ECE102, Fall 2023 **Homework #5**

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Due Monday, 13 Nov 2023, by 11:59pm to Gradescope.
100 points total.
This homework covers questions relate to Fourier series and Fourier transform.

1. (18 points) **Fourier Series**

   (a) (6 points) When the periodic signal $f(t)$ is real, we know that the fourier coefficients follow the following ,
   $Re(c_k) = Re(c_{-k})$, $Im(c_k) = -Im(c_{-k})$, $c_k^* = c_{-k}$, $|c_k| = |c_{-k}|$, $\angle c_k = -\angle c_k^*$.
   If $f(t)$ is purely imaginary, how do the above relations change ?, provide mathematical justification .

   (b) (12 points) Suppose we are given the following information about a signal $x(t)$, please write down the expression of $x(t)$:

   - $x(t)$ is real and even.
   - $x(t)$ is periodic with period $T = 16$ and has Fourier coefficients a_k.
   - $a_k \neq 0$ if and only if $|k| \leq 1$.
   - $\int_0^4 |x(t) - (\Sigma_{l=0}^{\infty} a_{2l})|^2 dt = 2$.
   - DC component of the signal is 5

   $a_k = 0$
   for $k < -1$
   for $k > 1$

   $\int_0^4 \left|\sum_{k=-1}^{1} a_k e^{jk\omega_0 t} - \sum_{l=-\infty}^{\infty} a_{2l}\right|^2$

   $\int_0^4 |\cancel{5} + a_1 e^{jk\omega_0 t} + a_{-1} e^{jk\omega_0 t} \cancel{-5}|^2 = 2$

   $\int_0^4 a_1 e^{jk\omega_0 t} + a_{-1} e^{-jk\omega_0 t} dt = 2$

   $a_0 = 5$

2. (32 points) **Symmetry properties of Fourier transform**

   (a) (16 points) Determine which of the signals, whose Fourier transforms are depicted in Fig. 1, satisfy each of the following:

   i. $x(t)$ is even
   ii. $x(t)$ is odd
   iii. $x(t)$ is real
   iv. $x(t)$ is complex (neither real, nor pure imaginary)
   v. $x(t)$ is real and even
   vi. $x(t)$ is imaginary and odd
   vii. $x(t)$ is imaginary and even
   viii. There exists a non-zero ω_0(ω_0 can be positive or negative) such that $e^{j\omega_0 t}x(t)$ is real and even

   (b) (8 points) Using the properties of Fourier transform, determine whether the assertions are true or false.

   i. The convolution of a real and even signal and a real and odd signal is odd.
   ii. The convolution of a imaginary and odd signal and the same signal time reversed is an real and odd signal.

   (c) (8 points) Show the following statements:

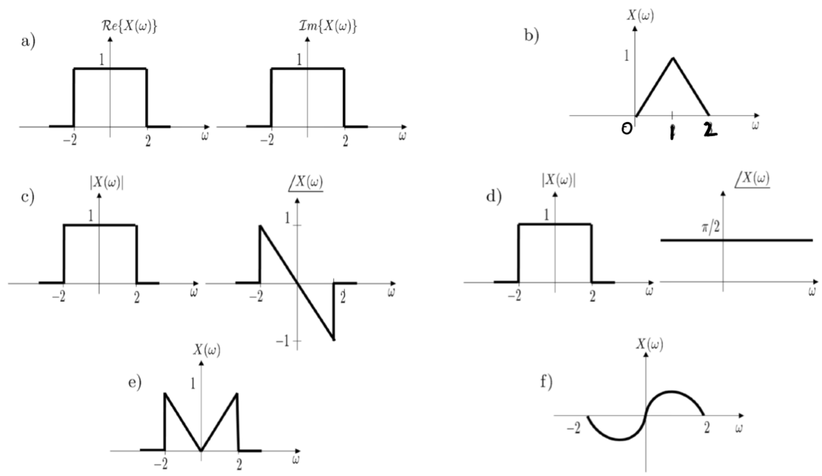


Figure 1: P2.a

i. If $x(t) = x^*(-t)$, then $X(j\omega)$ is real.

