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Bayesian population reconstruction of female populations for less developed and more developed countries

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We show that Bayesian population reconstruction, a recent method for estimating past populations by age, works for data of widely varying quality. Bayesian reconstruction simultaneously estimates age-specific population counts, fertility rates, mortality rates, and net international migration flows from fragmentary data, while formally accounting for measurement error. As inputs, Bayesian reconstruction uses initial bias-reduced estimates of standard demographic variables. We reconstruct the female populations of three countries: Laos, a country with little vital registration data where population estimation depends largely on surveys; Sri Lanka, a country with some vital registration data; and New Zealand, a country with a highly developed statistical system and good quality vital registration data. In addition, we extend the method to countries without censuses at regular intervals. We also use it to assess the consistency of results between model life tables and available census data, and hence to compare different model life table systems.

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Keywords: Bayesian hierarchical model; cohort reconstruction; demographic estimation; fertility; international migration; model life table; mortality; vital registration data

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Introduction

The release of *World Population Prospects 2010* (WPP 2010) (United Nations 2011a) coincided with a surge of interest in population statistics in both the popular and academic literature (e.g., Alberts 2011; Gillis and Dugger 2011; Nagarajan 2011; Phillips 2011; Reuters 2011; Scherbov et al. 2011). WPP 2010 was released in 2011, the year in which world population was predicted by the United Nations (UN) to reach 7 billion on 31 October. Despite the implied precision in the declaration of '7 billion day', there is considerable uncertainty about the size of national populations, past and present. In this paper we present evidence that Bayesian reconstruction, developed by Wheldon et al. (2010, 2012, 2013) for estimating past populations and vital rates by age, works for data of widely varying quality and could feasibly be used to reconstruct the female population of any country of the world.

Information about uncertainty can be conveyed by providing interval estimates, rather than simply point estimates as is done in many official statistical releases. Such intervals should have a probabilistic interpretation: they should contain the true value with some specified probability, conditional on the assumed statistical model. Wheldon et al.'s (2010, 2012, 2013) method produces such intervals. It reconstructs past population structures by embedding formal demographic relationships in a Bayesian hierarchical model. The outputs are joint probability distributions of demographic rates and population counts by age, from which fully probabilistic interval estimates can be derived in the form of Bayesian confidence intervals (or 'credible intervals'). The method was designed for use with the relatively unreliable data often encountered in less developed countries, but we show that it works well in reconstructing the populations of countries that vary in the quality of their data from poor to extremely good.

Wheldon et al. (2012, 2013) gave only a single example with real data, that of Burkina Faso. This country represents a 'medium' case: the available data are fragmentary, but independent estimates of adult and child mortality are available. It might be expected that in worse cases, there would be insufficient information to yield useful interval estimates, and that in better cases, there might be little to gain from a probabilistic model. Using a set of case studies chosen specifically to reflect the range and quality of data available across countries, we demonstrate that neither of these conclusions is true. In all cases, Bayesian reconstruction indicates where estimates of vital rates are inconsistent with census results, so that the method can be used to compare competing model life tables. We also extend the method to irregularly spaced censuses.

The remainder of the paper is structured as follows. In the next section we review existing methods of population reconstruction (for a review of Bayesian demography in general, see Bijak and Bryant 2016). Following that, we describe the method. Then we apply Bayesian reconstruction to the female populations of three countries: Laos, Sri Lanka, and New Zealand. The New Zealand case shows that the model performs sensibly for countries with very good data and the Laos case shows that it works with very fragmentary data. We use the case of Sri Lanka to demonstrate our extension to irregularly spaced censuses. For this case, Bayesian reconstruction detected inconsistencies between survey-based estimates of fertility and intercensal population changes, and provided a correction. There is relatively little mortality data for Laos, and we use this case to illustrate how Bayesian reconstruction can be used to choose between competing model life tables. We conclude with a discussion.

Methods of population reconstruction

Many human population reconstructions in the demography literature fall into one of two categories: reconstruction of populations of the distant past using data of the kind commonly found in European parish registers (e.g., Lee 1971, 1974; Wrigley and Schofield 1981; Oeppen 1993a, 1993b; Bertino and Sonnino 2003); and reconstruction of population dynamics after extreme crises, such as famine or genocide (e.g., Boyle and Ó Gráda 1986; Daponte et al. 1997; Heuveline 1998; Merli 1998; Goodkind and West 2001). A standard methodology has been developed for the former category, case-specific methods for the latter, but in one form or another the cohort component method of population projection (CCMPP) (Lewis 1942; Leslie 1945, 1948) is central to almost all methods of population reconstruction.

Two significant developments were Lee's (1971, 1974) method of 'inverse projection' and Wrigley and Schofield's (1981) method of 'back projection'. Inverse projection converts counts of births and deaths into the respective rates. Reconstruction proceeds forwards in time. Baseline population counts and model age patterns of fertility and mortality are also required. Where at least two independent estimates of population size are available, net migration can also be estimated (Lee 1985). In contrast, back projection takes counts at the terminal year and moves backward in time, reconstructing population counts and net migration along the way. Several iterations might be required to produce a satisfactory result. There was considerable debate about the efficacy of back projection, centred partly around identifiability issues that arise from trying to retrospectively separate members of the open-ended age group into separate age categories, and simultaneously estimate fertility, mortality, and migration rates (Lee 1985, 1993). Further developments are described by Barbi et al. (2004) and Oeppen (1993a, 1993b). Bonneuil and Fursa (2011) treat reconstruction as a high-dimensional optimization problem. The original methods of inverse projection and back projection and subsequent developments of them are all deterministic and produce point estimates only.

Stochastic inverse projection (SIP) was proposed by Bertino and Sonnino (2003). It incorporates a specific kind of stochastic variation into the reconstruction, taking inputs similar to those required by inverse projection. Model age patterns of fertility and mortality are treated as individual-level probabilities of death rather than fixed, population-level rates. Like its predecessors, SIP was designed to work with accurate time-series data of total births and deaths. The uncertainty in the final estimates comes only from modelling birth and death as stochastic processes at the level of the individual (Lee 1998 called this 'branching process uncertainty'). There is no allowance for measurement error in the data, nor is there any stochastic variation in the model fertility and mortality age patterns. For most less developed countries, information about births and deaths is not highly accurate, and age patterns of fertility and mortality are known only approximately. In these cases, the uncertainty is due mainly to measurement error. In fact, even for well-measured populations, at the national level, where counts are large, Lee (2004) and Cohen (2006) note that uncertainty attributable to stochastic vital rates is likely to be small relative to the uncertainty due to measurement error; see also Pollard (1968).

