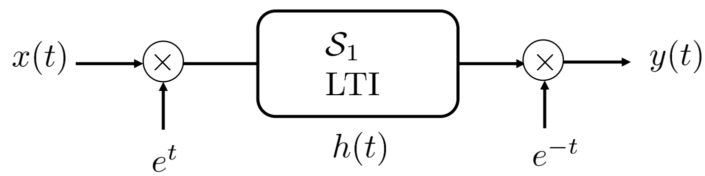
ECE 102, Fall 2023 **Homework #4**

Department of Electrical and Computer Engineering
University of California, Los Angeles
Prof. Jonathan C. Kao
TAs: Yang, Bruce, Shreyas

Due Monday, 6 November 2023, by 11:59 pm to Gradescope
Covers material up to lecture 9.
100 points total

1. (15 points) **LTI systems**

   Consider the following system:



   The system takes as input $x(t)$, it first multiplies the input with e^t, then sends it through an LTI system. The output of the LTI system gets multiplied by e^{-t} to form the output $y(t)$.

   (a) (5 points) Show that we can write $y(t)$ as follows:

   $$y(t) = \left[\left(e^t x(t)\right) * h(t)\right] e^{-t} \tag{1}$$

   (b) (5 points) Use the definition of convolution to show that (1) can be equivalently written as:

   $$y(t) = \int_{-\infty}^{\infty} h'(\tau) x(t-\tau) d\tau \tag{2}$$

   where $h'(t)$ is a function to define in terms of $h(t)$.

   (c) (5 points) Equation (2) represents a description of the equivalent system that maps $x(t)$ to $y(t)$. Show using (2) that the equivalent system is LTI and determine its impulse response $h_{eq}(t)$ in terms of $h(t)$.

2. (10 points) **Eigenfunctions and LTI systems**

   Determine if the following signals are eigenfunctions of LTI systems

   (a) (5 points) $f(t) = t^2$

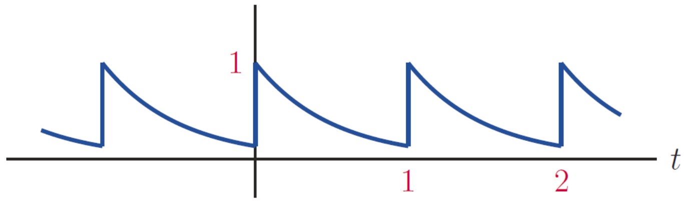
   (b) (5 points) $f(t) = e^{jwt} u(t)$

3. (28 points) **Fourier Series**

   (a) (18 points) Find the Fourier series coefficients for each of the following periodic signals:

i. (9 points) $f(t) = \sin(7\pi t) + \frac{1}{3}\cos(4\pi t)$

ii. (9 points) $f(t)$ is a periodic signal with period $T = 1$ s, where one period of the signal is defined as e^{-t} for $0 < t < 1$ s, as shown below.



(b) (10 points) Suppose you have two periodic signals $x(t)$ and $y(t)$, of periods T_1 and T_2 respectively. Let x_k and y_k be the Fourier series coefficients of $x(t)$ and $y(t)$.

i. (5 points) If $T_1 = T_2$, express the Fourier series coefficients of $z(t) = 3x(t) + 2y(t)$ in terms of x_k and y_k.

ii. (5 points) If $T_1 = 2T_2$, express the Fourier series coefficients of $w(t) = x(t) + y(t)$ in terms of x_k and y_k.

4. (34 points) **Fourier series of transformation of signals**

(a) (15 points) Suppose that $f(t)$ is a periodic signal with period T_0, with the following Fourier series:

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

Determine the period of each of the following signals, then express its Fourier series in terms of c_k:

$\sum_{k -\infty}^{\infty} c_k e^{jkwt}$

i. (5 points) $g(t) = 3f(t)$

ii. (5 points) $g(t) = f(-at)$ where a is a positive real number.

iii. (5 points) $g(t) = f(t - t_0)$

(b) (5 points) Given two periodic signals and their corresponding Fourier series representation as follows:

w

$-ak$

$w\, jk\, \underset{\sim}{-at}$

$a > 0$

$$x_1(t) = \sum_{k=-100}^{100} cos(k\pi) e^{jk\frac{2\pi}{50}t}$$

$\sum_{-\infty}^{\infty} ck\, e^{\underset{\sim}{aw}jkt}$

$$x_2(t) = \sum_{k=-100}^{100} jsin(\frac{k\pi}{2}) e^{jk\frac{2\pi}{50}t}$$

Identify whether the signals is/are even.