ii. If $x(t)$ is a real signal with $X(j\omega)$ its Fourier transform, then the Fourier transforms $X_e(j\omega)$ and $X_o(j\omega)$ of the even and odd components of $x(t)$ satisfy the following:

$$X_e(j\omega) = Re\{X(j\omega)\}$$

and

$$X_o(j\omega) = jIm\{X(j\omega)\}$$

3. (20 points) **Halloween Adventures with the Mystery Box**
On Halloween evening after ECE 102 lecture, you and your friend decide to head back home together. You are tired and are waiting for the pedestrian signal near Ronald Reagan Hospital, but it is taking forever... Suddenly your friend shouts “look what I found!” You see a weird box that has metallic contacts, a rugged structure with spider webs and a display monitor associated with it. Spooky!! You and your friend grow curious, and in the spirit of Halloween want to find out what this box all about. You decide to use a ideal signal generator from one of the labs in Engineering IV, and you observe the following.

(a) (3 points) When you input a $\delta(t)$, the mystery box displays a mathematical expression of the mystery function =1
When you input a sine wave of frequency 5 Hz the mystery box displays a mathematical expression for mystery function
Mystery function = $j\pi(\delta(\omega + 10\pi) - \delta(\omega - 10\pi)$ where w is in rad/s.

What operation that you learnt in ECE 102 do you think the Mystery Box is performing?

(b) (10 points) Happy with your conclusions, you and your friend get coffee from nearest coffee shop, and want to explore the mystery box again. But oops!! You spill coffee on the mystery device and spoil only its display, while keeping the rest of its functionality intact.

Consider your signal generator inputs time-domain signal $x(t)$ plotted below. Using the properties/definition of the Mystery function (let's call it $X(j\omega)$), you should be able able to compute all of the quantities required in each of the parts below, without explicitly looking at the Mystery function displayed. Indicate which properties you are using and clearly label your final answer for each part.

