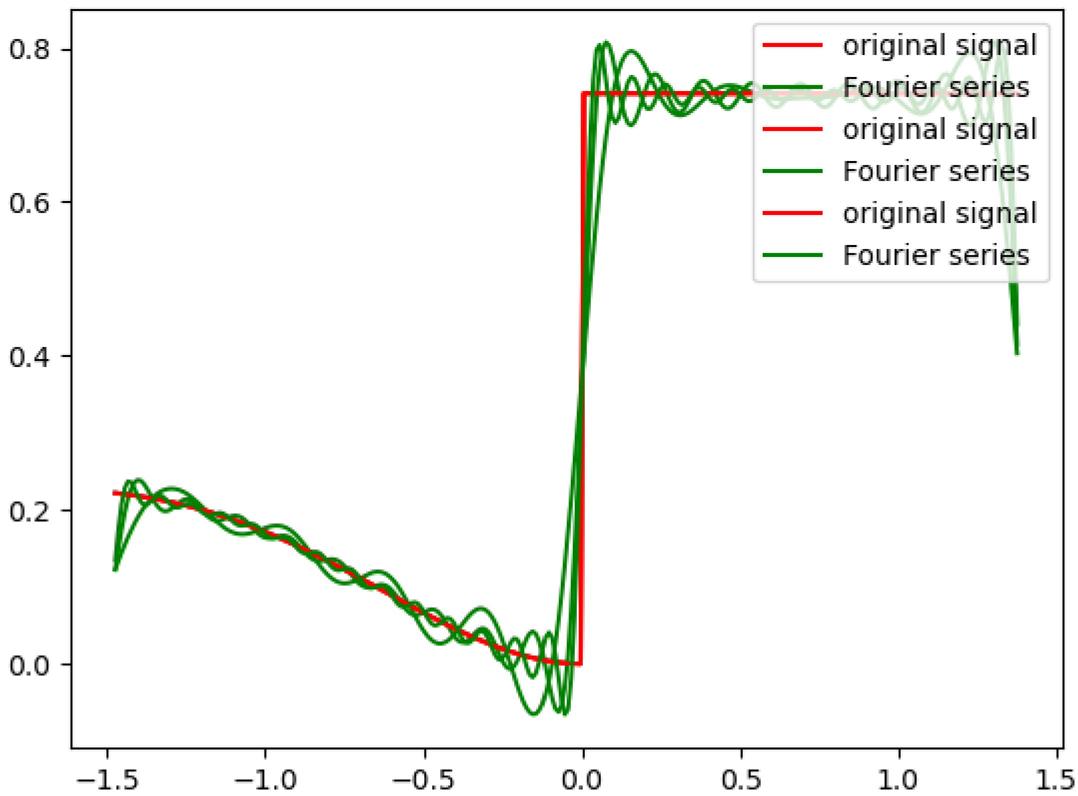
In [1]:
```
import numpy as np
import random
import math
import matplotlib.pyplot as plt
from scipy.signal import sawtooth
from scipy.signal import square
from scipy.signal import sawtooth
import matplotlib.pyplot as plt
from pylab import rcParams
import matplotlib.patches as patches
import cv2
```

Fourier series

In [2]:
```
def fseries(x, signal, K):

    """
    Returns an approximation of a given signal with a Fourier series a
    of K coefficients.

    Parameters:
    ----------
    x: independent variable (time)
    signal: function of x to be approximated
    K: number of harmonics to be used

    Returns:
    --------
    series: Fourier series of the signal
    a_0: coefficient with k=0
    a: array of coefficients for odd components (cosine)
    b: array of coefficients for even components (sine)


    """

    ## reconstructed signal
    #print(len(x))
    xr = np.zeros((len(x)))
    #print(xr)
    ## scaling factor
    c = (x[-1]-x[0])/len(x)

    a_0 = np.dot(signal, np.ones(len(x))) ## average of the signal ove
    a_0 = c*a_0/np.pi


    ## Fourier coefficients arrays
    a = np.zeros(K)
    b = np.zeros(K)

    for k in range(1,K+1):
        ## evaluates coefficients
        a_k = c*np.dot(signal,np.cos(k*x))*1/np.pi
        a[k-1]=a_k
        b_k = c*np.dot(signal,np.sin(k*x))*1/np.pi
        b[k-1]=b_k

        ## computes series
        xr = xr + a_k*np.cos(k*x) + b_k*np.sin(k*x)
        series = 1/2*a_0+xr

    return series, a_0, a, b
```

In [3]:
```
def best_k(x, signal, K, tol):


    ## compute function norm keeping into account the scaling factor
    c = (x[-1]-x[0])/len(x) ##(b-a)/n
    norm = c*np.dot(signal, signal)

    for k in range(1,K+1):
        reconstructed_signal, a_0, a, b = fseries(x,signal, k)

        ## Parseval
        approx_norm = np.pi*(np.sum(a**2) + np.sum(b**2) + a_0**2/4)

        delta = np.abs(approx_norm-norm)
        if (np.abs(delta) < tol):
            break

    #print('the number of used harmonics is %d' % k)

    ## plots the signal and its approximation
    plt.plot(x, signal, 'r', label='original signal')
    plt.plot(x, reconstructed_signal, 'g', label='Fourier series')
    plt.legend(loc='upper right')
    return delta
```