(c) (14 points) Suppose $x(t)$ is periodic with period T and is specified in the interval $0 < t < T/4$ as shown in figure 1.

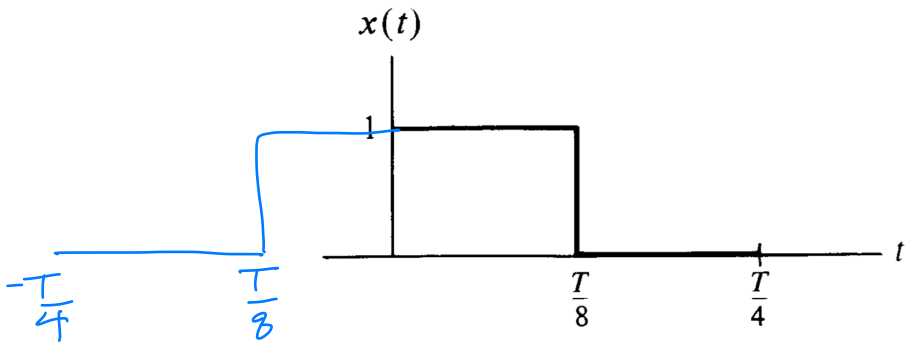


Figure 1: $x(t)$ in the interval $0 < t < T/4$

Sketch $x(t)$ in the interval $0 < t < T$ if

i. (7 points) the Fourier series has only odd harmonics and $x(t)$ is an even function

ii. (7 points) the Fourier series has only odd harmonics and $x(t)$ is an odd function

5. (13 points) **Python**

(a) (6 points) **Task 1**
Write an python function that takes a set of Fourier series coefficients, a fundamental frequency, and a vector of output times, and computes the truncated Fourier series evaluated at these times. The declaration and help for the python file might be:

```
  def myfs(Dn,omega0,t):
'''
  Evaluates the truncated Fourier Series at times t

  Dn     -- vector of Fourier series coefficients
  omega0 -- fundamental frequency
  t      --  vector of times for evaluation
'''
fn = myfs(Dn, omega0, t)
'''
  fn     --  truncated Fourier series evaluated at t}
'''
```

The output of the python function should be

$$f_N(t) = \Sigma_{n=-N}^{N} D_n e^{j\omega_0 n t}$$

The length of the vector Dn should be $2N + 1$. You will need to calculate N from the length of Dn.

(b) (7 points) **Task 2**
Verify the output of your routine by checking the Fourier series coefficients for the sawtooth waveform. The sawtooth signal is given by f(t) = t mod 1 described in the

class notes. Try for $N = 10$, $N = 50$. Use the python subplot command to put multiple plots on a page.

(b) (5 points) Use the definition of convolution to show that (1) can be equivalently written as:

$$y(t) = \int_{-\infty}^{\infty} h'(\tau)x(t-\tau)d\tau \qquad (2)$$

where $h'(t)$ is a function to define in terms of $h(t)$.

(c) (5 points) Equation (2) represents a description of the equivalent system that maps $x(t)$ to $y(t)$. Show using (2) that the equivalent system is LTI and determine its impulse response $h_{eq}(t)$ in terms of $h(t)$.

a) $S_1(e^t x(t)) = w(t)$

Since LTI $\Rightarrow w(t) = e^t x(t) * S_1(\delta(t-\tau))$

$w(t) = e^t x(t) * h(t)$

$y(t) = w(t) \cdot e^{-t}$

$\Rightarrow y(t) = [e^t x(t) * h(t)] e^{-t}$

b) $\Rightarrow \int_{-\infty}^{\infty} e^{\tau} x(\tau) h(t-\tau) d\tau \; e^{-t} \Rightarrow e^{-t} \int_{-\infty}^{\infty} e^{(t-\tau)} x(t-\tau) h(\tau) d\tau$

$\Rightarrow \int_{-\infty}^{\infty} e^{-\tau} x(t-\tau) h(\tau) d\tau \qquad \Rightarrow h'(\tau) = h(\tau) e^{-\tau}$

$\int_{-\infty}^{\infty} h'(\tau) x(t-\tau) d\tau$

c) $S_2 = \int_{-\infty}^{\infty} x(t-\tau) h'(\tau) d\tau = z(t)$ $\qquad m(t) = ax(t) + b\tilde{x}(t)$

$\Rightarrow S_2(m(t)) = \int_{-\infty}^{\infty} [ax(t-\tau) + b\tilde{x}(t-\tau)] h'(\tau) d\tau$

$\Rightarrow \int_{-\infty}^{\infty} ax(t-\tau) h'(\tau) + \int_{-\infty}^{\infty} b\tilde{x}(t-\tau) h'(\tau) d\tau$

$\Rightarrow \underbrace{a\int_{-\infty}^{\infty} x(t-\tau) h'(\tau)}_{aS_2(x(t))} + \underbrace{b\int_{-\infty}^{\infty} \tilde{x}(t-\tau) h'(\tau) d\tau}_{bS_2(\tilde{x}(t))}$ is linear.