The aim of Daponte et al. (1997) was to construct a history of the Iraqi Kurdish population from 1977 to 1990, a period during which it was the target of considerable state-sponsored violence. A Bayesian approach was taken in which vital rates and population counts were modelled as probability distributions. Prior distributions for fertility and mortality rates were based on survey data and beliefs about the uncertainty, founded on studies of the data sources, historical information, and knowledge of demographic processes. Conclusions from estimated posterior distributions took the form of fully probabilistic interval estimates. This approach took account of uncertainty attributable to measurement error, and made use of contextual knowledge to compensate for fragmentary, unreliable data. However, there were some restrictions, such as allowing mortality to vary only through the infant mortality rate and specifying fixed age patterns of fertility. Our approach is similar in spirit, but more flexible because no model age patterns are assumed to hold throughout the period of reconstruction.

Bryant and Graham (2013) proposed a Bayesian method for estimating and forecasting subnational population counts by combining official statistics with administrative data, such as tax records, electoral rolls, and school rolls. The data required to produce usable subnational estimates are unlikely to be available in many less developed countries, nor is access to historical administrative data. Therefore, like the UN, we focus on national-level estimates and do not use administrative data.

Method

Our overall goal is to show that Bayesian reconstruction is widely applicable; therefore, we give a conceptual overview of the method here and save the mathematical details for the supplementary material. All computation was done using *R*, the freely available statistical software package (R Development Core Team 2015); Bayesian population reconstruction is implemented in the *R* package ‘popReconstruct’.

Description of the model

Bayesian reconstruction reconciles two different sets of population estimates: those based on adjusted census counts (or similar data); and those derived by projecting initial estimates of the baseline population forward using initial estimates of vital rates. Adjusted census counts are raw counts which have been processed to reduce common biases such as under-counting and age heaping. Since projection is done using the CCMPP, the parameters for which we require initial point estimates are the CCMPP inputs. Specifically, population counts for the baseline year by age group, plus fertility rates, survival proportions, and the net number of migrants, all by age group, over the period of reconstruction. Migration is treated in the same way as fertility, mortality, and baseline population counts.

Initial judgements of the possible range of the variance in measurement error for each parameter are also required for the purpose of defining prior distributions. These can be based on expert judgement or data such as surveys to assess the coverage of vital registration systems. Sufficient data and expert knowledge to generate these inputs have been available for most countries from about 1960. The comparison is through a Bayesian hierarchical (or multi-level), statistical model which provides probabilistic posterior distributions of the inputs, as well as population counts at each projection step in the period of reconstruction.

Initial point estimates of the input parameters are derived from data. Baseline population estimates are derived from adjusted census counts (or similar sources), and fertility and mortality estimates from surveys, such as the Demographic and Health Surveys (DHSs), and from vital registration data. The model defines a joint prior distribution over these parameters which is parameterized by the initial point estimates and standard deviations. Typically, the initial point estimates will serve as the marginal medians of this distribution, but this is not a requirement. The standard deviations represent measurement uncertainty about the point estimates. These distributions induce a probability distribution on the population counts at the end of each projection step within the period of reconstruction. We impose the necessary condition that the joint prior distribution does not lead to negative projected population counts. In practice this truncates the marginal prior distribution for net migration by ruling out extreme negative values.

Uncertainty about the true population counts at the time of a census is also modelled by probability distributions. Census counts adjusted for bias are taken as the medians of these distributions, and the measurement uncertainty is represented analogously by standard deviations. There are separate standard deviation parameters for fertility, mortality, migration, and population counts. Therefore, the uncertainty remaining in each of these parameters after adjustment and bias reduction is explicitly accounted for in the model. We discuss bias and uncertainty about measurement error in greater detail below.

Calculation of initial estimates of fertility and survival may require the use of population counts at fixed points in time. It is important, however, that these initial counts (adjusted or otherwise) should not be used after the baseline year in either of the following ways. Intercensal survival rates should not be used to estimate mortality, and 'residual' counts, the difference between census counts and projected counts based on fertility and mortality alone, should not be used to estimate migration. These techniques use intercensal changes to estimate vital rates and migration. Bayesian reconstruction uses these changes in the process of updating initial estimates of the vital rates and migration. In consequence, estimating mortality, and migration, in these ways would be tantamount to using the intercensal changes twice: once to derive the initial estimates of vital rates, and again to update them. This could lead to an underestimate of uncertainty, possibly by a large extent.

By treating the induced distribution of projected counts as a prior distribution and basing the likelihood function on the distribution of measurement errors in the census counts, Bayesian reconstruction yields a posterior distribution of the inputs via Bayesian updating. This posterior distribution can be usefully summarized by marginal Bayesian confidence intervals for each input parameter which express the associated uncertainty probabilistically. Furthermore, confidence intervals for age-summarized parameters, such as the total fertility rate (TFR) and life expectancy at birth (e_0), can be obtained. Using simulation, Wheldon et al. (2010, 2012, 2013) found that Bayesian reconstruction produced well-calibrated marginal Bayesian confidence intervals. That is, p per cent Bayesian confidence intervals for each parameter of interest were found to contain the true value p per cent of the time.

Often, projected counts based on a sample from the joint prior distribution of the input parameters will not equal the adjusted census counts for the same year. This discrepancy is sometimes called an 'error of closure' (Preston et al. 2001), and could be reduced by manually making appropriate adjustments to any, or all, of the CCMPP input parameters and census counts. Many different combinations of adjustments will have the same effect on the discrepancy; for example, adding a migrant of age x has the same effect on the age-x population count as removing the death of a person of age x. We do not make such manual adjustments, but define a posterior distribution over the CCMPP input parameters that assigns greater probability to those combinations leading to larger reductions in the discrepancy. This means that each age/time-specific component of the input parameters is not affected equally, but according to the effect it has on the joint posterior.

Reconstruction can be undertaken where estimates of baseline population are available at the start of any period, and over which estimates of vital rates and international migration are available. However, when the period of reconstruction extends beyond the year of the most recent census, the posterior distribution for the period following that year will be heavily based on the prior distribution of the vital rates over that period. Essentially, it would be a projection with added uncertainty estimates, based predominantly on the prior distribution. Therefore, the information gain from reconstructing beyond the most recent census, over and above that of the information contained in the prior, is likely to be minimal. For this reason the periods of reconstruction in our case studies all end in the same year as the most recent census.