In [5]:
```
a = random.uniform(-2, -1)
b = random.uniform(1, 2)

p = random.uniform(0, 1)
q = random.uniform(0, 1)
r = random.uniform(0, 1)
s = random.uniform(0, 1)

t = np.arange(a, b, 0.01)

signal = []

for i in t:
    if a <= i <= 0:
        signal.append((p*np.exp(q*i))* np.sin(np.log(1+r*i**2)))
    elif 0 <= i <= b:
        signal.append(s)


k_array = [20, 40, 60]
for k in k_array:
    best_k(t, signal, k, 1/10)
```

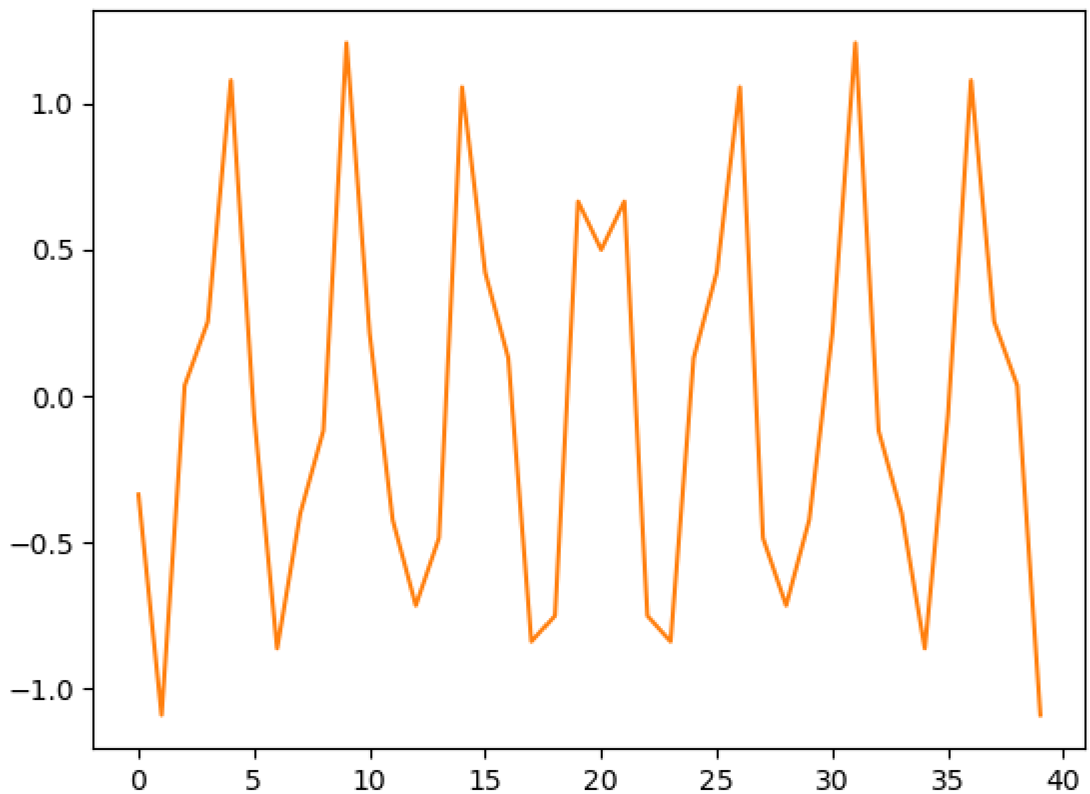


Comment: this plot concentrate that if k increases, the Gibbs phenomenon will decrease because much number of the combination sin and cos is near to real function. On the other hand, when n grows up, we have more overshoot and undershoot.

Fourier transform

In [6]:
```
alfa = random.uniform(-1, 1)
beta = random.uniform(-1, 1)
w0 = random.uniform(1, 5)
w1 = random.uniform(10, 20)

time = np.arange(-20, 20, 1)
y = (alfa * np.cos(w0 * time)) + (beta * np.cos(w1 * time))

plt.plot(0, 1, y)
```

