0.07427 \pm 0.00033Volt \tag{46}$$

Following formulas weill give us θ_{eq} and its uncertainty;

$$\theta_{eq} = \frac{\Delta\theta}{\Delta V} \cdot V_{eq} \tag{47}$$

$$\sigma_{\theta_{eq}} = \sqrt{\left(\frac{\Delta\theta}{\Delta V} \cdot \sigma_{V_{eq}}\right)^2 + \left(\frac{V_{eq}}{\Delta V} \cdot \sigma_{\Delta\theta}\right)^2 + \left(\frac{V_{eq} \cdot \Delta\theta}{\Delta V^2} \cdot \sigma_{\Delta V}\right)^2} \tag{48}$$

We calculate θ_{eq} as;

$$\theta_{eq} = 0.0027 \pm 0.0003rad \tag{49}$$

Now we have everything to calculate G constant. Other constant gather from Table 2;

$$G = \frac{\kappa\theta r^2}{MmL} \tag{50}$$

$$\sigma_G = \sqrt{\left(\frac{\kappa r^2}{MmL}\sigma_\theta\right)^2 + \left(\frac{\theta r^2}{MmL}\sigma_\kappa\right)^2 + \left(\frac{2\kappa\theta r}{MmL}\sigma_r\right)^2 + \left(\frac{\kappa\theta r^2}{M^2mL}\sigma_M\right)^2 + \left(\frac{\kappa\theta r^2}{Mm^2L}\sigma_m\right)^2 + \left(\frac{\kappa\theta r^2}{MmL^2}\sigma_L\right)^2} \tag{51}$$

In the end we found G constant as;

$$G = 3.5 \times 10^{-10}N.m^2.kg^{-2} \pm 0.4 \times 10^{-10}N.m^2.kg^{-2} \tag{52}$$

7 Conclusion

$$\chi^2 = \left(\frac{3.5\times10^{-10} - 6.673\times10^{-11}}{0.4\times10^{-10}}\right)^2 = 50 \tag{53}$$

To find χ^2 50 is show us that we made something wrong in experiment. Normally, in resonance part we expect every peak have same amplitude but we have very obvious fluctuation as seen in Figure 7. There are many reason and these are;

- We might be late to change position of the large masses. A few second delay can cause huge change in our data
- sunlight and heat have effect on the torsion constant κ. It is probable that torsion pendulum can make unexpected movement because of this reason
- We fit the data in the whole resonance part and we don't delete any data. If we don't consider some data and fit the proper looking data we might have get much good result.

The laser dot was very large and because of this we might make mistake while taking data from scale. Because we can't see exactly center of the dot.

We don't consider drag force because of the air in the system. It is also slightly change the result

Another problem might have occur because of our fitting method. We use python from scipy ODR(Orthogonal distance regression)[6] it work as a least square but least square of orthogonal distance. Since we have many data, it doesn't effect so much but have effect in any way.

There might be noise in the system and affect our voltage data.

References

[1] This Month in Physics History June 1798: Cavendish weighs the world [accessed date:29/03/2019]
$https://www.aps.org/publications/apsnews/200806/physicshistory.cfm$

[2] Cavendish Experiment: Weighing the World [accessed date:29/03/2019]
$http://ffden-2.phys.uaf.edu/211_fall2010.web.dir/smith_elliot/cav_world.html$

[3] Advanced Physics Experiments, Erhan Gulmez, Bogazici University Publications, 1999,ISBN 975-518-129-6

[4] Constants [accessed date:29/03/2019]
$https://physics.nist.gov/cuu/Constants/$

[5] TEL-Atomic Incorporated - Cavendish Balance Manuel [accessed date:29/03/2019]
$https://static1.squarespace.com/static/582c914c414fb5b0fbbf17f2/t/5852a59aff7c505c5f4c2812/$
$RP2111_Cavendish_Balance_manual.pdf$

[6] line fitting with orthogonal distance regression in data analysis (scipy.odr) [accessed date:29/03/2019]
$https://docs.scipy.org/doc/scipy/reference/odr.html$

[7] Thornton ,Marion -Classical Dynamics 5^{th} edition (2003)