Bias

Estimates of vital rates and population counts from surveys and censuses are susceptible to bias. For example, estimates of fertility rates based on birth histories suffer from the omission and misplacement of births, owing to recall error, and census counts may be biased owing to under-counts in certain age groups (Zitter and McArthur 1980; Preston et al. 2001). Bayesian reconstruction does not treat bias explicitly because demographic data differ markedly across parameters, time periods, and countries. Many methods for estimating and reducing these biases have been proposed. Methods appropriate for adjusting census data will not, in general, be applicable to vital registration or survey data. Even within these broad categories, there is great variation among countries and time periods which makes development of a general approach impractical. Therefore, the analyst applying Bayesian reconstruction will need to select bias reduction methods appropriate to the data being used.

Bias reduction for vital rates is often done using so-called 'indirect' methods such as the following: Brass's (1964, 1996) *P*/*F* ratio methods; adjustment of siblings and birth histories (e.g., Obermeyer et al. 2010; Schoumaker 2011, 2013); Alkema et al.'s (2012) method for TFR and child mortality (Alkema and New 2014); and others found in standard texts and manuals (e.g., Shryock et al. 1980; United Nations 1983; Preston et al. 2001; Rowland 2003; Moultrie 2013).

Population counts, typically based on censuses, play a central role in Bayesian reconstruction. Therefore, efforts should be made to reduce bias using, for example, post-enumeration surveys (PESs) (e.g., United Nations 2008, 2010). These are not available for many countries, however, making bias reduction more difficult. In these cases, the analyst will have to search for well-known examples of miscounting, such as under-counting of young adult males, over-counting of age groups at ages of eligibility for state pensions, and heaping at ages ending in '0' and '5'. The useful techniques available include analysis of counts by single years of age, calculating ratios of counts in adjacent age groups, and comparison with available data from administrative sources (Moultrie 2013). Intercensal survival ratios can be examined for plausibility, but the calculated ratios should not be used to derive initial estimates of survival. Where incorrect counts are indicated, methods of making defensible adjustments include taking plausible age or survival ratios, and age distributions, from similar populations. Age heaping might be reduced by applying statistical smoothing techniques (United Nations 1955; United States Bureau of the Census 1985; Booth and Gerland 2015).

Measurement error uncertainty

Bias-reduced initial point estimates of the CCMPP input parameters and adjusted population counts are still subject to measurement error, that is, variation which is non-systematic and cannot realistically be eliminated or otherwise modelled. In Bayesian reconstruction, measurement error is represented by the prior standard deviations of the initial estimates. In many cases there is not much data with which to estimate these parameters, but there is often relevant expert knowledge. This can be included by giving the variances themselves prior distributions and using the expert knowledge to set their fixed hyper-parameters, thus defining a hierarchical model. To do this, we require a value for p in statements of the form 'there is a 90 per cent probability that the true fertility rates are within plus-or-minus p per cent of the initial point estimates', and similarly for survival proportions, migration proportions, and population counts. We refer to p as the 'elicited relative error', and explain how we obtained it below.

Case studies

Our principal aim in this paper is to show that Bayesian reconstruction works across the range of data qualities found in practice. To this end, we selected three countries based on the quality of their mortality rate data: (i) New Zealand, with complete data on vital rates, based on vital registration; (ii) Sri Lanka, with reasonably good vital rate data requiring only small adjustments; and (iii) Laos, with only limited under-5 aged mortality estimates available, and fertility data from a few demographic surveys. In other words, we analysed New Zealand with excellent data, Sri Lanka with intermediate data, and Laos with poor data. Wheldon et al. (2012, 2013) analysed data for Burkina Faso which, in the availability and quality of data, lies between Laos and Sri Lanka, having survey data on both adult and under-5 aged mortality, but no vital registration data.

Each case is discussed separately below. We briefly describe the original data sources and the processes used to derive the initial estimates. We use the results to obtain a number of age-summarized demographic parameters such as TFR, net number of migrants, e_0, and under-5 mortality. We show the 95 per cent Bayesian confidence intervals with our initial estimates, and the posterior distributions of selected parameters using the notation: '(lower, upper)'. We compare our results for fertility and mortality with those published in WPP 2010 for years with comparable estimates. WPP 2010 was based on a different procedure but the same data, so the comparison is valid.

We cover the highlights here. More detailed descriptions of the data sources, initial estimates, and results are in the supplementary material, which also contains all results in comma-delimited files.

Laos, 1985–2005

Data and initial estimates. National censuses were conducted in 1985, 1995, and 2005. These data allowed us to reconstruct the female population between 1985 and 2005. We used the census year counts from the WPP 2010; there were no PESs, but the census counts were adjusted to compensate for some under-counting in certain age groups. Further details are in the supplementary material.

Initial estimates of age-specific fertility rates were based on direct and indirect estimates from the available surveys. Age-specific initial estimates were obtained by smoothing available TFR estimates over time and multiplying by similarly smoothed estimates of the age pattern of fertility. Owing to the small number of data points for TFR and the age pattern, smoothing was performed by taking medians across the data source for each age/time period combination.

The only available mortality data were for infants and under-5s. Therefore, our initial estimates came from Coale et al.'s (1983) West (CD West) model life tables with values of ${}_1q_0$ and ${}_5q_0$ close to those estimated from the available data.

We asked United Nations Population Division (UNPD) analysts to supply elicited relative errors for population counts, fertility, and mortality. These were 10 per cent on the population count scale, the fertility rate scale, and the survival proportion scale, respectively.

There was not much information on migration. To model it, we set initial point estimates to zero for all ages and time periods, but used an elicited relative error of 20 per cent for the net number of migrants. This means that the a priori central 90 per cent probability interval for net international migration ranges from −20 to +20 per cent of the population for each age group.

Results. Figure 1 shows our prior and posterior distributions for TFR and e_0, together with WPP 2010 estimates. The Bayesian reconstruction estimate of TFR differs from the initial estimates in the 5-year periods beginning 1985, 1990, and 2000. While both imply consistent decreases in fertility, the initial estimates appear to be too high in all but the third 5-year period. Our posterior intervals suggest a level of fertility more similar to WPP 2010, except that our estimates suggest that the acceleration in the decline began one 5-year period later.

Figure 1(a) shows that the posterior intervals are not constrained to lie inside the prior intervals. Moreover, the posterior intervals can be wider than the prior intervals, and this was found to be the case for the age-specific fertility rates for Laos.

The prior distributions of TFR and e_0 are asymmetric because the age-specific parameters from which they were calculated, namely, the age-specific fertility rates and survival proportions, were modelled on different scales. Age-specific fertility rates were modelled on the log scale and age-specific survival proportions were modelled on the log-odds (logit) scale. Further details are in the supplementary material.