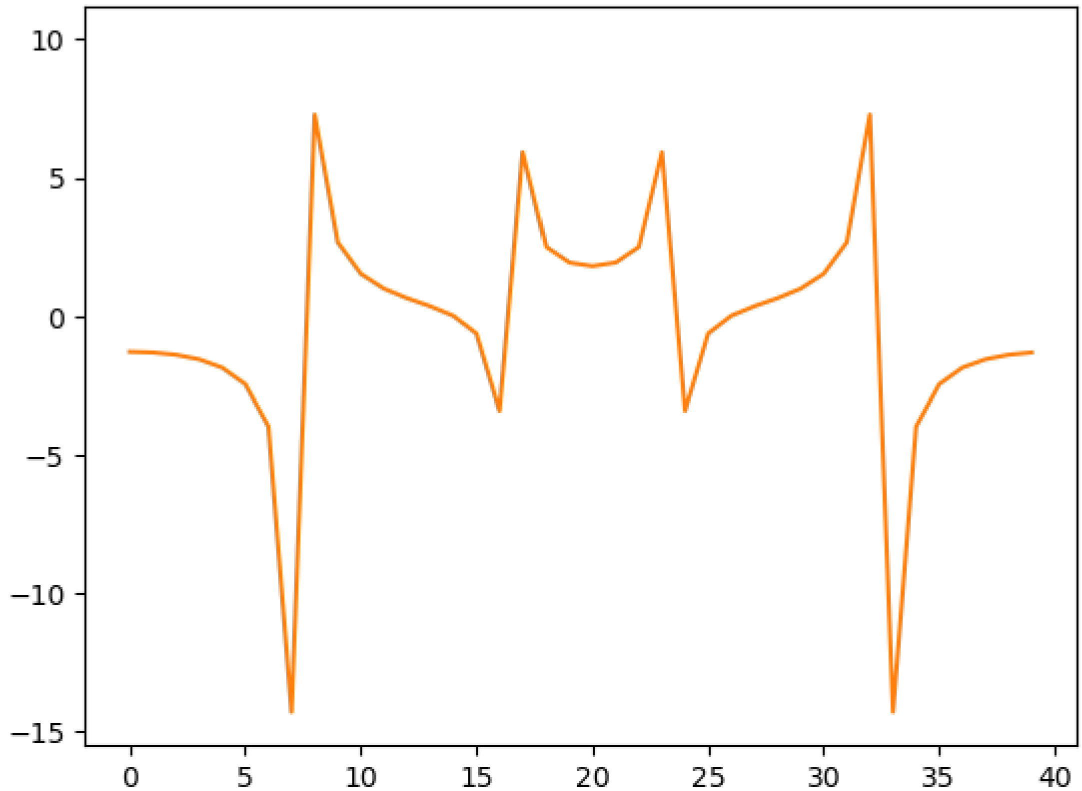
Out[6]:
```
[<matplotlib.lines.Line2D at 0x7fe865d41e50>,
 <matplotlib.lines.Line2D at 0x7fe865d41eb0>]
```



In [7]:
```
fft_y = np.fft.fft(y)

plt.plot(0, 10, fft_y[:40])
```

```
/home/sina/anaconda3/lib/python3.9/site-packages/matplotlib/cbook/__i
nit__.py:1298: ComplexWarning: Casting complex values to real discard
s the imaginary part
  return np.asarray(x, float)
```

Out[7]:
```
[<matplotlib.lines.Line2D at 0x7fe8658f74f0>,
 <matplotlib.lines.Line2D at 0x7fe8658f7550>]
```



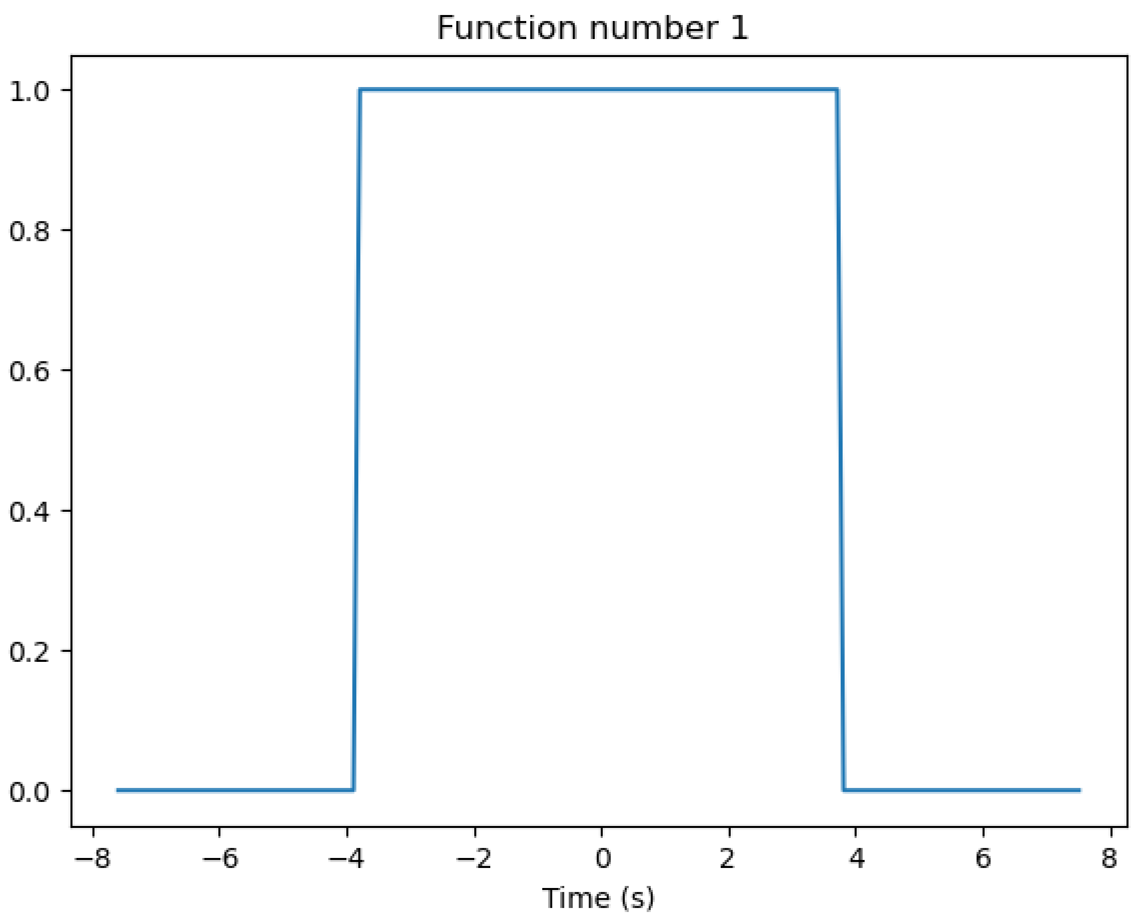
In terms of δ functions, the presence of a peak at a particular frequency in the Fourier transform can be interpreted as the presence of a δ function at that frequency in the signal. A peak in the Fourier transform indicates the presence of a frequency component in the input signal. The height or magnitude of the peak corresponds to the amplitude or strength of the frequency component.

