ECE 102, Fall 2023 **Homework #6**

Department of Electrical and Computer Engineering — Prof. Jonathan C. Kao

University of California, Los Angeles — TAs: Yang, Bruce, Shreyas

Due Monday, 27 Nov 2023, by 11:59pm to Gradescope

Covers material up to lecture 14.

100 points total

1. (32 points) **Frequency Response**

   (a) (18 points) Consider the LTI system depicted in figure 1 whose response to an unknown input, $x(t)$, is

   $$y(t) = \left(4e^{-t} - 4e^{-4t}\right) u(t)$$

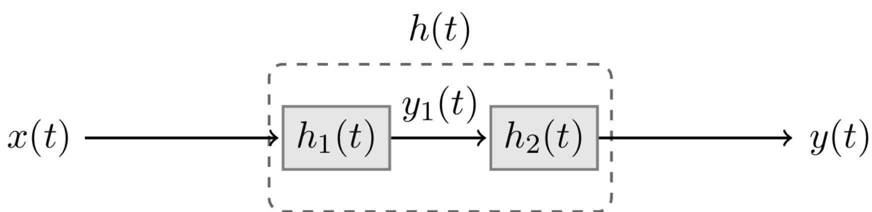


Figure 1: System for Problem 1(a)

   We know that for the same unknown input $x(t)$, the intermediate signal, $y_1(t)$, is given by:

   $$y_1(t) = 2e^{-t}u(t)$$

   The overall LTI system is described by the following differential equation:

   $$\frac{d^2}{dt^2}y(t) + 6\frac{d}{dt}y(t) + 8y(t) = 3x(t)$$

   i. Find the frequency response, $H(j\omega)$, of the overall system $h(t)$.
   ii. Find the frequency responses $H_1(j\omega)$ of the first LTI system and $H_2(j\omega)$ of the second LTI system.
   iii. Find the impulse responses $h(t)$, $h_1(t)$ and $h_2(t)$.

   (b) (8 points) Consider the system shown in the figure 2, where the frequency response $H(\omega)$ has magnitude and phase shown in figure 3.

   If,

   $$x_1(t) = \sin\left[\omega_1 t + \frac{\pi}{4}\right]$$
   $$x_2(t) = 2\cos\left[\omega_2 t - \frac{\pi}{3}\right]$$

   where $\omega_1 = \pi$ and $\omega_2 = 2\pi$, write an expression for $y(t)$.

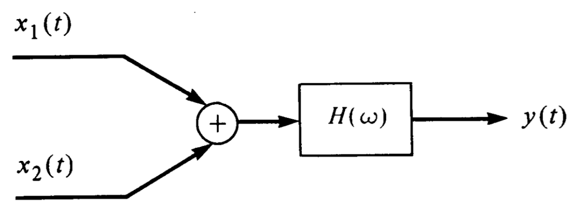


Figure 2: System for Problem 1(b)

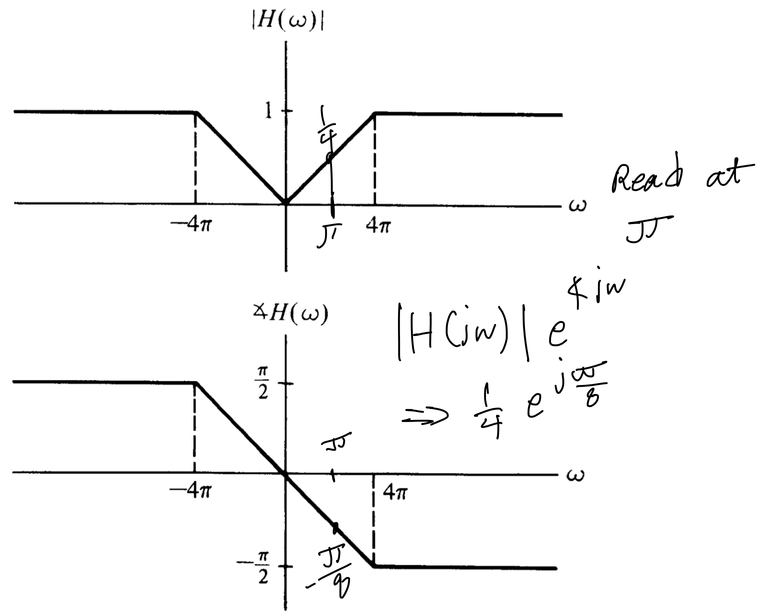


Figure 3: Frequency response for Problem 1(b)

(c) (6 points) Consider an LTI filter with $h(t)$ as impulse response. Its frequency response is given by:

$$H(j\omega) = \frac{1}{1 + j\omega}$$

i. Find the frequency range in Hertz in which the magnitude of the system function exhibits 1 percent or less deviation from its value at $\omega = 0$. Determine its cutoff frequency ω_c, where ω_c is defined below:

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}}|H(j0)|$$

ii. How can we transform $h(t)$ so that the cutoff frequency of the new filter becomes $5\omega_c$?