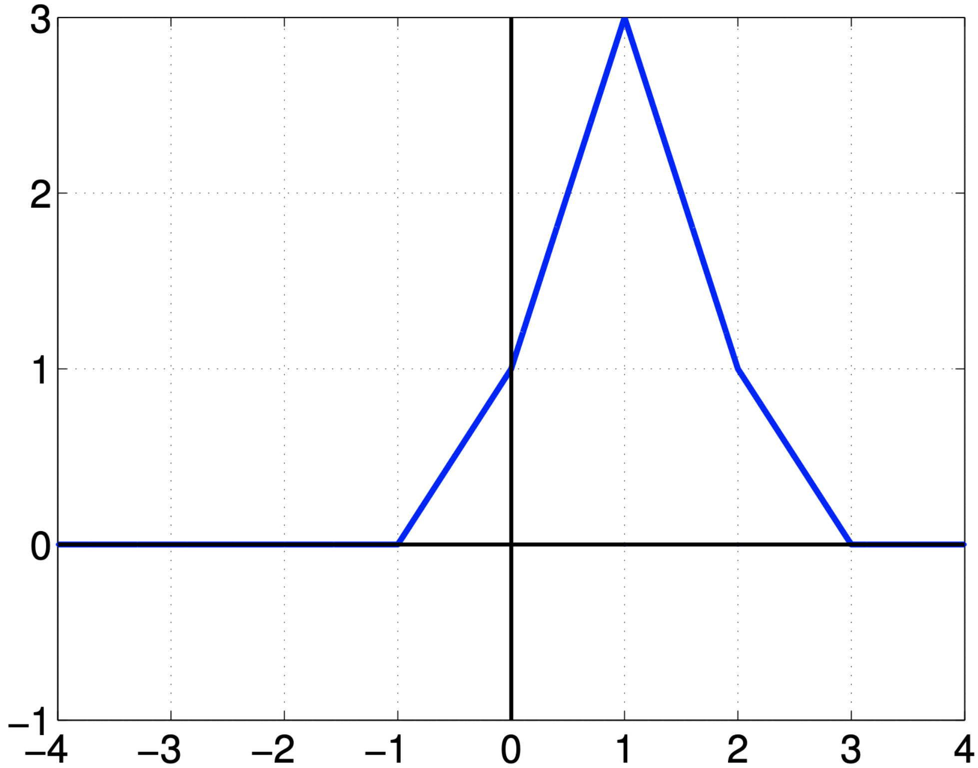


Figure 2: $x(t)$

i. Your friend claims that the value of $X(0)$ is 6 is your friend right ?

ii. Evaluate $\int_{-\infty}^{+\infty} X(j\omega)d\omega$

iii. Evaluate $\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$

iv. Compute the value of $Im(X(j\omega))$ i.e Imaginary component of mystery function When input from signal generator is $x(t+1) + x(-t+1)$?

v. We know that for the input $x(t)$ above mystery box outputs some $X(j\omega)$, express a new input $x(t)$ where the output of the mystery box is $\frac{1}{5}X(\frac{j\omega}{5})e^{j2\omega}$.

(c) Having explored the mystery box enough, you get more curious and feed $x(t) = t^4$ to the mystery box, and you start hearing a rumbling sound. The sound grows with time and you started getting concerned, it is Halloween after all... and then suddenly *smash*!! You see that the mystery box crashes and stops working once and for all, ending your adventure for that day :(. Disheartened you and your friend wonder what just happened? Can you think of a reason why the mystery box may have crashed and stopped working when you supplied it with this particular $x(t)$? What could you have done to avoid this crash? (7 points)

4. (30 points) **Fourier transform and its inverse**

(a) (21 points) Find the Fourier transform of each of the signals given below:
Hint: you may use Fourier Transforms derived in class.

i. (**optional**) $x_1(t) = 2\mathrm{rect}\left(\frac{-t-3}{2}\right)\cos(10\pi t)$

ii. $x_2(t) = e^{(2+3j)t}u(-t+1)$

iii. $x_3(t) = \begin{cases} 1+\cos(\pi t), & |t| < 1 \\ 0, & \text{otherwise} \end{cases}$

iv. $x_4(t) = te^{-2t}u(t)$

(b) (9 points) Two signals $f_1(t)$ and $f_2(t)$ are defined as

$$f_1(t) = \mathrm{sinc}(2t)$$
$$f_2(t) = \mathrm{sinc}(t)\cos(3.1\pi t)$$

Let the convolution of the two signals be

$$f(t) = (f_1 * f_2)(t)$$

i. Find $F(j\omega)$, the Fourier transform of $f(t)$.

ii. Find $f(t)$.

1. (18 points) **Fourier Series**

(a) (6 points) When the periodic signal $f(t)$ is real, we know that the fourier coefficients follow the following ,
$Re(c_k) = Re(c_{-k})$, $Im(c_k) = -Im(c_{-k})$, $c_k^* = c_{-k}$, $|c_k| = |c_{-k}|$, $\angle c_k = -\angle c_k^*$.
If $f(t)$ is purely imaginary, how do the above relations change ?, provide mathematical justification .

$$w_0 = \frac{2\pi}{T_0}$$

$$\underset{\text{Real}}{f(t)} = j \cdot \underset{\text{Real}}{h(t)}$$

$$\Rightarrow C_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} j \cdot h(t)\, e^{-jkw_0t}\, dt$$

$$\Rightarrow C_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} j \cdot h(t)\left[\cos\left(\frac{2\pi k}{T_0}t\right) - j\sin\left(\frac{2\pi k}{T_0}t\right)\right] dt$$

$$\Rightarrow C_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} j \cdot h(t)\cos\left(\frac{2\pi k}{T_0}t\right)dt + \frac{1}{T_0}\int_{t_0}^{t_0+T_0} h(t)\sin\left(\frac{2\pi k}{T_0}t\right) dt$$

$$Im(C_k) = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} j\, h(t)\cos\left(\frac{2\pi k}{T_0}t\right) dt$$