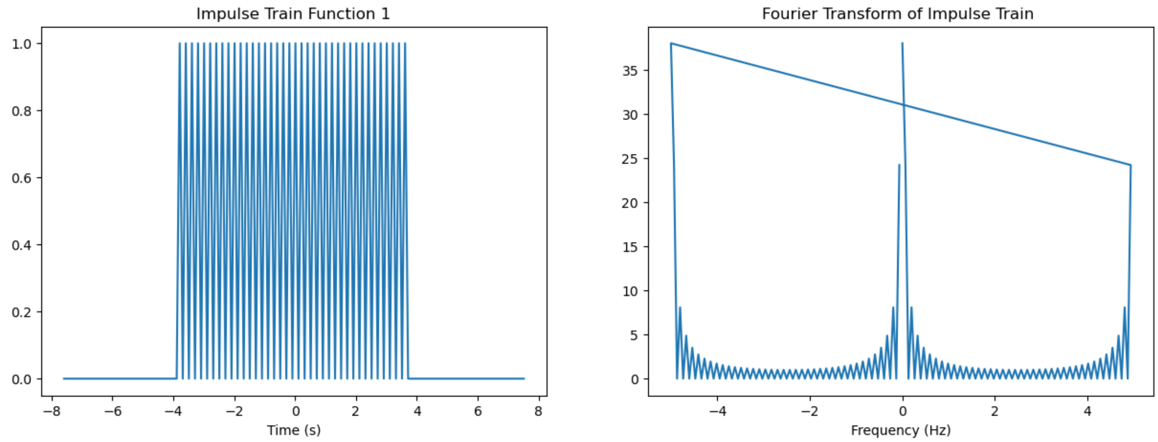
ailising

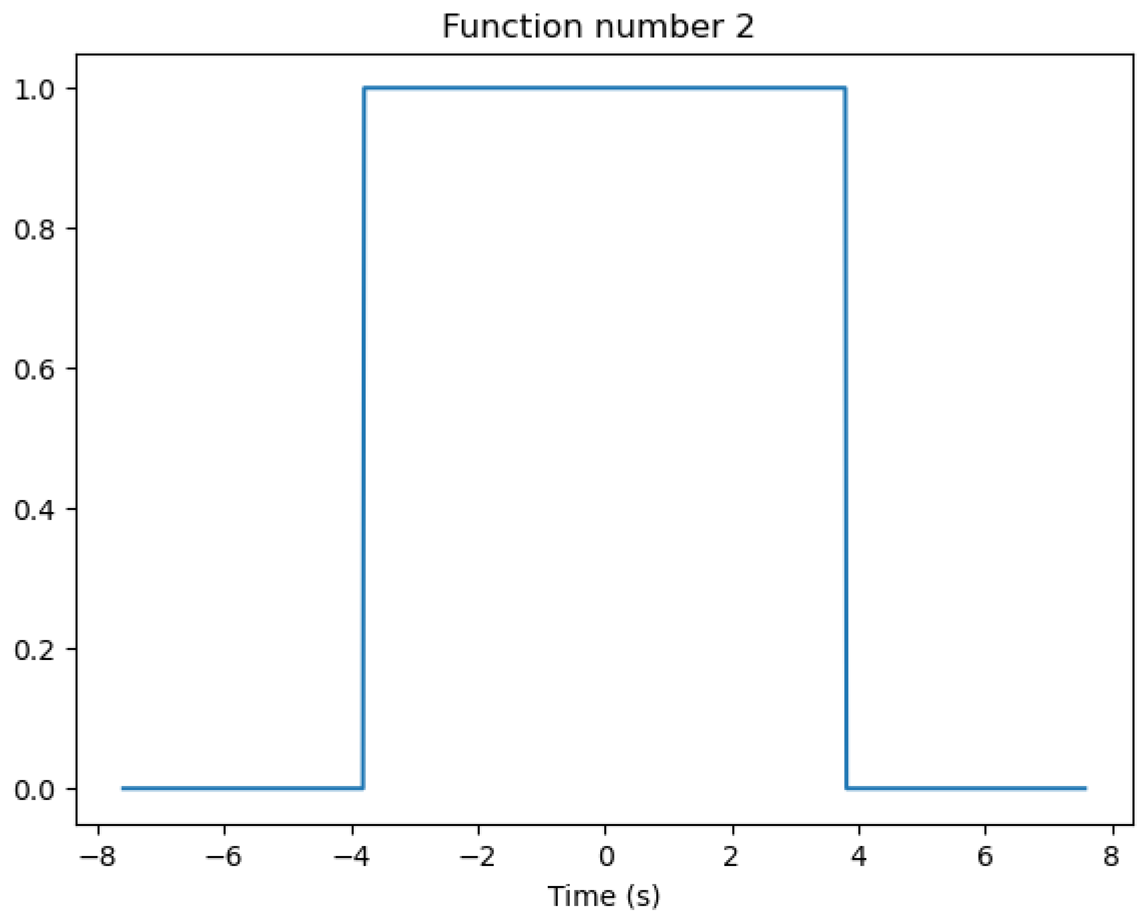
In [8]:
```
def pT(t, T):
    p = np.zeros_like(t)
    p[np.abs(t) <= T] = 1
    return p

fs1 = 10
fs2 = 50

T = np.random.uniform(1, 4)
t1 = np.arange(-2*T, 2*T, 1/fs1)
t2 = np.arange(-2*T, 2*T, 1/fs2)

signal1 = pT(t1, T)
signal2 = pT(t2, T)

# Calculate train of impulses
impulse_train1 = []
for count, t in enumerate(t1):
    if count % 2 == 0 and np.abs(t) <= T:
        impulse_train1.append(1)
    else:
        impulse_train1.append(0)

impulse_train2 = []
for count, t in enumerate(t2):
    if count % 100 == 0 and np.abs(t) <= T:
        impulse_train2.append(1)
    else:
        impulse_train2.append(0)


ft_impulse_train1 = np.fft.fft(impulse_train1)
ft_impulse_train2 = np.fft.fft(impulse_train2)

freq1 = np.fft.fftfreq(len(t1), 1/fs1)
freq2 = np.fft.fftfreq(len(t2), 1/fs2)

def temporal_sampling_rate(f, max_value, decay_percentage):
    """Calculate the temporal sampling rate for given decay percentage
    decay_value = max_value * (decay_percentage / 100)
    idx = np.argmin(np.abs(f - decay_value))
    return idx

max_value_1 = np.abs(ft_impulse_train1).max()
max_value_2 = np.abs(ft_impulse_train2).max()