TI: $z(t) = \int_{-\infty}^{\infty} x(t-\tau) h'(\tau) \, d\tau$

input shift $\Rightarrow \int_{-\infty}^{\infty} x(t-\tau-\alpha) h'(\tau) \, d\tau$

output shift $\Rightarrow z(t-\alpha) = \int_{-\infty}^{\infty} x(t-\tau-\alpha) h'(\tau) \, d\tau$

$\Rightarrow$ Time invariant and linear so LTI

$\Rightarrow \int_{-\infty}^{\infty} x(t-\tau) h'(\tau) \, d\tau \qquad \Rightarrow \quad x(t) = \delta(t)$

$\Rightarrow \int_{-\infty}^{\infty} \delta(t-\tau) h'(\tau) \, d\tau \qquad t = \tau$

$\Rightarrow h'(t) \int_{-\infty}^{\infty} \delta(t-\tau) \, d\tau = h'(t) = e^{-t} h(t)$

2. (10 points) **Eigenfunctions and LTI systems**

Determine if the following signals are eigenfunctions of LTI systems

(a) (5 points) $f(t) = t^2$

(b) (5 points) $f(t) = e^{jwt}u(t)$

a) $y(t) = \int_{-\infty}^{\infty} f(\tau) \cdot h(t-\tau)\, d\tau \Rightarrow \int_{-\infty}^{\infty} \tau^2 h(t-\tau)\, d\tau$

$\Rightarrow$ can't write is as a f(t) so not eigenfunction.

b) $f(t) = e^{jwt} u(t) \Rightarrow \int_{-\infty}^{\infty} e^{jw\tau} u(\tau)\, h(t-\tau) = \int_{-\infty}^{\infty} e^{jw(t-\tau)} u(t-\tau)\, h(\tau)\, d\tau$

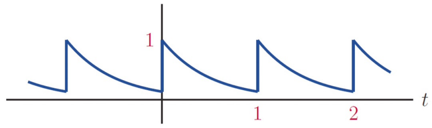
$\Rightarrow \cancel{\int_{-\infty}^{0} e^{jw(t-\tau)} u(t-\tau)\, h(\tau)\, d\tau}\ 0 + \int_{0}^{\infty} e^{jw(t-\tau)} u(t-\tau)\, h(\tau)\, d\tau$

$\Rightarrow \int_{0}^{\infty} e^{jwt} e^{-jw\tau} u(t-\tau)\, h(\tau)\, d\tau$

$e^{jwt} \int_{0}^{\infty} e^{-jw\tau} u(t-\tau)\, h(\tau)\, d\tau$

not eigenfuction. !

3) a)
i. (9 points) $f(t) = \sin(7\pi t) + \frac{1}{3}\cos(4\pi t)$
ii. (9 points) $f(t)$ is a periodic signal with period $T = 1$ s, where one period of the signal is defined as e^{-t} for $0 < t < 1$ s, as shown below.



$T_0 = \frac{2\pi}{w_0}$

$2\pi f_0 = w_0$

$\frac{2\pi}{T_0} = w_0 \Rightarrow \frac{2\pi}{w_0} = T_0$

1) $\sin(7\pi t) + \frac{1}{3}\cos(4\pi t) \Rightarrow \sum_{-\infty}^{\infty} c_{k_1} e^{jk_1 w_0 t} + \sum_{-\infty}^{\infty} C_{k_2} e^{jk_2 w_0 t}$

$W_1 = 7\pi$ $\qquad$ $W_2 = 4\pi$

$T_1 = \frac{2\pi}{7\pi} = \frac{2}{7}$ $\qquad$ $T_2 = \frac{1}{2}$

$\Rightarrow f(t) = \sum_{k=-\infty}^{\infty} C_k e^{jwkt}$

$\frac{2}{7}, \frac{4}{7}, \frac{6}{7}, \frac{8}{7}, \frac{10}{7}, \frac{12}{7}, \frac{14}{7} \leftarrow T_1$

$\frac{1}{2}, 1, \frac{3}{2}, 2 \leftarrow T_2$

$\Rightarrow \frac{1}{2j}\left[e^{jkw_0t} - e^{-jkw_0t}\right] + \frac{1}{6}\left[e^{kjw_0t} + e^{-jw_0tk}\right]$