$$Real(C_k) = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} h(t)\sin\left(\frac{2\pi k}{T_0}t\right)dt$$

$$Real(C_{-k}) = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} h(t)\sin\left(-\frac{2\pi k\, t}{T_0}\right) dt$$

$$\Rightarrow -\frac{1}{T_0}\int_{t_0}^{t_0+T_0} h(t)\sin\left(\frac{2\pi k}{T_0}t\right) dt$$

$$Real(C_{-k}) = -Real(C_k)$$

ii) $Im(C_{-k}) \Rightarrow \frac{1}{T_0}\int_{t_0}^{t_0+T_0} h(t)\, j\cos\left(\frac{-2\pi k}{T_0}t\right) dt$

$$= \frac{1}{T_0}\int_{t_0}^{t_0+T_0} h(t)\, j\cos\left(\frac{2\pi k t}{T_0}\right) dt$$

$Im(C_{-k}) = Im(C_k)$

iii) $C_k^* \Rightarrow \frac{1}{T_0}\int_{t_0}^{t_0+T_0} -h(t)\, j\cos\left(\frac{2\pi k t}{T_0}\right) dt + \frac{1}{T_0}\int_{t_0}^{t_0+T_0} h(t)\sin\left(\frac{2\pi k}{T_0}t\right)dt$

$$= -Im(C_k) + Real(C_k)$$

$$= -Im(C_{-k}) - Real(C_{-k})$$

$$= -\left(Real(C_{-k}) + Im(C_{-k})\right)$$

$$= -C_{-k}$$

$|A+B| \leq |A| + |B|$

IV) $C_k = Real(C_k) + Im(C_k)$

$C_{-k} = Real(C_{-k}) + Im(C_{-k}) \Rightarrow C_{-k} = -Real(C_k) + Im(C_k)$

$\Rightarrow |C_k| = |Real(C_k) + Im(C_k)| \Rightarrow |C_k| = |Real(C_k)| + |Im(C_k)|$

$|C_{-k}| = |-Real(C_k) + Im(C_k)| \Rightarrow |C_{-k}| = |-Real(C_k)| + |Im(C_k)|$

$|C_{-k}| = |Real(C_k)| + |Im(C_k)|$

$\Rightarrow |C_{-k}| = |C_k|$

V) $\angle C_k = \tan^{-1}\left(\frac{Im(C_k)}{Real(C_k)}\right)$ $\quad C_k^* = -C_{-k} =$

$C_{-k} = -Real(C_k) + Im(C_k)$

$-C_{-k} = Real(C_k) - Im(C_k)$

$$\measuredangle C_k^* = \measuredangle -C_{-k} = \tan^{-1}\left(\frac{-Im(C_k)}{Real(C_k)}\right)$$

$\Rightarrow \measuredangle C_k = -\measuredangle -C_{-k} \Rightarrow \measuredangle C_k = -\measuredangle C_k^*$

1b)

(b) (12 points) Suppose we are given the following information about a signal $x(t)$, please write down the expression of $x(t)$:

$T = 16 \quad w_0 = \frac{2\pi}{T_0} = \frac{2\pi}{16} = \frac{\pi}{8}$

- $x(t)$ is real and even.
- $x(t)$ is periodic with period $T = 16$ and has Fourier coefficients a_k.
- * $a_k \neq 0$ if and only if $|k| \leq 1$. $\longrightarrow$ $a_k = 0$ when $k > 1$, $k < -1$
- $\int_0^4 |x(t) - (\sum_{l=0}^{\infty} a_{2l})|^2 dt = 2.$
- DC component of the signal is 5 $\rightarrow a_0 = 5$