temporal_rate_1 = temporal_sampling_rate(np.abs(ft_impulse_train1), ma
temporal_rate_2 = temporal_sampling_rate(np.abs(ft_impulse_train2), ma
print("Temporal sampling rate for 10% decay:", t1[temporal_rate_1])
print("Temporal sampling rate for 1% decay:", t2[temporal_rate_2])


plt.figure(figsize=(15,5))
plt.subplot(121)
plt.title('Function number 1 ')
plt.plot(t1, signal1)
plt.xlabel('Time (s)')
```

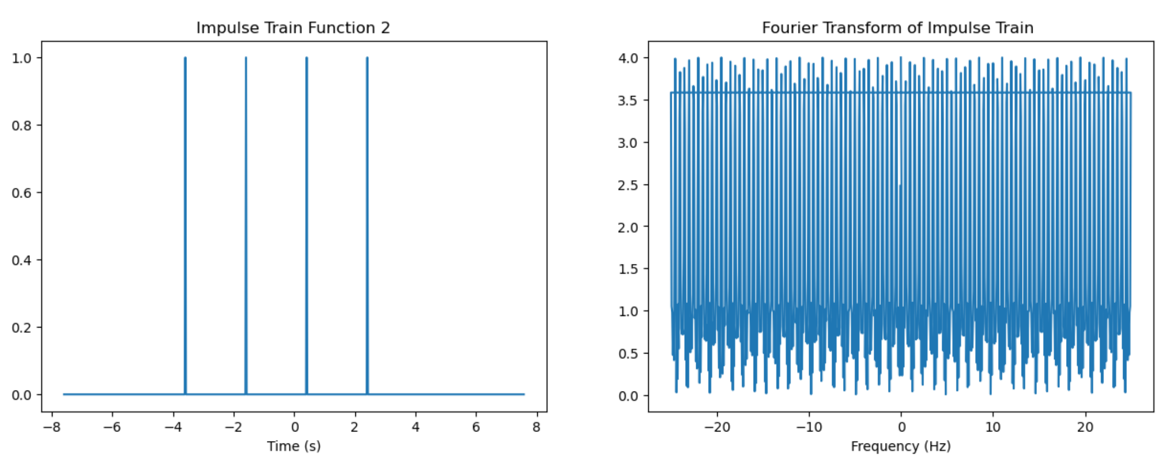
```
plt.figure(figsize=(15,5))
plt.subplot(121)
plt.title('Impulse Train Function 1')
plt.plot(t1, impulse_train1)
plt.xlabel('Time (s)')
plt.subplot(122)
plt.title('Fourier Transform of Impulse Train ')
plt.plot(freq1, np.abs(ft_impulse_train1))
plt.xlabel('Frequency (Hz)')
plt.figure(figsize=(15,5))
plt.subplot(121)
plt.title('Function number 2 ')
plt.plot(t2, signal2)
plt.xlabel('Time (s)')
plt.figure(figsize=(15,5))
plt.subplot(121)
plt.title('Impulse Train Function 2')
plt.plot(t2, impulse_train2)
plt.xlabel('Time (s)')
plt.subplot(122)
plt.title('Fourier Transform of Impulse Train ')
plt.plot(freq2, np.abs(ft_impulse_train2))
plt.xlabel('Frequency (Hz)')
plt.show()
```

```
Temporal sampling rate for 10% decay: 0.7132330586807729
Temporal sampling rate for 1% decay: 2.7732330586805816
```