2. (18 points) **Filters**

(a) (6 points) Consider an ideal low-pass filter $h_{LP,1}(t)$ with frequency response $H_{LP,1}(j\omega)$ depicted below in figure 4.

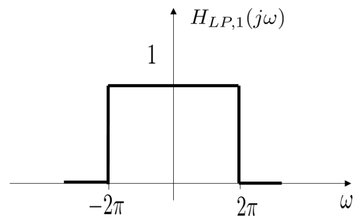


Figure 4: An ideal low pass filter

Using this filter, we construct the following new system:

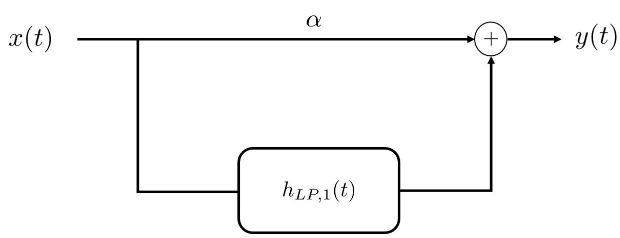


Figure 5: New system

We are given two choices for α: 1 or -1. Which value should we choose so that the new system is a high-pass filter? Does the new filter have any phase in its frequency response?

(b) (3 points) Why are the ideal filters non-realizable systems?

(c) (5 points) We want to design a causal non-ideal low-pass filter $h_{LP,2}(t)$, using the following frequency response:

$$H_{LP,2}(j\omega) = \frac{k}{\beta + j\omega}$$

Find k and β so that $H_{LP,2}(j0) = 1$ and its cutoff frequency is $\omega_0 = 2\pi$ rad/s, (i.e., the magnitude of $H_{LP,2}(j\omega)$ is $1/\sqrt{2}$ for $\omega = 2\pi$ rad/s).

(d) (4 points) We again consider the system of part (a) where instead of the ideal low-pass filter, we are going to use the non-ideal low-pass filter of part (c).

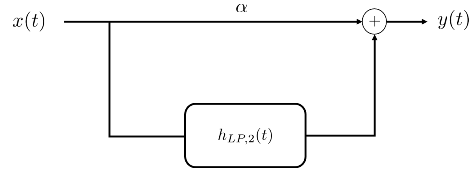
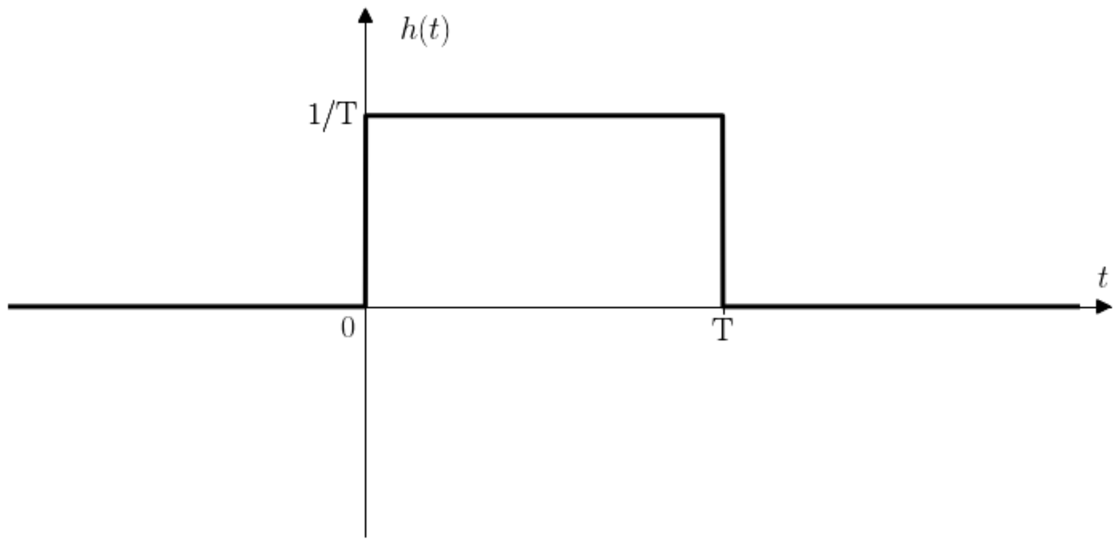


Figure 6: The system of part (a) with the non-ideal low pass filter

For the same value of α you found in part (a), find the frequency response of the equivalent system. Explain if the new system behaves as a high-pass filter.

3. (25 points) **Moving average filters**

We now consider the moving average filter, also known as a "boxcar" filter, and is one of the most primitive filters in practice. Its impulse response is shown below:



(a) (6 points) What is the frequency response $H(j\omega)$ of this filter?

(b) (6 points) Sketch the amplitude response $|H(j\omega)|$ of the filter. What happens to $|H(j\omega)|$ as $\omega \to \infty$?