$\Rightarrow 2 = T_{12}$

$W_{12} = \pi$ $\qquad$ $k=7$ $\qquad$ $k=4$

$\Rightarrow W_{12} = \frac{2\pi}{2} = \boxed{\pi}$ common

$\Rightarrow \underbrace{\frac{1}{2j}\left[e^{jk\pi t} - e^{-jk\pi t}\right]} + \frac{1}{6}\left[e^{kj\pi t} + e^{-jk\pi t}\right]$

$C_7 = \frac{1}{2j}, \; C_{-7} = -\frac{1}{2j}, \; K_4 = \frac{1}{6}, \; K_{-4} = \frac{1}{6}$ and rest of $C_k = 0$

3a ii) $\frac{1}{T_0}\int_0^{T_0} f(t) e^{-jkw_0t}\, dt \Rightarrow \frac{1}{1}\int_0^1 e^{-t} e^{-jk2\pi t}\, dt$

$\frac{2\pi}{T_0} = w_0$

$w_0 = \frac{2\pi}{1} = 2\pi$

$\Rightarrow \int_0^1 e^{-t(1+2\pi jk)}\, dt$

$\Rightarrow -\frac{e^{-t(1+2\pi jk)}}{1+2\pi jK}\Bigg|_0^1 = \frac{-e^{-1-2\pi jk}}{1+2\pi jK} + \frac{1}{1+2\pi jK}$

$\Rightarrow \frac{1-e^{-1-2\pi jk}}{1+2\pi jk}$

$$e^{-1-2\pi jk} = e^{-1} e^{-2\pi jk} = e^{-1}\left[\cancel{\cos}^{1}(2\pi) - j\cancel{\sin}^{0}(2\pi)\right]$$

$$= e^{-1}$$

$$\Rightarrow \frac{1-e^{-1}}{1+2\pi jk} = C_k$$

3b)

(b) (10 points) Suppose you have two periodic signals $x(t)$ and $y(t)$, of periods T_1 and T_2 respectively. Let x_k and y_k be the Fourier series coefficients of $x(t)$ and $y(t)$.

i. (5 points) If $T_1 = T_2$, express the Fourier series coefficients of $z(t) = 3x(t) + 2y(t)$ in terms of x_k and y_k.

ii. (5 points) If $T_1 = 2T_2$, express the Fourier series coefficients of $w(t) = x(t) + y(t)$ in terms of x_k and y_k.

$$\Rightarrow z(t) = \sum_{k=-\infty}^{\infty} z_k e^{jkw_0 t}$$

$T_1 = T_2$

$\Rightarrow w_1 = w_2 = w_0$

$$\Rightarrow x(t) = \sum_{k=-\infty}^{\infty} x_k e^{jkw_0 t} \Rightarrow 3x(t) = \sum_{k=-\infty}^{\infty} 3x_k e^{jkw_0 t}$$

$$y(t) = \sum_{k=-\infty}^{\infty} y_k e^{jkw_0 t} \Rightarrow 2y(t) = \sum_{k=-\infty}^{\infty} 2y_k e^{jkw_0 t}$$

$$\Rightarrow z(t) = \sum_{k=-\infty}^{\infty} 3x_k e^{jkw_0 t} + \sum_{k=-\infty}^{\infty} 2y_k e^{jkw_0 t}$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} 3x_k e^{jkw_0 t} + 2y_k e^{jkw_0 t} \Rightarrow \sum_{k=-\infty}^{\infty} z_k e^{jkw_0 t}$$

$$\Rightarrow \boxed{z_k = 3x_k + 2y_k}$$

3bii) $T_1 = 2T_2$

$$w_0 = \frac{2\pi}{T_0}$$

$$\Rightarrow \frac{T_1}{2} = T_2$$

$$\Rightarrow w_1 = \frac{2\pi}{T_1}$$

$$\Rightarrow \frac{w_2}{w_1} = \frac{\frac{4\pi}{T_1}}{\frac{2\pi}{T_1}} = 2$$

$$w_2 = \frac{2\pi}{T_2} = \frac{2\pi}{\frac{T_1}{2}} = \frac{4\pi}{T_1}$$

$$\Rightarrow w_2 = 2w_1$$

$$w(t) = x(t) + y(t)$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} w_k e^{jkw_0 t} = \sum_{k=-\infty}^{\infty} x_k e^{jkw_1 t} + \sum_{k=-\infty}^{\infty} y_k e^{jkw_2 t}$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} x_k e^{jkw_1 t} + \sum_{k=-\infty}^{\infty} y_k e^{jk2w_1 t}$$