$a_2 = 0$

2. (32 points) **Symmetry properties of Fourier transform**

$$\sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t} \Rightarrow \quad k = -1, 1, 0$$

$$\Rightarrow a_1 e^{jw_0 t} + a_{-1} e^{-jw_0 t} + a_0 = x(t)$$

$a_1 = a_{-1}$ ($x(t)$ is real and even)

$$\int_0^4 \left| x(t) - \sum_{l=0}^{\infty} a_{2l} \right|^2 dt = 2 \Rightarrow \int_0^4 \left| \cancel{a_0} + a_1 e^{jw_0 t} + a_{-1} e^{-jw_0 t} - \cancel{a_0} \right|^2 dt = 2$$

$$\Rightarrow \int_0^4 \left| a_1 e^{jw_0 t} + a_1 e^{-jw_0 t} \right|^2 dt = 2$$

$$\Rightarrow \int_0^4 \left| a_1 \left(e^{jw_0 t} + e^{-jw_0 t} \right) \right|^2 dt = 2$$

$$\int_0^4 \left| a_1 (2\cos(w_0 t)) \right|^2 dt = 2$$

$$\Rightarrow \int_0^4 4a_1^2 \cos^2(w_0 t)\, dt = 2$$

$$a_1^2 \int_0^4 \cos^2\left(\frac{\pi}{8} t\right) dt = \frac{1}{2}$$

$$a_1^2 \int_0^4 \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi}{4} t\right) dt = \frac{1}{2}$$

$u = \frac{\pi}{4} t$

$du = \frac{\pi}{4} dt \quad \frac{4\,du}{\pi} = dt$

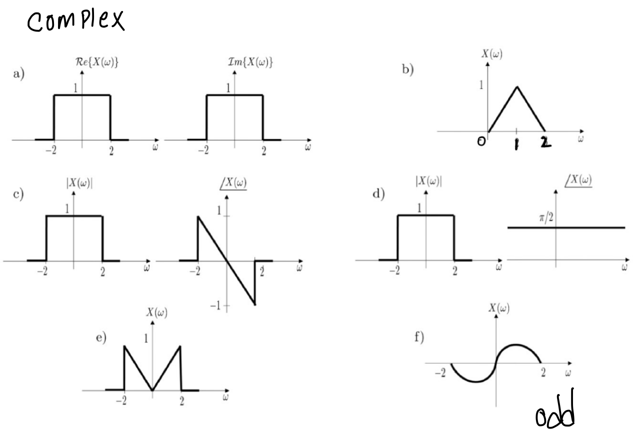
$$2a_1^2 + \frac{2}{\pi} \sin\left(\frac{\pi}{4} t\right) \Big|_0^4$$

$$\Rightarrow 2a_1^2 = \frac{1}{2} \Rightarrow a_1^2 = \frac{1}{4} \Rightarrow a_1 = \pm \frac{1}{2}$$

$$\Rightarrow x(t) = 5 + \frac{1}{2} e^{jw_0 t} + \frac{1}{2} e^{-jw_0 t}$$

$$\Rightarrow 5 + \frac{1}{2}\left[e^{jw_0 t} + e^{-jw_0 t} \right] = 5 + \cos(w_0 t) = 5 + \cos\left(\frac{\pi}{8} t\right)$$

2a)



$X(w) = |X(w)| e^{j\measuredangle X(w)}$

i) a, D, E are even. $X(-jw) = X(jw)$

if $X(jw)$ even $\Rightarrow$ $X(t)$ is even.

ii) F magnitude and Phase is odd.

iii) if $X(t)$ Real $\Rightarrow X(-jw) = X^*(jw)$ $\Rightarrow$ C and e

IV) a and b

V) $X(t)$ Real and eve if $X(jw)$ is Real and even

so only $\boxed{C}$

VI) $X(t)$ imaginary and odd if $X(jw)$ is real and odd.

so $\boxed{F}$

VII) $X(t)$ imaginary and even if $X(jw)$ is imaginary and even.

so only $\boxed{D}$

VIII) $e^{jw_0 t} x(t) \Leftrightarrow F(jw - jw_0)$

Shifted by w_0

$X(t)$ is Real and even if $X(jw)$ is real and even.