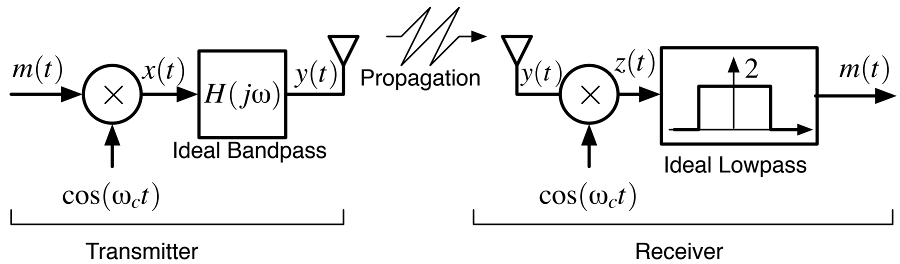
(c) (6 points) For non-constant **periodic signal** $x(t)$ with what period does $y(t) = x(t) * h(t) = C$ for some constant C (**Hint**: think about Fourier series and its Fourier transform.)?

(d) (7 points) Suppose we have a baseband signal $f(t)$ such that $F(j\omega) \neq 0$ for $-2\pi B < \omega < 2\pi B$, and $F(j\omega) = 0$ for $|\omega| > 2\pi B$. What is the maximum value of B (Hz) such that every frequency in $f(t)$ is retained in the output (i.e. $Y(j\omega) \neq 0$ for $-2\pi B < \omega < 2\pi B$)? Within this frequency range, what is the group delay, where group delay $t_d(\omega)$ is defined as

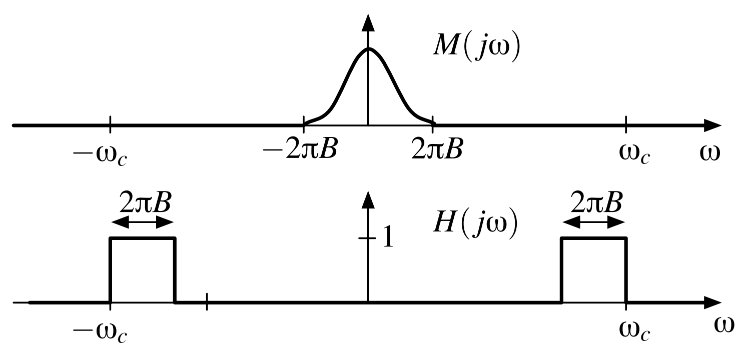
$$t_d(\omega) = -\frac{d}{d\omega}\angle H(j\omega)$$

4. (25 points) **Modulation and Demodulation**

(a) (10 points) Consider the communication system shown below:



The signal $m(t)$ is first modulated by $\cos(\omega_c t)$, and then passed through an ideal bandpass filter. The spectrum of the input $M(j\omega)$ and the frequency response of the ideal bandpass filter $H(j\omega)$ are:

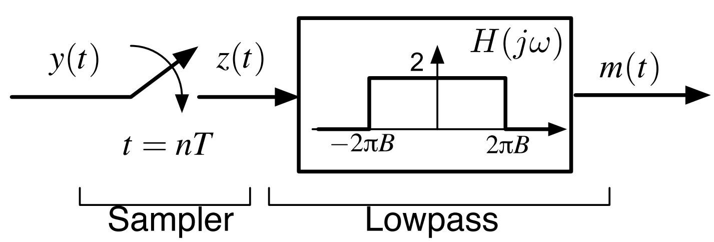


The modulated signal is $x(t)$, and the output of the ideal bandpass is $y(t)$. This signal is transmitted through a channel. We assume that this channel does not introduce distortion into $y(t)$. The received signal $y(t)$ is then processed by a receiver. Sketch the signal spectrum at

i. the output of the modulator, i.e., $X(j\omega)$,
ii. the output of the ideal bandpass, $Y(j\omega)$, and
iii. the output of the demodulator, $Z(j\omega)$

Does this system recover $m(t)$?

(b) (15 points) In the first part of this problem, you have seen that to demodulate the received signal, we multiply $y(t)$ by $\cos(\omega_c t)$, and then to recover $m(t)$, we low-pass filter the result. In this part, you will show that you can achieve the same effect with an ideal sampler. In other words, we assume instead the following block diagram of the receiver: where the ideal sampler is drawn as a switch that closes instantaneously every T seconds to acquire a new sample. Show that we can recover $m(t)$ if the ideal sampler operates at a frequency ω_c (i.e. samples at a rate of $\omega_c/2\pi$ samples/s). Draw the spectrum of the signal right before the lowpass filter $Z(j\omega)$.