Sri Lanka, 1951–2001

Data and initial estimates. Censuses were conducted in Sri Lanka in 1953, 1963, 1971, 1981, and 2001, and we were therefore able to reconstruct the female population between 1953 and 2001. We took population counts from WPP 2010, which were adjusted to account for under-enumeration. Initial estimates of age-specific fertility rates were derived in a manner similar to that used for Laos, although at the level of TFR we used loess (Cleveland 1979; Cleveland et al. 1992) to smooth multiple data points across time. Initial estimates of age-specific survival proportions were based on abridged national life tables, calculated from death registrations and available surveys. Elicited relative errors for all of these parameters were obtained in the same way as for Laos, and set at 10 per cent on the same scales as described above.

We used the same default initial estimate of international migration as that used for Laos. Luther et al. (1987) have provided age-specific estimates for the periods 1971–75 and 1976–80 using census data as well as information about vital rates. Their results are not suitable as a basis for initial estimates because they were partly derived from intercensal changes in population counts; we used them for comparison instead. These results are in the supplementary material.

Interpolation to handle irregular census intervals. Wheldon et al. (2010, 2012, 2013) assumed that censuses were taken at regular intervals but there is an irregular interval between the 1963 and 1971 censuses. Therefore, we propose interpolating the CCMPP outputs on the growth rate scale such that they coincide with the census years. We explain with an example.

Consider the number in the population aged $[x, x+5]$ for which we have a 1963 census-based estimate, and another for 1971. Initial estimates for vital rates are available for 1963, 1968, 1973, and 5-yearly subsequently. The CCMPP can be used with these data to derive projected counts for this age group in 1968 and 1973. To compare the CCMPP output with the 1971 census counts, we assumed that the growth rate for this age group, $r_{x,1968}$, was constant between 1968 and 1973, and estimated it from the projected counts. The estimate was then used to interpolate the CCMPP output to 1971. Using a 'hat' (^) to

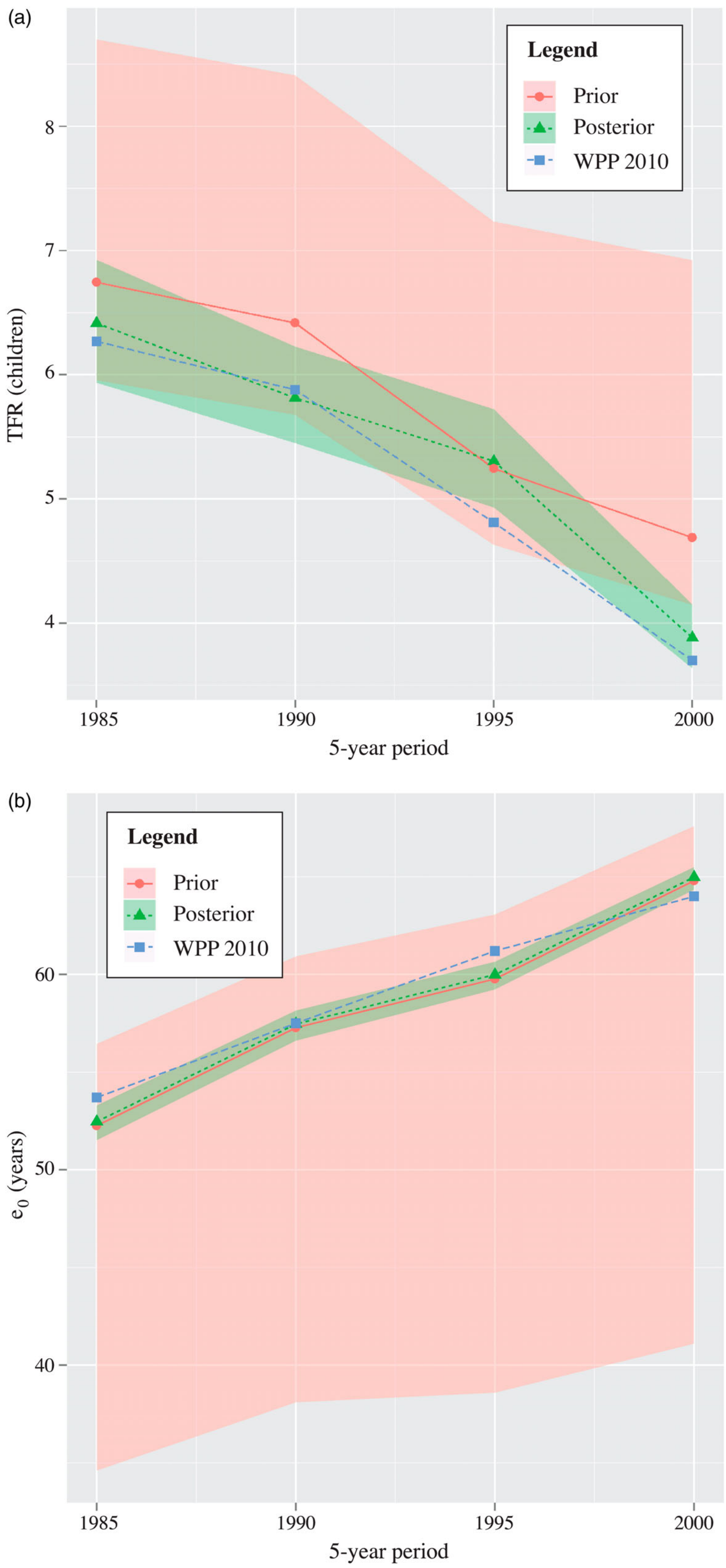

Figure 1 Prior and posterior medians and 95 per cent Bayesian confidence intervals for selected parameters in the reconstructed female population of Laos, 1985–2005. Prior medians correspond to initial estimates. (a) Total fertility rate. (b) Life expectancy at birth of females

Source: Coale et al. (1983); 1986–88 Multi-Round Survey; 1993 Laos Social Indicator Survey; 1994 Fertility and Birth Spacing Survey; 1995 and 2005 censuses; the 2000 and 2005 Lao Reproductive Health Surveys; United Nations (2011a); Bureau of East Asian and Pacific Affairs (2011); 2006 MICS3 Survey.