So only b

(b) (8 points) Using the properties of Fourier transform, determine whether the assertions are true or false.

i. The convolution of a real and even signal and a real and odd signal is odd. True.

ii. The convolution of a imaginary and odd signal and the same signal time reversed is an real and odd signal. False

2b)

Real and even $= f(t)$

Real and odd $= g(t)$

i) $F[(f*g)(t)] = F(jw)\cdot G(jw)$

$\Rightarrow F(f(t)) = \text{Real and even} = h_1(jw) \Rightarrow h_1(-jw) = h_1(jw)$

$F(g(t)) = \text{Imaginary and odd} = F(g(t)) \Rightarrow h_2(jw)\cdot j$

$\Rightarrow j\, h_1(jw)\cdot h_2(jw) = H(jw)$ $\qquad h_2(-jw) = -h_2(jw)$

$\Rightarrow H(-jw) = -j\, h_1(jw)\, h_2(jw)$ odd and Imaginary.

$F^{-1}(H(jw)) = \text{Real and odd}$ $\boxed{\text{True.}}$

ii) $f(t) = \text{Imaginary and odd}$

$g(t) = \text{Imaginary and odd} = f(-t)$

$F[(f*g)(t)] = F(jw)\cdot G(-jw)$

$x_o(-t) = -x_o(t)$

$F(jw) = \text{Real and odd} = h_1(jw) =$

$G(-jw) = \text{Real and odd} \Rightarrow = h_2(-jw)$

$\Rightarrow h_1(jw)\cdot h_2(-jw) = H(jw) \Rightarrow H(jw)$ is real and even.

$\Rightarrow H(t) = \text{Real and odd} \qquad \mathcal{F}^{-1}(H(t)) = \text{imaginary and odd}$

2c)

i. If $x(t) = x^*(-t)$, then $X(j\omega)$ is real.

ii. If $x(t)$ is a real signal with $X(j\omega)$ its Fourier transform, then the Fourier transforms $X_e(j\omega)$ and $X_o(j\omega)$ of the even and odd components of $x(t)$ satisfy the following:

$$X_e(j\omega) = Re\{X(j\omega)\}$$

and

$$X_o(j\omega) = jIm\{X(j\omega)\}$$

i) $X(jw)$ is real if $X(jw) = X^*(jw)$

$$X^*(jw) = \left[\int x(t) e^{-jwt} dt\right]^*$$

$t = -u \qquad t = \infty \Rightarrow u = -\infty$

$dt = -du \qquad t = -\infty \Rightarrow u = +\infty$

$$= \int_{-\infty}^{\infty} x^*(t) e^{jwt} dt$$

$$= \int_{-\infty}^{\infty} x^*(-u) e^{-jwu} \cdot (-du)$$

$x^*(-u) = x(u)$

$$= -\int_{+\infty}^{-\infty} x^*(-u) e^{-jwu} du = \int_{-\infty}^{\infty} x^*(-u) e^{-jwu} du$$

$$= \int_{-\infty}^{\infty} x(u) e^{-jwu} du$$

$$= X(jw)$$

$x(t) \Longleftrightarrow X(jw)$

$X^*(jw) = X(jw)$

ii) $x(t)$ is real $\qquad X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

$$\Rightarrow \int_{-\infty}^{\infty} x_e(t)\, e^{-j\omega t}\, dt + \int_{-\infty}^{\infty} x_o(t)\, e^{-j\omega t}\, dt$$

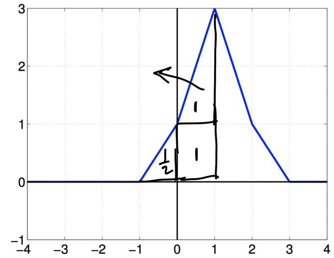
$$= \int_{-\infty}^{\infty} x_e(t) \cos(\omega t) \cancelto{0}{- j x_e \sin(\omega t)\, dt} + \int_{-\infty}^{\infty} \cancelto{0}{x_o(t) \cos(\omega t)} - x_o(t)\, j \sin(\omega t)\, dt$$

$$\Rightarrow \underbrace{\int_{-\infty}^{\infty} x_e(t) \cos(\omega t)}_{X_e(j\omega)} \underbrace{- \int_{-\infty}^{\infty} j x_o(t) \sin(\omega t)}_{X_o(j\omega)}$$