Impulse Train Function 1
Time (s)
Fourier Transform of Impulse Train
Frequency (Hz)


Function number 2
Time (s)


Impulse Train Function 2
Time (s)
Fourier Transform of Impulse Train
Frequency (Hz)

High sampling rate means that we can have better representation of the signal. In first case (decay=10) we have high frequency. lower temporal sampling rate will result in a wider frequency spectrum with higher frequency components

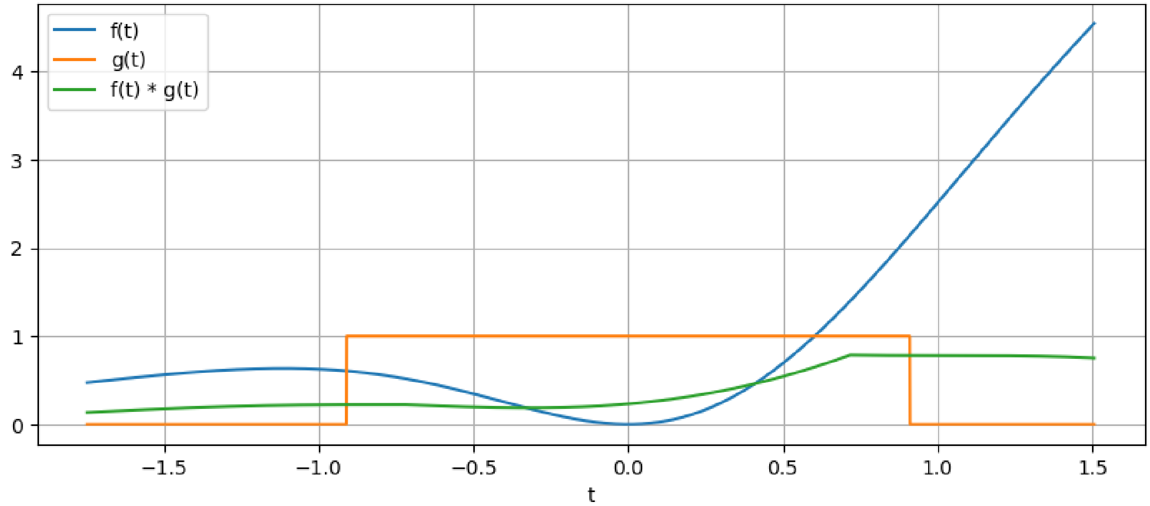
LTI

In [9]:
```
a = random.uniform(-2,-1)
b = random.uniform(1, 2)

p = random.uniform(0, 2)
q = random.uniform(0, 2)
r = random.uniform(0, 2)
T = random.uniform(0, 2)

t = np.arange(a, b, 0.001)
```

In [10]:
```
def f_t(t):
    return p * np.exp(q * t) * np.sin(np.log(1 + r * t**2))

def g_t(t):
    g_t_list = []
    for i in t:
        if abs(i) <= T:
            g_t_list.append(1)
        else:
            g_t_list.append(0)
    return g_t_list
```