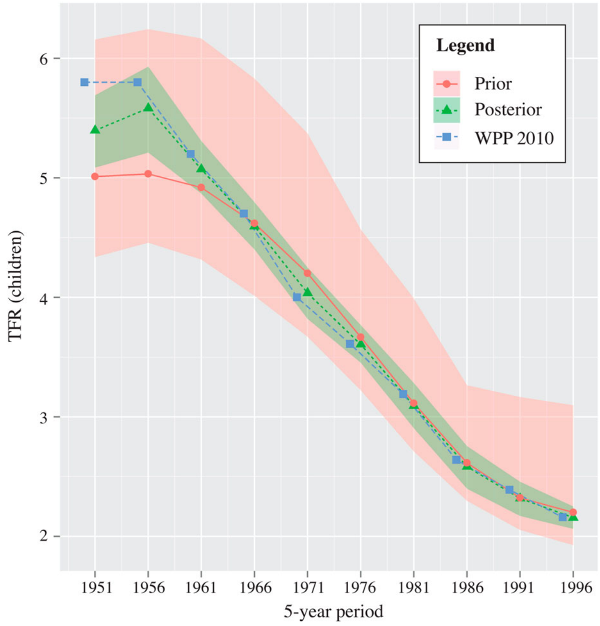

Figure 2 Prior and posterior medians and 95 per cent Bayesian confidence intervals and WPP 2010 estimates of TFR in the reconstructed female population of Sri Lanka, 1950–2000. Prior medians correspond to initial estimates
Source: 1953–2001 censuses; 1972–2006 vital registration; 1975 Sri Lanka WFS; 1987, 1993, 2000, 2006–07 Sri Lanka DHSs; United Nations (2011a).

denote ‘estimate’, this is compactly expressed as

$$\hat{r}_{x,1968} = \frac{1}{5}\log_e\left(\frac{n_{x,1973}}{n_{x,1968}}\right)$$
$$\hat{n}_{x,1971} = (n_{x,1968})e^{3\hat{r}_{x,1968}}.$$

We used a similar method to extrapolate the population counts from the 1953 census back to 1951 using the 1953–63 growth rate. Interpolating in this manner is adequate for periods of less than 5 years.

Results. Posterior distributions for the mortality and migration parameters are summarized in the supplementary material. Our posterior estimates of mortality and migration agree closely with those in WPP 2010 and Luther et al. (1987).

Applying Bayesian reconstruction suggests, however, that the sources upon which the initial estimates of fertility rates were based are inconsistent with intercensal changes in the number of births (Figure 2). The posterior estimates of TFR from Bayesian reconstruction differ noticeably from the initial estimates in the periods 1951–56 and 1956–61 (posterior intervals (5.09, 5.69) and (5.21, 5.93); initial estimates 5.01 and 5.03 children per woman, respectively). Our method has automatically provided a correction which, in this case, yields results similar to the WPP 2010 estimates.

New Zealand, 1961–2006

Data and initial estimates. Census counts were used from national censuses conducted every 5 years between 1961 and 2006. Initial estimates of fertility rates were calculated from published age-specific fertility rates (Statistics New Zealand 2011a) and numbers of births (Statistics New Zealand 2012) by age group of mother by year. Initial estimates for survival proportions were calculated from New Zealand life tables (Statistics New Zealand 2011b).

Information on the measurement errors of these parameters was available in the form of PESs and estimates of the coverage achieved by the birth and death registration systems (Statistics New Zealand 1998, 2010a). Using this information, elicited relative errors were set by the authors at 2.5, 1, and 1 per cent for population counts, fertility, and mortality, respectively.

Information on international migration is more reliable than in most other countries because New Zealand is a small island nation with a well-resourced official statistics system. The basis of our initial estimates of international migration were counts of permanent and long-term (PLT) migrants taken from arrival and departure cards (Statistics New Zealand 2010b). The largest source of error in these estimates of international migration is the discrepancy between the stated intentions and actual behaviour of travellers. To reflect this, we set the elicited relative error of the migration input parameter to 5 per cent.

Results. The posterior distributions for TFR, e_0, and under-5 mortality are summarized in the supplementary material. Our posterior estimates of mortality and fertility follow the initial estimates closely. This is not unexpected because the initial estimates were based on data of high quality and coverage. The least reliable data, a priori, were those for migration. Our posterior intervals suggest corrections in some time periods (Figure 3). The initial estimates for periods between 1961 and 1974 appear to be too high, while those for periods between 1976 and 1989 are too low.

Choosing between alternative initial estimates of mortality

In the application of the method to Laos we derived initial estimates of over-5 mortality from the CD

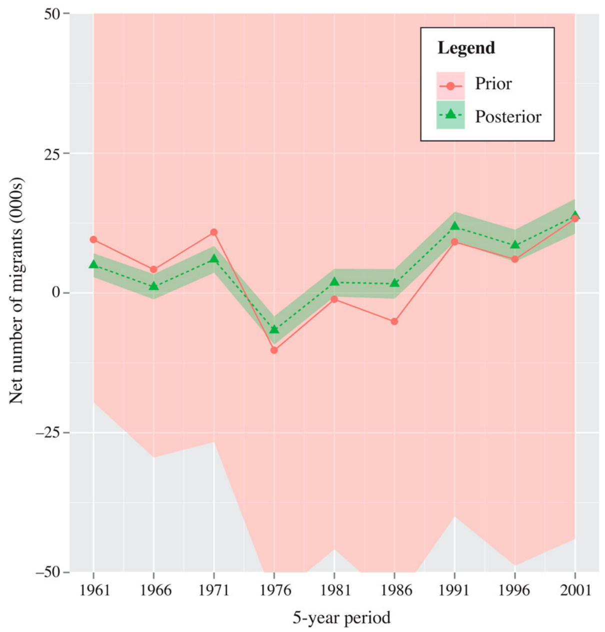

Figure 3 Prior and posterior medians and 95 per cent Bayesian confidence intervals of the total net number of female migrants (average annual) in the reconstructed female population of New Zealand, 1961–2006. Prior medians correspond to initial estimates

Source: 1961–2006 censuses; 1961–2005 vital registration and international arrivals and departures; 1996, 2001, 2006 PESs; Statistics New Zealand (1998, 2010b).