$y(t)$
$z(t)$
$t = nT$
$H(j\omega)$
2
$-2\pi B$
$2\pi B$
$m(t)$
Sampler
Lowpass

1. (32 points) **Frequency Response**

(a) (18 points) Consider the LTI system depicted in figure 1 whose response to an unknown input, $x(t)$, is

$$y(t) = \left(4e^{-t} - 4e^{-4t}\right) u(t)$$

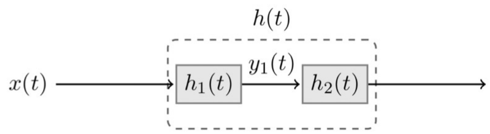


Figure 1: System for Problem 1(a)

We know that for the same unknown input $x(t)$, the intermediate signal, $y_1(t)$, is given by:

$$y_1(t) = 2e^{-t}u(t)$$

The overall LTI system is described by the following differential equation:

$$\frac{d^2}{dt^2}y(t) + 6\frac{d}{dt}y(t) + 8y(t) = 3x(t)$$

i. Find the frequency response, $H(j\omega)$, of the overall system $h(t)$.

ii. Find the frequency responses $H_1(j\omega)$ of the first LTI system and $H_2(j\omega)$ of the second LTI system.

iii. Find the impulse responses $h(t)$, $h_1(t)$ and $h_2(t)$.

1) $-w^2 Y(jw) + 6jw Y(jw) + 8 Y(jw) = 3X(jw)$

ai) $\Rightarrow Y(jw)\left[-w^2 + 6jw + 8\right] = 3X(jw)$

$\Rightarrow H(jw) = \dfrac{Y(jw)}{X(jw)} \Rightarrow \dfrac{3}{-w^2+6jw+8} = \dfrac{3}{(jw)^2+6(jw)+8} = \dfrac{3}{(4+jw)(2+jw)}$

$be^{-at}u(t) \Leftrightarrow \dfrac{b}{a+jw}$

ii) $H_1(jw) = \dfrac{Y_1(jw)}{X(jw)}$ $\quad H(jw) = H_1(jw)H_2(jw)$

$Y(jw) = F\left[\left(4e^{-t} - 4e^{-4t}\right)u(t)\right] \Rightarrow \dfrac{4}{1+jw} - \dfrac{4}{4+jw}$

$\Rightarrow \dfrac{16+4jw-4-4jw}{(1+jw)(4+jw)} = \dfrac{12}{(1+jw)(4+jw)}$

$X_2(jw) = Y_1(jw) \Rightarrow Y_1(jw) = \dfrac{2}{1+jw}$

$\Rightarrow H_2(jw) = \dfrac{Y(jw)}{X_2(jw)} = \dfrac{\frac{12}{(\cancel{1+jw})(4+jw)}}{\frac{2}{(\cancel{1+jw})}} = \dfrac{6}{(4+jw)}$

$\Rightarrow H_1(jw) = \dfrac{H(jw)}{H_2(jw)} = \dfrac{\frac{3}{(4+jw)(2+jw)}}{\frac{6}{(4+jw)}} = \dfrac{1}{2(2+jw)}$

iii) $h(t) = \mathcal{F}^{-1}\left[\frac{3}{(4+jw)(2+jw)}\right]$

$$\Rightarrow \frac{A}{4+jw} + \frac{B}{2+jw} = \frac{3}{(4+jw)(2+jw)}$$

$$A(2+jw) + B(4+jw) = 3$$

$$(A+B)(jw) = 0 \quad \Rightarrow A = -B$$

$$2A + 4B = 3 \quad \Rightarrow 2(-B) + 4B = 3 \Rightarrow B = \frac{3}{2}$$

$$A = -\frac{3}{2}$$

$$\Rightarrow -\frac{3}{2(4+jw)} + \frac{3}{2(2+jw)}$$

$$\Rightarrow h(t) = -\frac{3}{2} e^{-4t} u(t) + \frac{3}{2} e^{2t} u(t)$$

$$h_2(t) = \mathcal{F}^{-1}\left[\frac{6}{4+jw}\right] = 6e^{-4t} u(t)$$

$$\Rightarrow h_1(t) = \mathcal{F}^{-1}\left[\frac{1}{2(2+jw)}\right] = \frac{1}{2} e^{-2t} u(t)$$

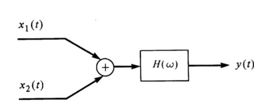


Figure 2: System for Problem 1(b)

b)

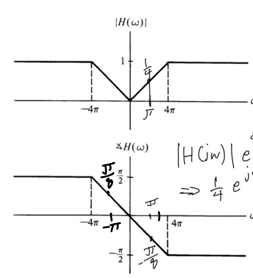
(b) (8 points) Consider the system shown in the figure 2, where the frequency response $H(\omega)$ has magnitude and phase shown in figure 3.
If,

$$x_1(t) = \sin\left[\omega_1 t + \frac{\pi}{4}\right]$$

$$x_2(t) = 2\cos\left[\omega_2 t - \frac{\pi}{3}\right]$$

where $\omega_1 = \pi$ and $\omega_2 = 2\pi$, write an expression for $y(t)$.