3)

a) fourier transform

$2+\frac{1}{2} = \frac{5}{2}$



X(t)

b) i) $X(0) = 6$ $\quad \int_{-\infty}^{\infty} x(t) e^{-jwt} \, dt$

$$\Rightarrow \int_{-\infty}^{\infty} x(t) e^{-j(0)t} \, dt \Rightarrow \int_{-\infty}^{\infty} x(t) \, dt = 5$$

Area under the curve.

they are wrong.

ii) $\int_{-\infty}^{\infty} X(jw) \, dw \Rightarrow x(t) = \frac{1}{2\pi} \int_{-\infty} X(jw) e^{jwt} \, dw$

$$2\pi x(0) = \int_{-\infty}^{\infty} X(jw) \, dw$$

$$2\pi(1) = \int_{-\infty}^{\infty} X(jw) \, dw$$

$$= 2\pi$$

iii) $\int_{-\infty}^{\infty} |X(jw)|^2 \, dw = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 \, dt$

$$2\pi \left[2\int_{-1}^{0} |(t+1)|^2 + 2\int_{0}^{1} |(2t+1)|^2 \, dt \right.$$

$$2\pi \left[2\int_{-1}^{0} t^2 + 2t + 1 \, dt + 2\int_{0}^{1} 4t^2 + 4t + 1 \, dt \right]$$

$$2\pi \left[\frac{2}{3} + \frac{26}{3} \right] = \frac{56\pi}{3}$$

$\overset{\nearrow f(t)}{}$ $\overset{\nearrow g(t)}{}$

IV) $x(t+1) + x(-t+1)$

$x(t+1)$: even and real; $x(-t+1)$: even and real

$f(t) \Rightarrow F(jw)$ — even and real

$g(t) \Rightarrow G(jw)$ — even and real

$F(jw) + G(jw) = X(jw)$

Real Real

$\Rightarrow Im(X(jw)) = 0$

V) $\frac{1}{5} X\left(\frac{jw}{5}\right) e^{j2w}$

$f(at) \Leftrightarrow \frac{1}{|a|} F\left(j\frac{w}{a}\right)$

$f(t-\alpha) \Leftrightarrow e^{-j\alpha w} F(jw)$

$\Rightarrow a = 5$

$= \alpha = +2 \quad \Rightarrow \quad X(5(t+2))$

c) fourier trans form of a signal such as $x(t) = t^4$ means a signal that has infinite energy as $X(jw) = \int_{-\infty}^{\infty} t^4 e^{-jwt} dt$ will not converge so we can't have a value that the mystery box could output. we could prevent this by using energy signals that have finite energy or $X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$ converges.

4a) ii) $X_2(t) = e^{(2+3j)t} u(-t+1)$

$$\Rightarrow e^{3jt} e^{2t} u(-(t-1)) = e^{3jt} e^{2(t-1)} e^{2} u(-(t-1))$$

$$e^{-at} u(t) \Leftrightarrow \frac{1}{a+jw} \quad , \quad e^{2t} u(-t) \Leftrightarrow \frac{1}{2-jw}$$

$$e^{2(t-1)} u(-(t-1)) \Leftrightarrow \frac{e^{-jw}}{2-jw}$$

$$e^{3jt} e^{2(t-1)} u(-(t-1)) \Leftrightarrow \frac{e^{-j(w-3)}}{2-j(w-3)}$$

$$X_2(t) \Leftrightarrow e^{2} \frac{e^{-j(w-3)}}{2-j(w-3)}$$

iii) $X_3(t) = \begin{cases} 1+\cos(\pi t), & |t| < 1 \\ 0, & \text{otherwise} \end{cases}$