$$x_1 e^{jw_1 t} + x_2 e^{2jw_1 t} + x_3 e^{3jw_1 t} + x_4 e^{4jw_1 t} + x_5 e^{5jw_1 t} + x_6 e^{6jw_1 t}$$

$$y_1 e^{2jw_1 t} + y_2 e^{4jw_1 t} + y_3 e^{6jw_1 t} + y_4 e^{8jw_1 t} + \cdots$$

$$\Rightarrow \sum_{k_{odd}=-\infty}^{\infty} x_k e^{kjw_1 t} + \sum_{k_{even}=-\infty}^{\infty} x_k e^{jkw_1 t} + y_{\frac{k}{2}} e^{jmw_1 t}$$

$$c_{k'} = x_k + y_{\frac{k}{2}}$$

$$\Rightarrow \sum_{k_{odd}=-\infty}^{\infty} x_k e^{kjw_1 t} + \sum_{k'=-\infty}^{\infty} c_{k'} e^{jk'w_1 t}$$

$$w_k = \text{for}(-\infty < k < \infty) \begin{cases} k_{odd}, & [x_k] \\ k_{even}, & [x_k + y_{\frac{k}{2}}] \end{cases}$$

4a)

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \qquad T_0$$

i) $g(t) = 3f(t) \Rightarrow T_g = T_0$

$$\sum_{k=-\infty}^{\infty} \underbrace{g_k} e^{jk\omega_0 t} \Rightarrow 3f(t) = \sum_{k=-\infty}^{\infty} \underbrace{3f_k} e^{jk\omega_0 t}$$

$$\boxed{\Rightarrow g_k = 3f_k}$$

ii) $g(t) = f(-at)$, $a > 0$

$$\omega_0 = \frac{2\pi}{T_0} \Rightarrow T_g = \frac{2\pi}{a\omega_0} = \frac{T_0}{a}$$

T_0

$$\Rightarrow \omega_g = a\omega_0$$

$$\sum_{k=-\infty}^{\infty} g_k e^{jk\omega_0 t}$$

$$f(-at) = \sum_{k=-\infty}^{\infty} f_k e^{-jk\omega_0 at}$$

$a\omega_0 = \omega_g$

$k' = -k$

$$\Rightarrow \sum_{k'=\infty}^{-\infty} f_{k'} e^{jk'\omega_g t}$$

$$\boxed{\Rightarrow g_k = f_{k'} = f_{-k}}$$

iii) $g(t) = f(t - t_0)$

Period stays the same since it is only time shift. Proved in HW 1.

$$g(t) = \sum_{k=-\infty}^{\infty} g_k e^{jk\omega_0 t}$$

$$\Rightarrow f(t-t_0) = \sum_{k=-\infty}^{\infty} f_k e^{jk\omega_0 (t-t_0)}$$

$$= \sum_{k=-\infty}^{\infty} f_k e^{jk\omega_0 t} e^{-jk\omega t_0} = \sum_{k=-\infty}^{\infty} f_k e^{-jk\omega_0 t_0} e^{jk\omega_0 t}$$

$$\boxed{\Rightarrow g_k = f_k e^{-jk\omega_0 t_0}}$$

4b) $x_1(t) = \sum_{k=-100}^{100} \cos(k\pi) e^{jk\frac{2\pi}{50}t}$

$\Rightarrow x_1(-t) = x_1(t) \quad \Rightarrow \sum_{k=-100}^{100} \cos(k\pi) e^{jk\frac{2\pi}{50}(-t)}$

$\Rightarrow \sum_{k=-100}^{100} \cos(k\pi) e^{j(-k)\frac{2\pi}{50}t}$

$\cos(-x) = \cos(x) \quad \Rightarrow \sum_{k=100}^{-100} \cos(-k\pi) e^{jk\frac{2\pi}{50}t}$

$\sum_{k=100}^{-100} \cos(k\pi) e^{jk\frac{2\pi}{50}t} = \sum_{k=-100} \cos(k\pi) e^{jk\frac{2\pi}{50}t}$

So even.