$\measuredangle Y = \measuredangle H + \measuredangle X$



Read at π

$|H(jw)| e^{j\measuredangle w}$

$\Rightarrow \frac{1}{4} e^{j\frac{\pi}{8}}$

Figure 3: Frequency response for Problem 1(b)

$$x_1(t) = \sin\left[w_1 t + \frac{\pi}{4}\right]$$

$$\Rightarrow \frac{1}{2j}\left[e^{j(w_1 t + \frac{\pi}{4})} - e^{-j(w_1 t + \frac{\pi}{4})}\right]$$

$$X_1(t) + X_2(t) \rightarrow \boxed{H(w)} \rightarrow y(t)$$

$$\Rightarrow \frac{1}{8j}\left[e^{j(\pi t + \frac{\pi}{4} - \frac{\pi}{8})} - e^{-j(\pi t + \frac{\pi}{4} - \frac{\pi}{8})}\right.$$

$$\frac{1}{8j}\left[e^{j(\pi t + \frac{\pi}{8})} - e^{-j(\pi t + \frac{\pi}{8})}\right]$$

$$= \frac{1}{4}\sin\left(\pi t + \frac{\pi}{8}\right)$$

$$X_2(t) = 2\cos\left[w_2 t - \frac{\pi}{3}\right]$$

at 2π $\measuredangle H = \frac{-\pi}{4}$ and $|H| = \frac{1}{2}$

$$\Rightarrow 2\left[\frac{1}{2}\left[e^{j(w_2 t - \frac{\pi}{3})} + e^{-j(w_2 t - \frac{\pi}{3})}\right]\right]$$

$$\Rightarrow \frac{1}{2}\left[e^{j(2\pi t - \frac{\pi}{3} - \frac{\pi}{4})} + e^{-j(2\pi t - \frac{\pi}{3} - \frac{\pi}{4})}\right]$$

$$\Rightarrow \cos\left(2\pi t - \frac{7\pi}{12}\right) \qquad \Rightarrow y(t) = \cos\left(2\pi t - \frac{7\pi}{12}\right) + \frac{1}{4}\sin\left(\pi t + \frac{\pi}{8}\right)$$

c)

(c) (6 points) Consider an LTI filter with $h(t)$ as impulse response. Its frequency response is given by:

$$H(j\omega) = \frac{1}{1 + j\omega}$$

i. Find the frequency range in Hertz in which the magnitude of the system function exhibits 1 percent or less deviation from its value at $\omega = 0$. Determine its cutoff frequency ω_c, where ω_c is defined below:

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}}|H(j0)|$$

ii. How can we transform $h(t)$ so that the cutoff frequency of the new filter becomes $5\omega_c$?

$$|H(jw_c)| = \frac{1}{\sqrt{2}}|H(j\cdot 0)| \qquad H(jw) = \frac{1}{1+jw}$$

$$\Rightarrow |H(jw)| = \frac{1}{\sqrt{1+w^2}}$$

$$\Rightarrow |H(j\cdot 0)| = \frac{1}{\sqrt{1+0^2}} = 1$$

$$\frac{1}{\sqrt{1+w_c^2}} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \frac{1}{\sqrt{1+w_i^2}} = 0.99 \qquad w_i = 0.14249 \text{ Hz}$$

ii) in order to do $5w_c$ we need to mutiply by 5 in time domaine and compress it by 5 as well.

So $5h(5t)$

2)

(a) (6 points) Consider an ideal low-pass filter $h_{LP,1}(t)$ with frequency response $H_{LP,1}(j\omega)$ depicted below in figure 4.

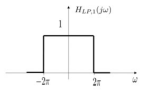


Figure 4: An ideal low pass filter

Using this filter, we construct the following new system:

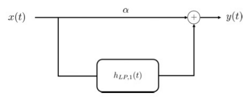


Figure 5: New system

We are given two choices for α: 1 or -1. Which value should we choose so that the new system is a high-pass filter? Does the new filter have any phase in its frequency response?

(b) (3 points) Why are the ideal filters non-realizable systems?

(c) (5 points) We want to design a causal non-ideal low-pass filter $h_{LP,2}(t)$, using the following frequency response:

$$H_{LP,2}(j\omega) = \frac{k}{\beta + j\omega}$$

Find k and β so that $H_{LP,2}(j0) = 1$ and its cutoff frequency is $\omega_0 = 2\pi$ rad/s, (i.e., the magnitude of $H_{LP,2}(j\omega)$ is $1/\sqrt{2}$ for $\omega = 2\pi$ rad/s).

(d) (4 points) We again consider the system of part (a) where instead of the ideal low-pass filter, we are going to use the non-ideal low-pass filter of part (c).

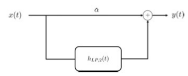


Figure 6: The system of part (a) with the non-ideal low pass filter

For the same value of α you found in part (a), find the frequency response of the equivalent system. Explain if the new system behaves as a high-pass filter.