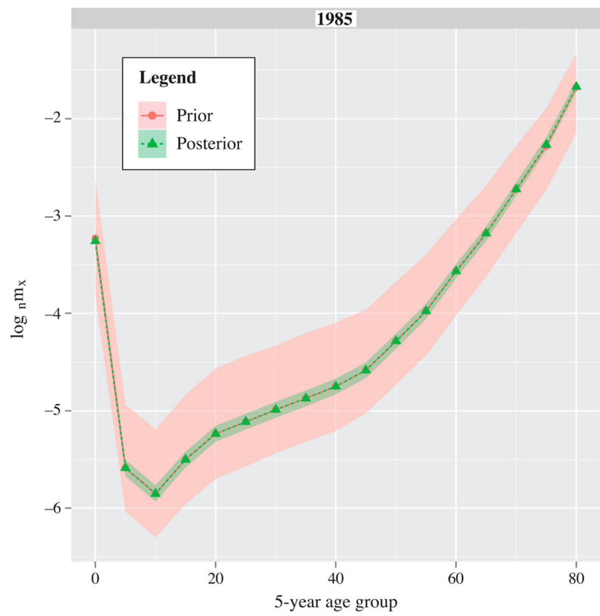

Figure 4 Prior and posterior medians and 95 per cent Bayesian confidence intervals of the age-specific log mortality rates in the reconstructed female population of Laos, 1985–90. Prior medians correspond to initial estimates, which were calculated using the CD West model life table. Prior and posterior medians coincide almost exactly. Note that the posterior Bayesian confidence intervals are the darker, inner bands in this plot

Source: As for Figure 1(a).

West model life table. This choice was made by UNPD analysts, who drew on previous studies (Hartman 1996a, 1996b; United Nations 2011b). However, other approaches were possible. We therefore compared the results above with those given by an alternative set of initial estimates of survival based on a different model life table, and we use the comparison to explain why the CD West model should be preferred. To do this, we look at the age-specific mortality rates rather than e_0.

The posterior distribution of e_0 in Figure 1(b) was computed from the posterior distribution of the age-specific survival proportions, ${}_5S_x[t, t+5]$, which are an output of Bayesian reconstruction. These were converted into age-specific annual mortality rates using the separation factors implicit in the CD West life table. Medians and limits of 95 per cent Bayesian confidence intervals for the marginal posterior distributions of these parameters for the period 1985–89 are shown in Figure 4 on a logarithmic scale. Posterior uncertainty about these quantities is very low in this and all subsequent reconstructed periods (see the supplementary material); the mean half-widths of the 95 per cent Bayesian confidence intervals of mortality rate over age, within year, are all less than 0.002 (raw scale). Mean half-widths of confidence intervals were derived by taking half the difference between the upper and lower limits of the intervals for each age group and time period, and then averaging over age groups and time periods.

An alternative set of initial estimates for the ${}_5S_x[t, t+5]$ was generated from the same data on under-5 mortality, but adult mortality was estimated using the Brass two-parameter relational logit model with the United Nations South Asian (UNSA) model life table, $e_0 = 57.5$ years (United Nations 1982). Figure 5 gives the initial estimates and marginal posteriors of the log mortality rates using these alternative survival estimates, but keeping the initial estimates of all other parameters the same. The posterior intervals of mortality rate are much wider under this set of initial estimates; the mean half-widths over age, within year, are between 0.015 and 0.025 (raw scale), a large increase.

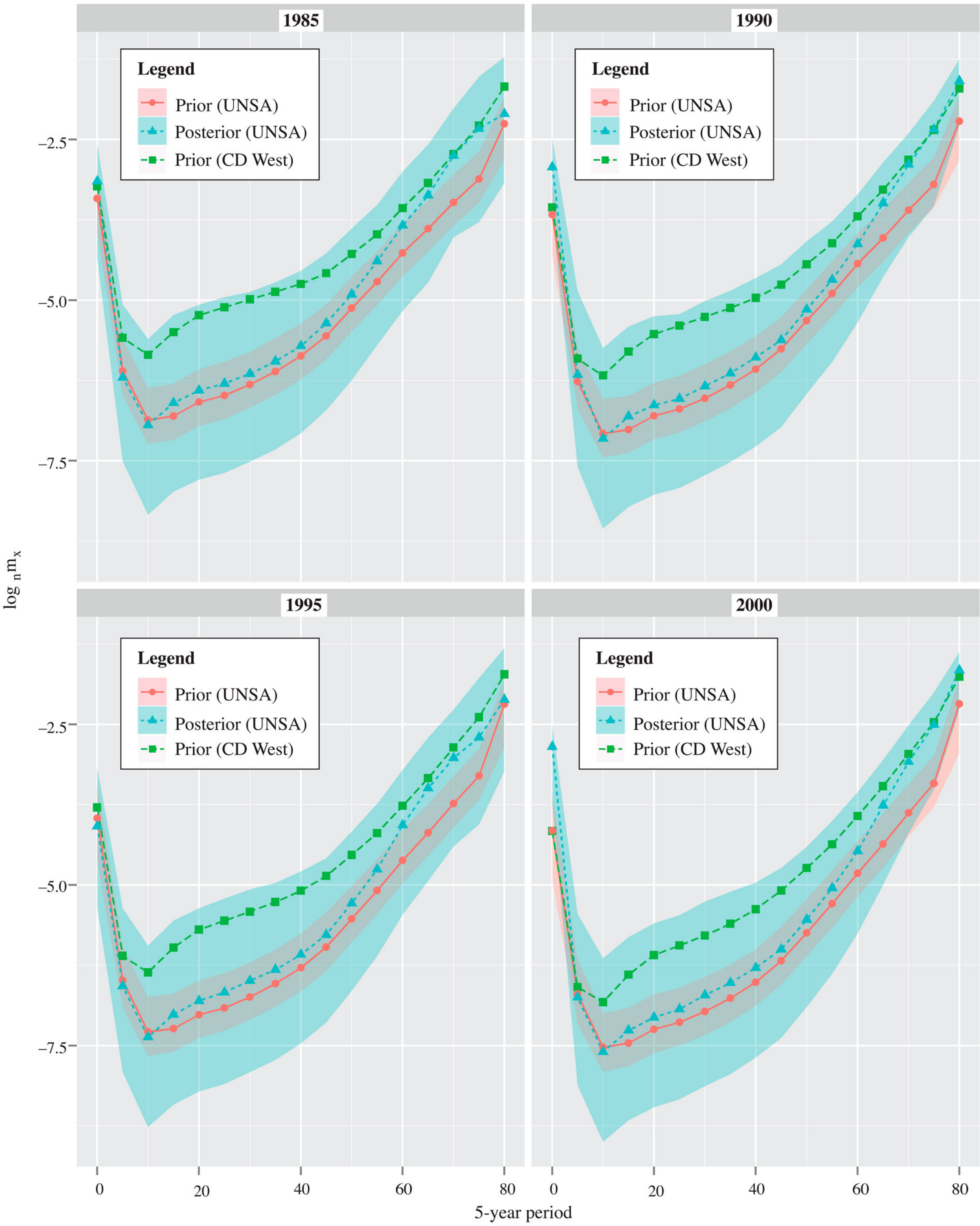

Figure 5 Prior and posterior medians and 95 per cent Bayesian confidence intervals of age-specific log mortality rates in the reconstructed female population of Laos, 1985–2005. Prior medians correspond to initial estimates. Initial estimates and posterior distributions were calculated using the Brass two-parameter relational logit model with the United Nations South Asian (UNSA) model life table. Note that the posterior Bayesian confidence intervals are the lighter, wider bands in this plot

Source: As for Figure 1(a); United Nations (1982).