$x_2(t) = \sum_{k=-\infty}^{\infty} j\sin\left(\frac{k\pi}{2}\right) e^{jk\frac{2\pi}{50}t}$

$x_2(-t) = \sum_{k=-\infty}^{\infty} j\sin\left(\frac{k\pi}{2}\right) e^{jk\frac{2\pi}{50}(-t)} \quad \Rightarrow \sum_{k'=\infty}^{-\infty} j\sin\left(\frac{-k'\pi}{2}\right) e^{jk'\frac{2\pi}{50}t}$

$-k' = k$

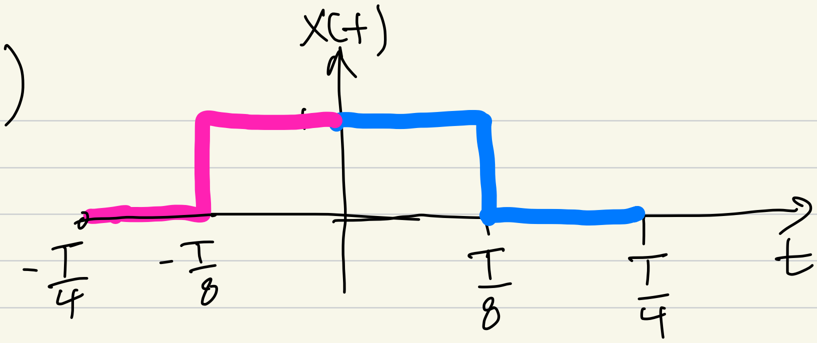
$-k = k'$

$\sin(-ax) = -\sin(ax)$

$\Rightarrow \sum_{k=\infty}^{-\infty} -j\sin\left(\frac{k'\pi}{2}\right) e^{jk'\frac{2\pi}{50}t}$

$x_2(-t) = -x_2(t)$ not eved.

4c)



i) odd harmonics and x(t) even. $x(t) = -x(t - \frac{T}{2})$

$$x(t) = \sum_{k\,odd=-\infty}^{\infty} C_k e^{jk\omega_0 t} \Rightarrow x(t - \tfrac{T}{2}) = \sum_{k\,odd=-\infty}^{\infty} C_k e^{jk\omega_0 (t - \frac{T}{2})}$$

$$\Rightarrow \sum_{k\,odd}^{\infty} C_k e^{jk\omega_0 t} \underbrace{e^{-jk\omega_0 \frac{T}{2}}}$$

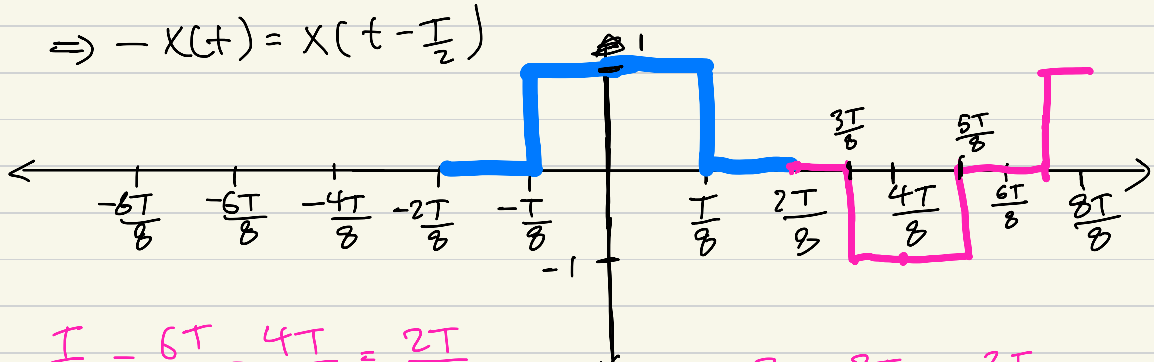
$\omega_0 = \frac{2\pi}{T}$

$$\Rightarrow e^{-jk\frac{2\pi}{T}\cdot\frac{T}{2}} \Rightarrow e^{-jk\pi}$$

$$= \sum_{k\,odd} \underbrace{\cos(k\pi)}_{-1} - \cancel{j\sin(k\pi)}^{0}$$

$$\Rightarrow \sum_{k\,odd}^{\infty} C_k e^{jk\omega_0 t}(-1) = -\sum_{k\,odd}^{\infty} C_k e^{jk\omega_0 t}$$

$$\Rightarrow -x(t) = x(t - \tfrac{T}{2})$$



$\frac{6T}{8} - \frac{T}{2} = \frac{6T}{8} - \frac{4T}{8} = \frac{2T}{8}$

$\frac{5T}{8} - \frac{3T}{8} = \frac{2T}{8}$

$\frac{2T}{8} - \frac{T}{2} = \frac{2T}{8} - \frac{4T}{8} = -\frac{2T}{8} = -\frac{T}{4}$ $= \frac{T}{4}$