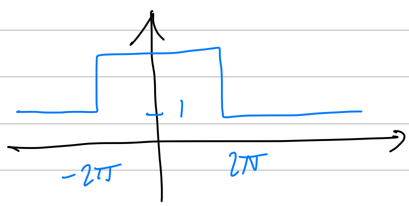
$$Y(jw) = \alpha X(jw) + H(jw)X(jw)$$

$$H(jw) = \frac{Y(jw)}{X(jw)}$$

$$\Rightarrow Y(jw) = X(jw)\left[H(jw) + \alpha\right]$$

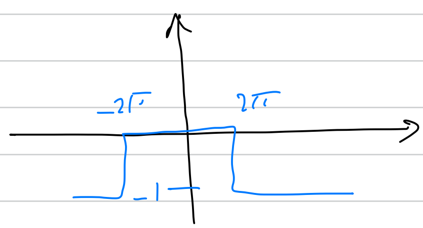
$$\frac{Y(jw)}{X(jw)} = H(jw) + \alpha$$

if $\alpha = 1$ $\Rightarrow$



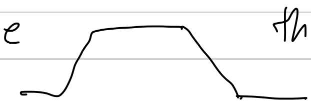
it doesn't work as a High Pass filter

if $\alpha = -1$ $\Rightarrow$



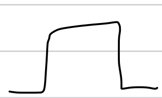
this works as a high Pass filter as it will filter low frequencies and flip the magnitude of high frequencies by -1 but it will still let high frequencies to Pass through.

b) the only difference between ideal and non ideal filters is that non ideals look like



this

and ideal look like



this and the difference as pictures show is so small that makes them unrealizable.

$$H_{LP2}(j\omega) = \frac{K}{\beta + j\omega} \quad \Rightarrow \omega = 0 \quad H_{LP2}(j0) = 1$$

c)

$$= \frac{K}{\beta + j(0)} = 1 \qquad |H_{LP2}(2\pi)| = \frac{1}{\sqrt{2}}$$

$$\Rightarrow K = \beta \qquad \Rightarrow |H_{LP2}(2\pi)| = \frac{1}{\sqrt{2}} = \left| \frac{K}{\beta + 2\pi j} \right|$$

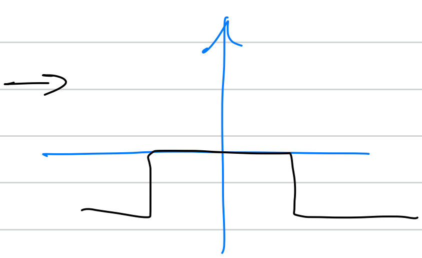
$$= \frac{K^2}{\beta^2 + 4\pi^2} = \frac{1}{2} \Rightarrow \frac{\beta^2}{\beta^2 + 4\pi^2} =$$

$\Rightarrow \beta = 2\pi$ and -2π

d) $\alpha X(j\omega) + H(j\omega) X(j\omega) = Y(j\omega)$

$$H(j\omega) + \alpha = \frac{Y(j\omega)}{X(j\omega)}$$

$$\frac{2\pi}{2\pi + j\omega} - 1 = \frac{Y(j\omega)}{X(j\omega)} \Rightarrow \frac{-j\omega}{2\pi + j\omega}$$



as $\omega \to \infty \Rightarrow -1$

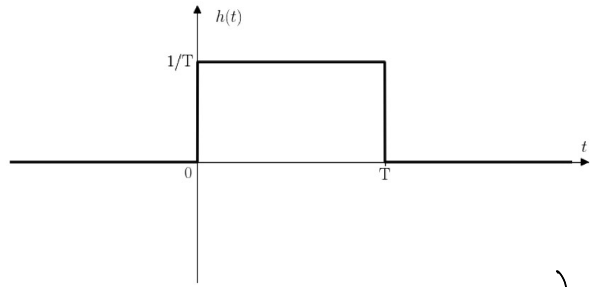
as $\omega \to 0 \Rightarrow 0$

So like a high pass filter.

3)

3. (25 points) **Moving average filters**

We now consider the moving average filter, also known as a "boxcar" filter, and is one of the most primitive filters in practice. Its impulse response is shown below:



(a) (6 points) What is the frequency response $H(j\omega)$ of this filter?

(b) (6 points) Sketch the amplitude response $|H(j\omega)|$ of the filter. What happens to $|H(j\omega)|$ as $\omega \to \infty$?

(c) (6 points) For non-constant **periodic signal** $x(t)$ with what period does $y(t) = x(t) * h(t) = C$ for some constant C (**Hint**: think about Fourier series and its Fourier transform.)?