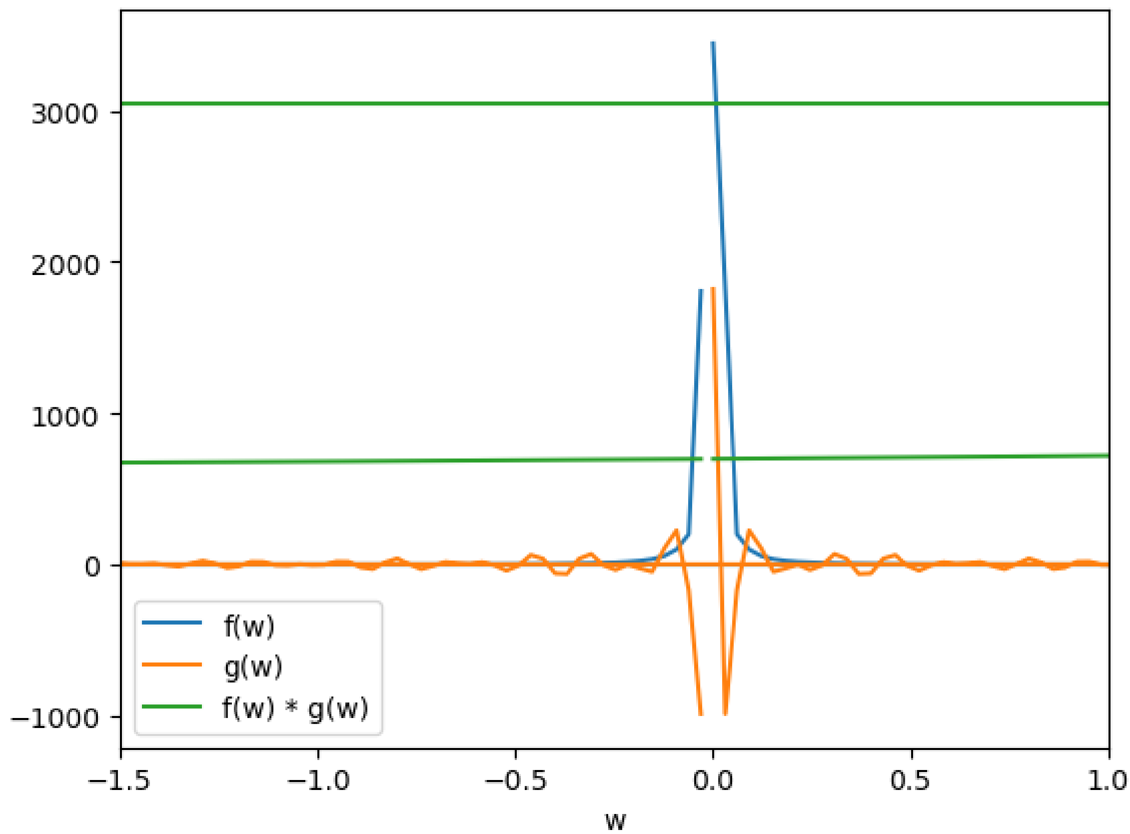
In [11]:
```
f_t = f_t(t)
g_t = g_t(t)
c = np.convolve(f_t, g_t, mode='same')
c_normalize = c / np.sum(f_t)
```

In [12]:
```
plt.figure(figsize=(10, 4))
plt.plot(t, f_t, label='f(t)')
plt.plot(t, g_t, label='g(t)')
plt.plot(t, c_normalize, label='f(t) * g(t)')
plt.xlabel('t')
plt.legend()
plt.grid()
```



In [13]:
```
freq = np.fft.fftfreq(len(t), 0.01)

f_t_fft = np.fft.fft(f_t)
g_t_fft = np.fft.fft(g_t)
c_fft = f_t_fft * g_t_fft
c = np.fft.ifft(c_fft).real

plt.plot(freq, f_t_fft, label='f(w)')
plt.plot(freq, g_t_fft, label='g(w)')
plt.plot(freq, c, label='f(w) * g(w)')
plt.xlabel('w')
plt.legend()
plt.xlim(-1.5, 1)
```

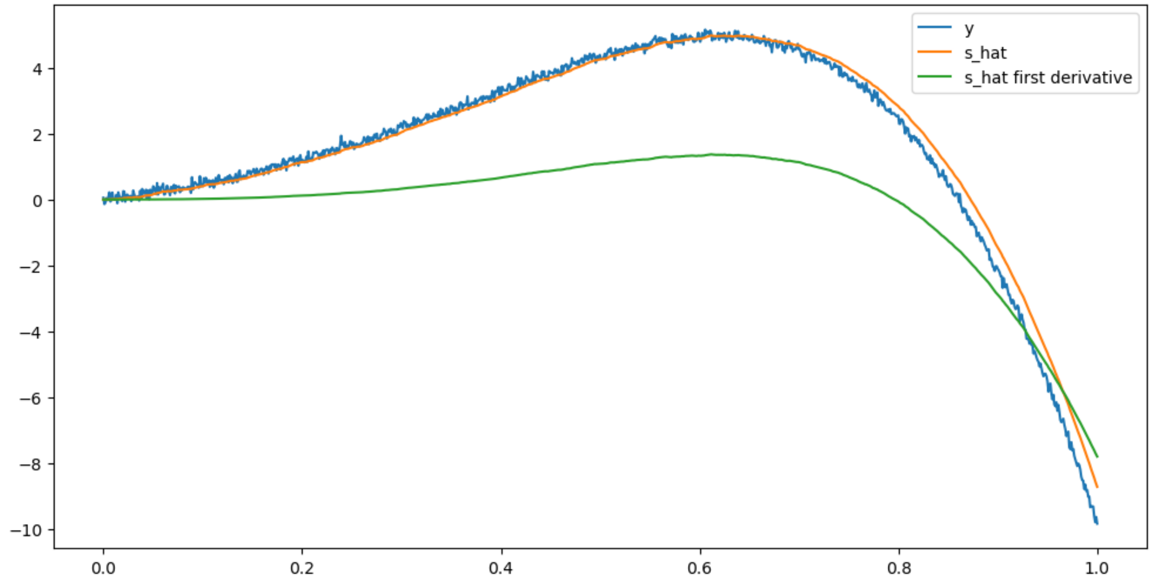
Out[13]: (-1.5, 1.0)