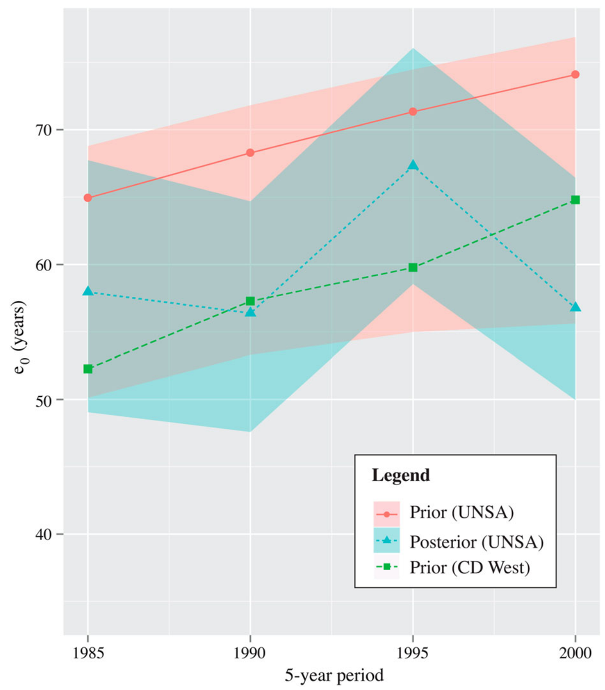

Figure 6 Initial and posterior estimates of e_0 for Laos females, 1985–2005, using the Brass two-parameter logit model and the United Nations South Asian (UNSA) model life table. This figure summarizes the results shown in Figure 5. Note that the posterior Bayesian confidence intervals are the darker bands in this plot
Source: As for Figure 5.

The wider intervals show that using the alternative initial estimates greatly increases posterior uncertainty. In addition, for many of the older age groups, the posterior medians are actually closer to the CD West initial point estimates than those used to fit the model. While not a formal statistical test, this does suggest that the initial estimates based on the CD West life tables are more consistent with the intercensal changes in population counts, given the initial estimates for the other parameters, and that they are preferable to the UNSA-derived initial estimates.

Looking at e_0 in Figure 6 leads to the same conclusion. Again, uncertainty is much greater under the alternative set of initial estimates (cf. Figure 1 (b)). The posterior distribution has shifted away from the initial estimates used to fit the model toward those derived from the CD West model life table. In fact, all the CD West initial point estimates are contained within the 95 per cent posterior interval based on the alternative estimates, while this is not the case for the initial estimates used to fit the model.

We emphasize that our preferred set of initial estimates are those generated using the CD West standard. Our purpose here is not to advocate the UNSA standard, or the Brass two-parameter logit model, but to present an alternative, plausible, set of initial estimates which we can use to generate an alternative set of posterior estimates for use in a comparative analysis.

Discussion

We have shown that Bayesian reconstruction (Wheldon et al. 2010, 2012, 2013) works well when applied to data sets that vary in quality from different countries. For Laos, the only mortality data are for the under-5s and come from surveys. New Zealand has complete period life tables based on vital registration. Burkina Faso (analysed in Wheldon et al. 2012, 2013) and Sri Lanka lie between these extremes. Furthermore, the magnitude of the estimated uncertainty reflected the quality of the available data. For instance, posterior intervals for New Zealand are much narrower than those for Laos, reflecting the greater accuracy and coverage of the New Zealand data.

From the single example presented by Wheldon et al. (2012, 2013), it was not clear that Bayesian reconstruction would produce useful estimates in cases of very fragmentary data, such as the data for Laos. Our results show that, even in a case where data on only infant and under-5 mortality are available, the posterior uncertainty is small enough to make the results useful. New Zealand, in contrast, has very good data, but even here there is uncertainty that Bayesian reconstruction is able to estimate. That the method performed well in such a diverse range of cases suggests that it could feasibly be used to reconstruct the female population of any country. Bayesian reconstruction is likely to be of greatest value for those countries which lack comprehensive, reliable vital registration data, and where the uncertainty is therefore greater. Roughly one half of all the countries and areas included in the WPP fall into this category (United Nations 2011a). Where data are even more fragmentary than for Laos, the uncertainty could be greater than that shown in our examples. Ultimately, of course, the quality of the outputs will be dependent on the quality of the inputs, including the bias reduction process, and elicited relative errors.

Bayesian reconstruction embeds the standard CCMPP in a hierarchical statistical model which takes initial estimates of vital rates and population counts as inputs, together with expert opinion about their relative error (informed by data if available), and yields fully probabilistic interval estimates for all of the inputs. International migration is handled in the same way as the other inputs. The approach is Bayesian because the initial estimates serve as the medians of informative priors for CCMPP input parameters, hence also population counts, which are then updated using observed population counts over the period of reconstruction. These observed population counts, typically derived from available census data, play a central role in the reconstruction, but they are not assumed to be error-free. Rather, uncertainty caused by measurement error is reflected in the variance parameters in the same way as for the vital rates.

The age patterns in the initial estimates will be plausible a priori by construction, or possibly flat in the case of net migration, but there are no further impositions on age patterns during the modelling process. The posterior estimates will depart from the initial estimates only if there is strong evidence in the data. We did not observe any such departure in our case studies, but we prefer not to exclude this possibility completely. In general, posterior distributions for age-specific parameters can be inspected for plausibility. We give these distributions for the case studies in the supplementary material.

The posterior distributions will be somewhat sensitive to the elicited relative errors because the initial estimates themselves do not contain much information about measurement error. However, in cases like that of New Zealand, the elicited relative errors are likely to be heavily based on data from register-coverage studies and post-census surveys. In countries without such studies, expert opinion and knowledge about the situation in neighbouring countries with similar conditions provide an important source of information. Elicited relative errors specified for each component can vary depending on the reliability of the information available. In both cases, it is appropriate that these sources of information have some bearing on the resulting estimates.

In our case studies, the periods of reconstruction were delimited by the earliest and most recent censuses. Reconstruction can be undertaken beyond the year of the most recent census if initial estimates of vital rates and international migration are available, but the posterior distribution for this latter period will not be updated and will depend heavily on the prior distributions. In practice, therefore, a minimum of two sets of population counts are required for a useful reconstruction.