(d) (7 points) Suppose we have a baseband signal $f(t)$ such that $F(j\omega) \neq 0$ for $-2\pi B < \omega < 2\pi B$, and $F(j\omega) = 0$ for $|\omega| > 2\pi B$. What is the maximum value of B (Hz) such that every frequency in $f(t)$ is retained in the output (i.e. $Y(j\omega) \neq 0$ for $-2\pi B < \omega < 2\pi B$)? Within this frequency range, what is the group delay, where group delay $t_d(\omega)$ is defined as

$$t_d(\omega) = -\frac{d}{d\omega}\angle H(j\omega)$$

$$h(t) = \frac{1}{T}\,\text{rect}\left(\frac{1}{T}\left(t-\frac{T}{2}\right)\right)$$

$$\text{rect}\left(\frac{t}{T}\right) \Longleftrightarrow T\,\text{sinc}\left(\frac{T\omega}{2\pi}\right)$$

a)

$$\frac{1}{T}\,\text{rect}\left(\frac{t}{T}\right) \Longleftrightarrow \text{sinc}\left(\frac{T\omega}{2\pi}\right)$$

$$\frac{1}{T}\,\text{rect}\left(\frac{t-\frac{T}{2}}{T}\right) \Longleftrightarrow e^{-j\omega\frac{T}{2}}\,\text{sinc}\left(\frac{T\omega}{2\pi}\right)$$

$$|H(j\omega)| = \left|e^{-j\omega\frac{T}{2}}\,\text{sinc}\left(\frac{T\omega}{2\pi}\right)\right|$$

$$= \left|e^{-j\omega\frac{T}{2}}\right|\left|\text{sinc}\left(\frac{T\omega}{2\pi}\right)\right|$$

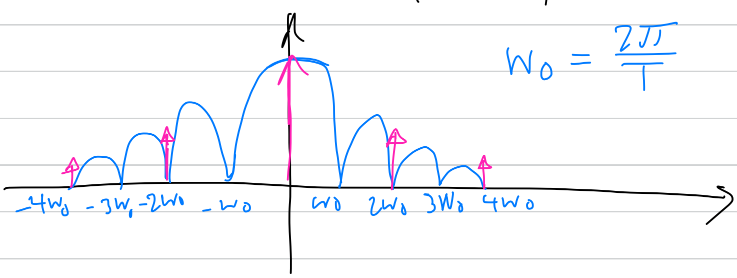
$$\left|e^{j\theta}\right| = 1$$

$$\Rightarrow |H(j\omega)| = \left|\text{sinc}\left(\frac{T\omega}{2\pi}\right)\right|$$

b)

$|H(j\omega)|$

$$\omega_0 = \frac{2\pi}{T}$$



c) $y(t) = x(t) * h(t) = C$

$Y(j\omega) = X(j\omega)\cdot H(j\omega) = C \cdot 2\pi\delta(\omega)$

$X_2(j\omega)$ is discrete.

$$\omega_x = \frac{2\pi}{T_x}$$

$$\omega_x = k\omega_0 = \frac{2\pi}{T}$$

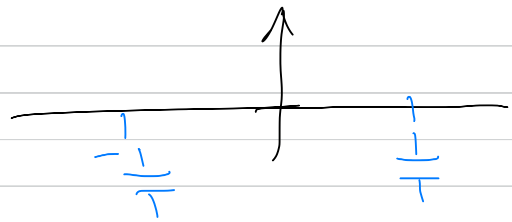
$$T_x = \frac{T}{k}$$

$$\Rightarrow \omega_x = 2\omega_0$$

d) $Y(jw) = 0$ when $|w| > 2\pi B$

$$\left|\frac{2\pi}{T}\right| > 2\pi B$$

$$\Rightarrow \left.\begin{aligned} \frac{2\pi}{T} &> 2\pi B \\ -\frac{2\pi}{T} &< 2\pi B \end{aligned}\right\} \begin{aligned} \frac{1}{T} &> B \\ -\frac{1}{T} &< B \end{aligned} \Rightarrow |B| < \frac{1}{T}$$



max $B = \frac{1}{T}$

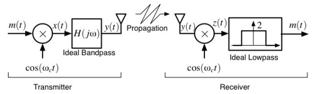
$$\measuredangle H(jw) = -\frac{wT}{2}$$

$$\frac{d}{dw}\left(\measuredangle H(jw)\right) = \boxed{-\frac{T}{2}}$$

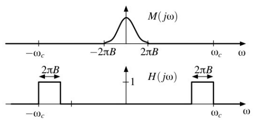
4)

4. (25 points) **Modulation and Demodulation**

(a) (10 points) Consider the communication system shown below:



The signal $m(t)$ is first modulated by $\cos(\omega_c t)$, and then passed through an ideal bandpass filter. The spectrum of the input $M(j\omega)$ and the frequency response of the ideal bandpass filter $H(j\omega)$ are:



The modulated signal is $x(t)$, and the output of the ideal bandpass is $y(t)$. This signal is transmitted through a channel. We assume that this channel does not introduce distortion into $y(t)$. The received signal $y(t)$ is then processed by a receiver. Sketch the signal spectrum at

i. the output of the modulator, i.e., $X(j\omega)$,
ii. the output of the ideal bandpass, $Y(j\omega)$, and
iii. the output of the demodulator, $Z(j\omega)$

Does this system recover $m(t)$?