$$X(jw) = \int_{-\infty}^{\infty} X_3(t) e^{-jwt} dt = \int_{-1}^{1} (1+\cos(\pi t)) e^{-jwt} dt$$

$$\Rightarrow \int_{-1}^{1} e^{-jwt} dt + \int_{-1}^{1} \cos(\pi t) e^{-jwt} dt$$

$$\int_{-1}^{1} e^{-jwt} dt = \frac{e^{-jwt}}{-jw} \Big|_{-1}^{1} = \frac{e^{-jw}}{-jw} - \frac{e^{jw}}{-jw} = \frac{e^{-jw} - e^{jw}}{-jw} = \frac{2}{w} \sin(w) = 2\text{sinc}(w)$$

$$\int_{-1}^{1} e^{-jwt}\cos(\pi t)\,dt = \int_{-1}^{1} \frac{1}{2}\left[e^{j\pi t} + e^{-j\pi t}\right] e^{-jwt}\,dt =$$

$$= \frac{1}{2}\int_{-1}^{1} e^{jt(\pi - w)}\,dt + \frac{1}{2}\int_{-1}^{1} e^{-jt(\pi + w)}\,dt$$

$$= \frac{1}{2}\left[\frac{e^{jt(\pi-w)}}{j(\pi-w)}\right]\Bigg|_{-1}^{1} + \frac{1}{2}\left[\frac{e^{-jt(\pi+w)}}{-j(\pi+w)}\right]\Bigg|_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{e^{j(\pi-w)} - e^{-j(\pi-w)}}{j(\pi-w)}\right] + \frac{1}{2}\left[\frac{e^{-j(\pi+w)} - e^{j(\pi+w)}}{-j(\pi+w)}\right]$$

$$\frac{\sin(\pi-w)}{\pi-w} - \frac{\sin(\pi+w)}{\pi+w} = \text{sinc}(\pi-w) + \text{sinc}(\pi+w)$$

$$\Rightarrow 2\,\text{sinc}(w) + \text{sinc}(\pi-w) + \text{sinc}(\pi+w)$$

IV) $x_4(t) = te^{-2t}u(t)$

$$-jt\,f(t) \Longleftrightarrow F'(jw)$$

$$t\,f(t) \Longleftrightarrow -\frac{1}{j}F'(jw)$$

$$e^{-2t}u(t) \Longleftrightarrow \frac{1}{2+jw}$$

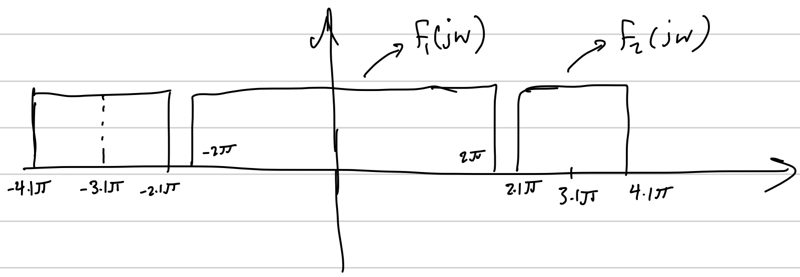
$$te^{-2t}u(t) \Longleftrightarrow -\frac{1}{j}\frac{d}{dw}\left[\frac{1}{2+jw}\right]$$

$$= -\frac{1}{j}\left(-\frac{j}{(2+jw)^2}\right) = \frac{1}{(2+jw)^2}$$

$$x_4(t) \Longleftrightarrow \frac{1}{(2+jw)^2}$$

b) i) $F_1(jw)\,F_2(jw) = F(jw)$ $\quad f_1(t) = \text{Sinc}(2t)$

$f_2(t) = \text{Sinc}(t)\cos(3.1\pi t)$



i) $\Rightarrow \mathcal{F}[(f_1 * f_2)(t)] = F_1(jw)\,F_2(jw) = 0]$

ii) since $F(jw) = 0$ $\quad \mathcal{F}^{-1}[F(jw)] = f(t)$

$\mathcal{F}^{-1}[0] = 0 \Rightarrow f(t) = 0$