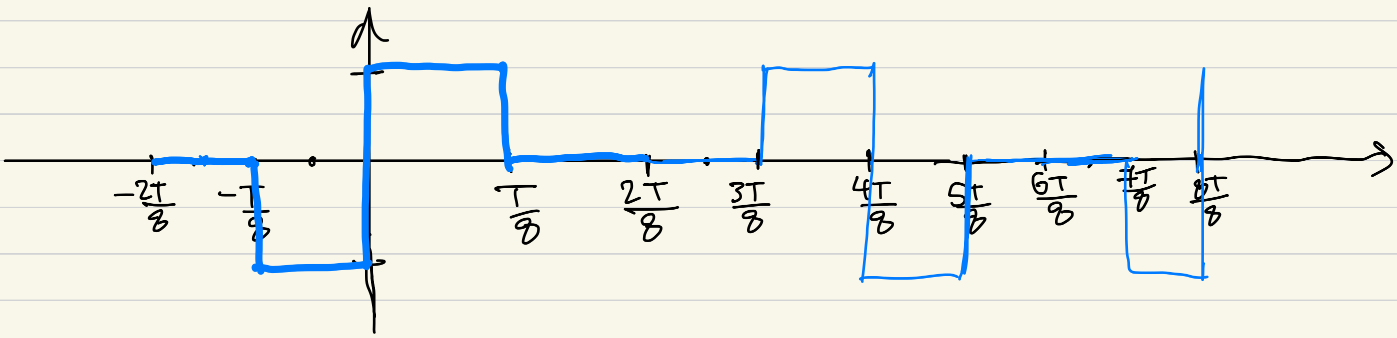
$\frac{T}{8} + \frac{T}{8} = \frac{2T}{8}$

$\frac{8T}{8} - \frac{T}{2} = \frac{8T}{8} - \frac{4T}{8} = \frac{T}{2}\,(-1) \Rightarrow (1)$

$\frac{3T}{8} - \frac{T}{2} = \frac{3T}{8} - \frac{4T}{8} = -\frac{T}{8}$ (0)

$\frac{5T}{8} - \frac{4T}{8} = \frac{T}{8}$ (0)

(ii) odd harmonics and x(t) odd.



$\Rightarrow x(t) = -x(t - \frac{T}{2})$

$\Rightarrow \frac{3T}{8} - \frac{4T}{8} = -\frac{T}{8}$

$\Rightarrow \frac{4T}{8} - \frac{4t}{8} = 0$

$\frac{2.5T}{8} - \frac{4T}{8} = -\frac{1.5T}{8}$ (0)

$\frac{3.5T}{8} - \frac{4T}{8} = \frac{-0.5T}{8}$

$\frac{6.5T}{8} - \frac{4T}{8} = \frac{2.5T}{8}$

$\frac{7T}{8} - \frac{4T}{8} = \frac{3T}{8}$

In [259]:
```
import matplotlib.pyplot as plt
import numpy as np
import math
plt.style.use('ggplot')
```

In [260]:
```

def coefficient(k, w_0):
    y = np.zeros(len(k), dtype=complex)
    for i in range(len(k)):
#         print(k[i], "index k")
        if (k[i]==0):
            y[i] = 0.5
        else:
            y[i] = ((1j*np.exp(-1j*k[i]*w_0))/(k[i]*w_0)) + (((np.exp(-1
    return y
# len(k2)
# len(k)
```