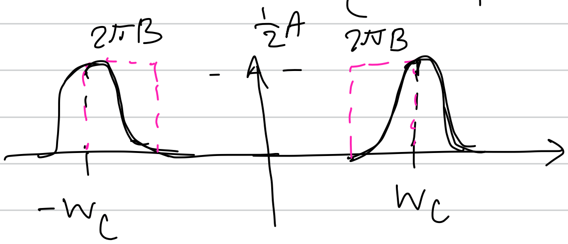
(b) (15 points) In the first part of this problem, you have seen that to demodulate the received signal, we multiply $y(t)$ by $\cos(\omega_c t)$, and then to recover $m(t)$, we low-pass filter the result. In this part, you will show that you can achieve the same effect with an ideal sampler. In other words, we assume instead the following block diagram of the receiver: where the ideal sampler is drawn as a switch that closes instantaneously every T seconds to acquire a new sample. Show that we can recover $m(t)$ if the ideal sampler operates at a frequency ω_c (i.e. samples at a rate of $\omega_c/2\pi$ samples/s). Draw the spectrum of the signal right before the lowpass filter $Z(j\omega)$.

a i) $m(t)\cos(W_c t) = m(t)\left[\frac{1}{2}\left(e^{jW_c t} + e^{-jW_c t}\right)\right]$

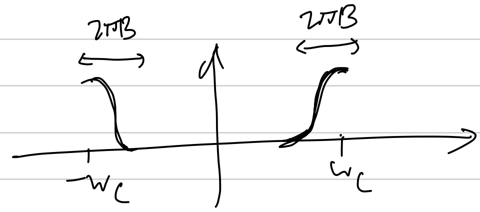
$$\frac{1}{2}m(t)e^{jW_c t} + \frac{1}{2}m(t)e^{-jW_c t} \Longleftrightarrow \frac{1}{2}M(j(W+W_c)) + \frac{1}{2}M(j(W-W_c))$$

$\Rightarrow$ shifted by W_c and $-W_c$ and scaled by $\frac{1}{2}$.

a ii)



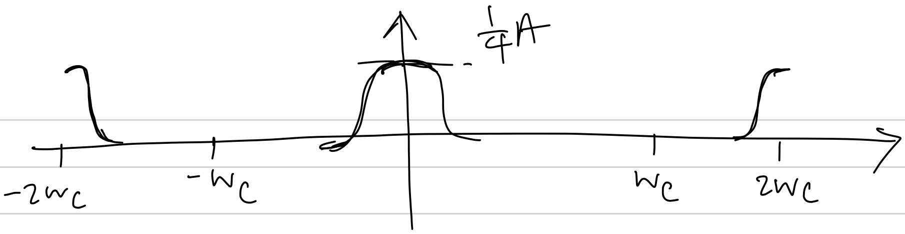
$\Rightarrow$



a iii) $y(t)\cos(W_c t) \Rightarrow \frac{1}{2}y(t)e^{jW_c t} + \frac{1}{2}y(t)e^{-jW_c t}$

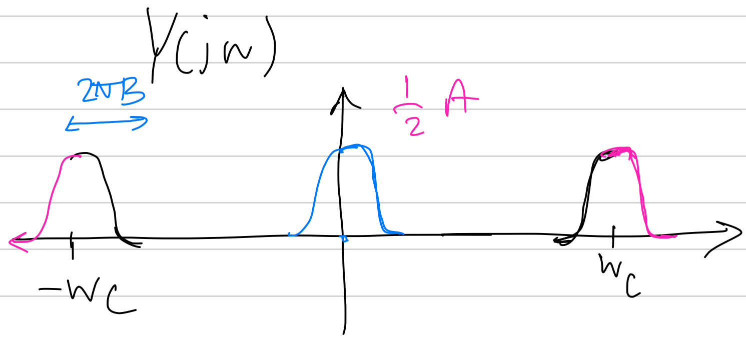
$$\Rightarrow \frac{1}{2}\left[\frac{1}{2}M(j(W+W_c+W_c)) + \frac{1}{2}M(j(W-W_c+W_c))\right]$$

$$+\frac{1}{2}\left[\frac{1}{2}M(j(W-W_c-W_c)) + \frac{1}{2}M(j(W+W_c-W_c))\right]$$

$\Rightarrow$ 

$\Rightarrow 2 \times \frac{1}{4} = \frac{1}{2} m(t)$ it will recover the signal but with $\frac{1}{2}$ of the amplitude.

b)

$\Rightarrow$ 

$\Rightarrow$ at T see samples so $W = \frac{2\pi}{T}$

$\Rightarrow T = \frac{2\pi}{W} \Rightarrow \frac{W_c}{2\pi}$ $\frac{\text{samples}}{s}$

since $Z(jw) = \sum_{n=-\infty}^{\infty} Y(j(w - nw_c))$ so they get sampled at each w_c