Appendices

```
###Whole data####
import numpy as np
import matplotlib.pyplot as plt
import pylab
import pandas

colnames = ['A1', 'B1']
data = pandas.read_excel("rez.xlsx", names=colnames)
icc = data.A1.tolist()
vcc = data.B1.tolist()
time=np.array(icc)
volt=np.array(vcc)

plt.xlabel('Time(sec)')
plt.ylabel('Voltage(V)')
plt.axvline(x=1500,ls = "--", color='black', alpha=0.7)
plt.axvline(x=3800,ls = "--", color='black', alpha=0.7)
plt.text(2500,-0.48, 'Resonance State',horizontalalignment='center')
plt.text(4800,-0.48, 'Damping State',horizontalalignment='center')
plt.plot(time, volt)
###Output####
#graph#

###resonance part####
import numpy as np
import matplotlib.pyplot as plt
import pylab
import pandas
from scipy.odr import *
import math

colnames = ['A1', 'B1']
data = pandas.read_excel("rez.xlsx", names=colnames)
icc = data.A1.tolist()
vcc = data.B1.tolist()
time=np.array(icc)
volt=np.array(vcc)

res_t = time[2800:7800]
res_v = volt[2800:7800]
plt.xlabel('Time(sec)')
plt.ylabel('Voltage(V)')
plt.plot(res_t,res_v)

def fit_func(p, x):
    a,w,k,c= p
    return a*(np.cos((w*x))+k) +c
```

```
linear = Model(fit_func)

data = RealData(res_t, res_v)

odr= ODR(data, linear, beta0=[0., 2e-2,0.,np.mean(res_v)])

out = odr.run()

out.pprint()

x_fit = np.linspace(res_t[0], res_t[-1], 1000)
y_fit = fit_func(out.beta, x_fit)

plt.plot(x_fit, y_fit,color='orange')

print(len(y_fit))
plt.show()
print(2*out.beta[0])
print(2*out.sd_beta[0])
###OUTPUT###

#graph#
Beta: [-0.07411692  0.02021926  -0.50966985  -0.6396483 ]
Beta Std Error: [4.97856593e-04 2.46726542e-06 3.33862410e+03 2.47448525e+02]
Beta Covariance: [[ 3.97996127e-04  1.17167211e-08  1.59722617e+01  1.1840197
 [ 1.17167211e-08  9.77466907e-09 -1.88441905e-01 -1.39667597e-02]
 [ 1.59722617e+01 -1.88441905e-01  1.78980357e+10  1.32654722e+09]
 [ 1.18401978e+00 -1.39667597e-02  1.32654722e+09  9.83195903e+07]]
Residual Variance: 0.0006227728619940781
Inverse Condition #: 7.385330805198425e-11
Reason(s) for Halting:
  Sum of squares convergence
1000
-0.14823383541067378
0.0009957131857129453

####DAMPING PART####
import numpy as np
import matplotlib.pyplot as plt
import pylab
import pandas
from scipy.odr import *

colnames = ['A1', 'B1']
data = pandas.read_excel("rez.xlsx", names=colnames)
icc = data.A1.tolist()
vcc = data.B1.tolist()
time=np.array(icc)
volt=np.array(vcc)

dam_t = time[7800:10500]
dam_v = volt[7800:10500]
```

```
plt.xlabel('Time(sec)')
plt.ylabel('Voltage(V)')
plt.plot(res_t,res_v)

def fit_func(p, x):
    a,w,k,b,c= p
    return a*(np.cos((w*x))+k)*np.exp(+b*x) +c

linear = Model(fit_func)

data = RealData(dam_t, dam_v)

odr= ODR(data, linear, beta0=[1.,0.0295,0.,0.,0.])

out = odr.run()

out.pprint()

x_fit = np.linspace(res_t[0], res_t[-1], 1000)
y_fit = fit_func(out.beta, x_fit)

plt.plot(x_fit, y_fit,color='orange')

plt.show()
omega=np.sqrt(out.beta[1]**2+out.beta[3]**2)
omega_sd = np.sqrt(((out.beta[1]*out.sd_beta[1])/(np.sqrt(out.beta[1]
    **2+out.beta[3]**2)))**2+((out.beta[3]*out.sd_beta[3])/
    (np.sqrt(out.beta[1]**2+out.beta[3]**2)))**2)

print(omega)
print(omega_sd)
iner= 0.000143
iner_sd= 0.000001
kappa= (omega**2)*iner
kappa_sd = np.sqrt((2*omega*iner*omega_sd)**2+((omega**2)*iner_sd)**2)
print(kappa)
print(kappa_sd)

####output####
#graph#

Beta: [ 1.27130626e+00  2.94598969e-02  3.25143834e-01 -9.08894651e-04
 -5.65381308e-01]
Beta Std Error: [6.77239692e-02 9.61883887e-07 9.09256920e-03 1.22563215e-05
 2.06342403e-04]
Beta Covariance: [[ 3.96989611e+02 -2.40134749e-04 -1.21918832e+01 -7.1628922
   3.16042461e-01]
 [-2.40134749e-04  8.00828709e-08 -1.91866743e-05  4.10766177e-08
   4.55121366e-07]
 [-1.21918832e+01 -1.91866743e-05  7.15595440e+00  2.17987049e-03
  -1.52581860e-01]
 [-7.16289225e-02  4.10766177e-08  2.17987049e-03  1.30021336e-05
```

```
   -5.86406975e-05]
  [ 3.16042461e-01  4.55121366e-07 -1.52581860e-01 -5.86406975e-05
    3.68528688e-03]]
Residual Variance: 1.1553289748834903e-05
Inverse Condition #: 1.0593923444731156e-05
Reason(s) for Halting:
  Sum of squares convergence
0.029473914120065982
1.0330479148681002e-06
1.2422576073865455e-07
8.687552583152468e-10
```

```
####Calculation Part####

import numpy as np
import uncertainties
from uncertainties import ufloat
from uncertainties.umath import *
s=ufloat(0.85,0.1) #delta S over 2
L=ufloat(161.3,0.1)
a=atan(s/L) #theta
v=ufloat(0.14823,0.00010)
veq=ufloat(0.0747,0.00033)
print('theta:',a)
print('alpha:',a/v)
teq= (veq*a)/v #senin theta eq i gir
print('theta eq:',teq)
k=ufloat(kappa,kappa_sd) #senin kappa degerini gir
r=ufloat(0.0461025,0.000158)
mm=ufloat(1.0385,0.001)
m=ufloat(0.014559,0.000001)
LD =ufloat(0.066653,3.71e-5)
g=(k*teq*(r**2))/(2.*mm*m*LD)
print(g)
print(2*LD)

####OUTPUT####

theta: 0.0053+/-0.0006
alpha: 0.036+/-0.004
theta eq: 0.00266+/-0.00031
(3.5+/-0.4)e-10
0.13331+/-0.00007
```