We have presented 95 per cent Bayesian confidence intervals for the marginal distributions of TFR, total net number of migrants, e_0, and under-5 mortality; 95 per cent intervals cover the range of the most likely values. Results for TFR and age-specific fertility for Laos show that the posterior intervals are not constrained to lie inside prior intervals, nor are they necessarily narrower than prior intervals. Our posterior estimates of TFR for Laos and Sri Lanka suggest that, in some years, the initial estimates, based mainly on surveys, are inconsistent with intercensal changes in number of births, and Bayesian reconstruction is able to provide an appropriate correction.

In our studies of Laos and Sri Lanka, the a priori central 90 per cent probability interval for net international migration ranged from −20 to +20 per cent of the population for each age group. This is a wide range and reduces the influence of the migration prior on the posterior relative to the priors of the other vital rate parameters and of the population counts. Posterior Bayesian confidence intervals are much narrower, however, and posterior medians display a noticeable age pattern, despite an age pattern being absent from the prior. Further discussion, including that on age-specific posterior distributions for migration, appears in the supplementary material.

The method as described in Wheldon et al. (2010, 2012, 2013) was limited by the fact that it required census data at regular intervals. Here, we relaxed this requirement by showing that linearly interpolating census counts on the growth rate scale produces good results.

We have also shown how Bayesian reconstruction might be used to help choose between two sets of initial mortality estimates. We compared the posterior distributions of age-specific mortality rates for Laos derived from initial estimates based on the CD West model life table and the Brass two-parameter relational logit with the UNSA model life table. In the latter case, the interval widths were much greater. This implies that the CD West-based initial estimates agree more closely with the data on fertility, mortality, and population counts, and that they should be preferred.

Model life tables are a useful source of initial estimates of mortality in cases where data are fragmentary, but they do not need to be used in isolation. Where the analyst has additional information about mortality patterns, independent of intercensal population changes, this can be used in the reconstruction by incorporating it into the initial estimates. For example, knowledge about how wars and epidemics affect mortality age patterns could be used in cases where such events are known to have occurred. In countries experiencing substantial HIV/AIDS mortality, mortality rates from all causes of death based on vital registration or retrospective information from surveys can be used.

Bias and variance in measurement error are handled separately under Bayesian reconstruction. Existing demographic techniques, such as indirect estimation via *P*/*F* ratios and model life tables, and adjustments using PESs, based on raw data collected from surveys, vital registration, and censuses, are used to reduce bias in initial point estimates and population counts. The nature of bias varies greatly across parameters, time period, and country, so we do not propose a general method to replace the many existing techniques. Instead, the analyst is able to select the most appropriate technique for the data at hand. In the case of population counts, this may be difficult to achieve if PESs are not available, and the analyst will have to search for well-known examples of miscounting, such as under-counting of young adult males, over-counting at ages of eligibility for state pensions, and heaping of ages ending in '0' and '5'.

Measurement error variance is accounted for through the variance parameters (equivalently, standard deviations) of the initial point estimates. Expert opinion is used a priori to set reasonable ranges for measurement error uncertainty. Separate ranges for fertility, mortality, migration, and population counts are used to account for a priori uncertainty in initial estimates and population counts that remains after adjustment and bias reduction. These are then updated using all the available initial estimates and census counts.

Cross-sectional population counts over the period of reconstruction may be needed to construct initial estimates of vital rates and migration, but intercensal changes should not be used for this purpose. Forward or reverse-survival methods, methods of estimating migration using 'residuals' after projecting using fertility and mortality alone, and other techniques that use intercensal changes to estimate vital rates and migration, are not compatible with Bayesian reconstruction. Using any of these could lead to an under-estimate of uncertainty. If no reliable migration data are available, the default initial point estimates should be centred at zero for all ages and time periods, with a large elicited relative error. If no reliable mortality data are available, a model life table must be chosen from a family of standards or derived using data for another country. In both cases, the resulting increase in uncertainty can be fed into the model via the elicited relative error. Since a key aim of Bayesian reconstruction is to quantify the uncertainty in population reconstructions, the usefulness of the method, per se, will not be compromised because it will still be possible to estimate posterior uncertainty, and this, itself, is important information.

The remaining bias in cross-sectional population counts used to construct initial estimates of vital rates and migration may affect the initial estimates themselves. For example, if death and population counts are used to produce initial estimates of survival, and population is under-counted but deaths are not, the resulting survival proportions will be too low. However, Bayesian reconstruction is undertaken simultaneously across all age groups and time periods, and population under-count tends to be limited to specific age ranges. In this case, the cohort will eventually age into an age group where bias is not significant. At this point, there will be an error of closure owing to a discrepancy between the observed, un-biased population count and the counts based on the projection of the under-counted earlier population. Bayesian reconstruction will respond by exploring all possible combinations of changes in fertility, survival, and migration across the age range and period of the reconstruction, assigning higher posterior probability to those combinations which lead to larger reductions in the discrepancy. Changes in the survival proportions for the period of population under-count will be included in this exploration and adjustments will be made according to the impact on the joint posterior.

Bayesian reconstruction was developed and demonstrated for female-only populations. In Wheldon et al. (2015) we extended the method to two-sex populations. A further potential refinement is to use single-year age groups and time periods.

A great deal of attention has already been directed at the estimation of uncertainty in demographic forecasts, as opposed to estimates about the past which we focus upon here. The study of stochastic models for forecasting dates back at least to Pollard (1966) and Sykes (1969). Further developments were reviewed by Booth (2006) with more recent additions in Hyndman and Booth (2008), Alkema et al. (2011), and Scherbov et al. (2011). One component of error in forecasts of population size is the error in estimates of population size and the vital rates in the baseline year. While the ergodic theorems of demography (Lotka and Sharpe 1911; Lopez 1961) imply that these become irrelevant if one forecasts far enough into the future, short-term forecasts can be significantly affected (e.g., Keilman 1998; National Research Council 2000). It is possible, then, that

Bayesian reconstructions could contribute to improved forecasting methods by providing important information about the uncertainty in estimates of baseline populations.

The fact that official statistical estimates are not perfect is undisputed. The UNPD acknowledges this fact both explicitly (United Nations 2011a) and implicitly in that the WPP are revised biannually as new sources of data become available and methods are improved. Therefore, augmenting point estimates with quantitative estimates of their uncertainty is an important contribution. For many countries, the available data are fragmentary and subject to bias and measurement error, and so the expert opinions of demographers are valuable. A Bayesian approach is especially appropriate (Bijak and Bryant 2016) since this knowledge can be used in conjunction with the available data in a statistically coherent manner.

Notes

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