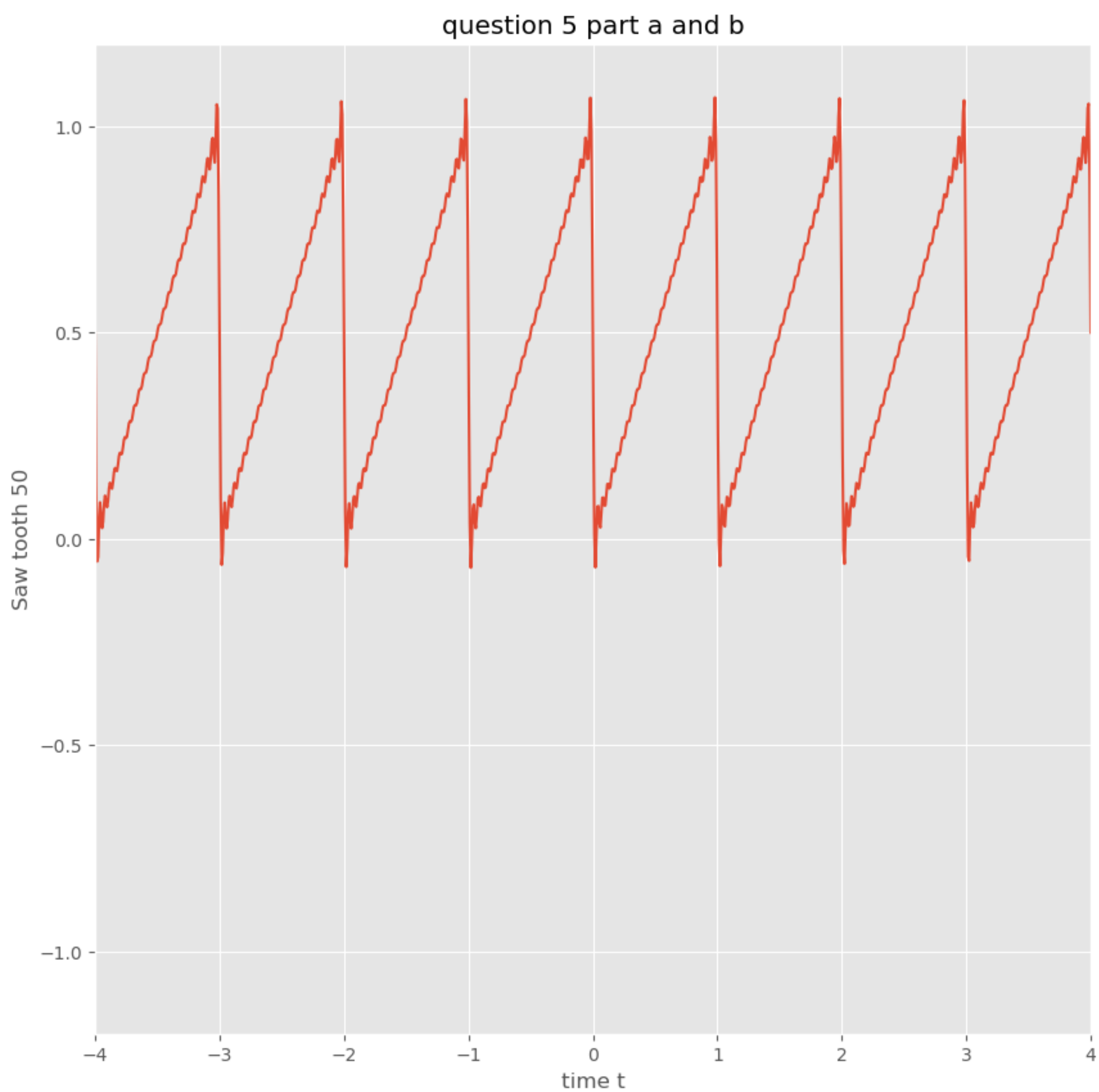
In [261]:
```
def fourier(C_k, w_0, t):
    coeff = (len(C_k)-1)//2
    y = np.zeros(len(t), dtype=complex)
    for i in range(-coeff,coeff+1):
        y += (C_k[i+coeff]) * (np.exp(1j*w_0*i*t))
#         print(i+coeff)
    return y
```

In [262]:
```
k = np.arange(-25,25+1,1)
k2 = np.arange(-5,5+1,1)
t = np.linspace(-4,4,1000)
saw_co50 = coefficient(k, 2*np.pi)
saw_co10 = coefficient(k2, 2*np.pi)
saw_tooth_50 = fourier(saw_co50, (2*np.pi), t)
saw_tooth_10 = fourier(saw_co10, (2*np.pi), t)
```

In [263]:

```
fig, saw_tooth_with_50 = plt.subplots(figsize=(10,10))
saw_tooth_with_50.plot(t, saw_tooth_50)
saw_tooth_with_50.set_xlabel("time t")
saw_tooth_with_50.set_ylabel("Saw tooth 50")
saw_tooth_with_50.set_xlim([-4, 4])
saw_tooth_with_50.set_ylim([-1.2,1.2])
saw_tooth_with_50.set_title("question 5 part a and b")
plt.show()
```

```
/Applications/anaconda3/lib/python3.11/site-packages/matplotlib/cbook/_
_init__.py:1335: ComplexWarning: Casting complex values to real discard
s the imaginary part
  return np.asarray(x, float)
```



In [264]:
```
fig, saw_tooth_with_10 = plt.subplots(figsize=(10,10))
saw_tooth_with_10.plot(t, saw_tooth_10)
saw_tooth_with_10.set_xlabel("time t")
saw_tooth_with_10.set_ylabel("Saw tooth 10")
saw_tooth_with_10.set_xlim([-4, 4])
saw_tooth_with_10.set_ylim([-0.1,1.1])
saw_tooth_with_10.set_title("question 5 part a and b")
plt.show()
```