The convolution for two signal should be equal to convolution for fourier transform of f and g convolution

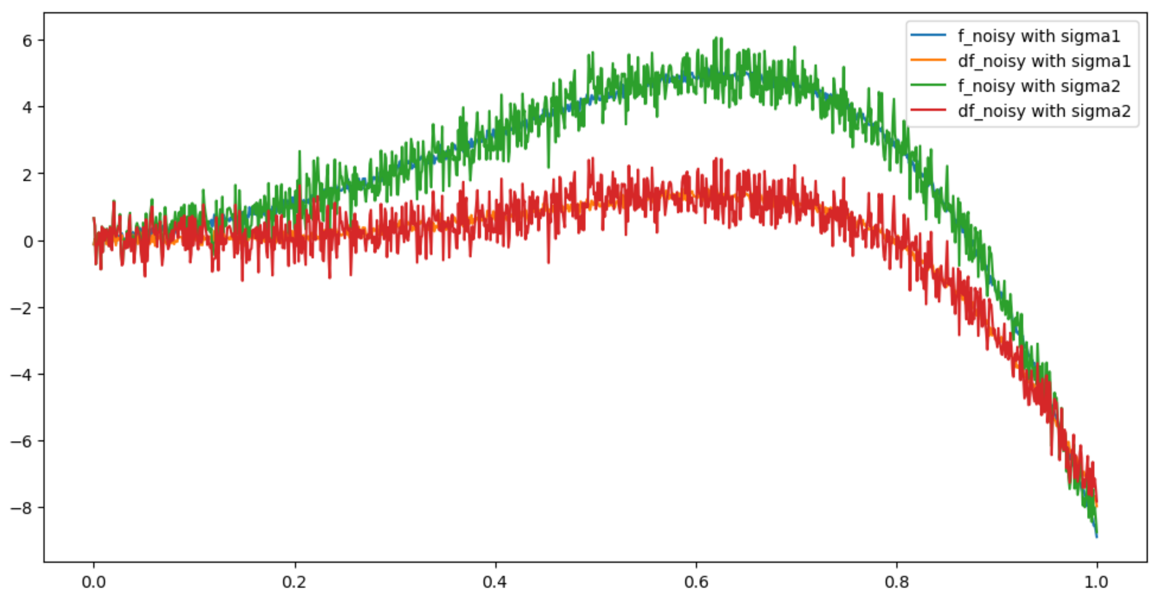
Kalman filtering

In [14]:
```
n = 1000
T = np.linspace(0, 1, n)
p = np.random.uniform(2, 4)
q = np.random.uniform(2, 4)

y = np.sin(p*T) * np.exp(q*T) + np.random.normal(0, 0.1, len(T))

def kalman_filter():
    n = 2
    m = 1
    phi = np.array([[1, T[1]-T[0]], [0, 1]])
    H = np.array([1, 0])[None, :]
    Q = np.array([[0.05, 0], [0, 0.05]])
    R = 10

    s = np.zeros((n, 1))
    P = np.zeros((n, n))

    s_est = np.zeros((n, len(T)))

    for i in range(len(T)):
        s_pred = np.dot(phi, s)
        P = np.dot(np.dot(phi, P), phi.T) + Q
        K = np.dot(P, H.T) / (np.dot(np.dot(H, P), H.T) + R)

        s = s_pred + np.dot(K, y[i] - np.dot(H, s_pred))

        P = np.dot(np.eye(n) - np.dot(K, H), P)

        s_est[:, i] = s[:, 0].flatten()

    return s_est

s_est = kalman_filter()
f = s_est[0, :]
df = s_est[1, :]

plt.figure(figsize=(12, 6))
plt.plot(T, y, label="y")
plt.plot(T, f, label="s_hat")
plt.plot(T, df, label="s_hat first derivative")
plt.legend()
plt.show()
```



In [15]:

```
sigma1 = 0.1
sigma2 = 0.5

epsilon1 = np.random.normal(0, sigma1, n)
epsilon2 = np.random.normal(0, sigma2, n)

f_noisy1 = f + epsilon1
df_noisy1 = df + epsilon1

f_noisy2 = f + epsilon2
df_noisy2 = df + epsilon2

plt.figure(figsize=(12, 6))
plt.plot(T, f_noisy1, label="f_noisy with sigma1")
plt.plot(T, df_noisy1, label="df_noisy with sigma1")
plt.plot(T, f_noisy2, label="f_noisy with sigma2")
plt.plot(T, df_noisy2, label="df_noisy with sigma2")
plt.legend()
plt.show()
```



This code will generate two sets of plots for the two different values of the noise process covariance (sigma = 0.1 and sigma = 0.5), showing the estimates against the values for f and f' computed analytically.

wavelet

In [16]:
```
def haar_wavelet_transform(img):
    n, m = img.shape

    h0 = np.array([1/np.sqrt(2), 1/np.sqrt(2)])
    h1 = np.array([1/np.sqrt(2), -1/np.sqrt(2)])

    new_array = np.zeros_like(img)
    detail_array = np.zeros_like(img)

    for i in range(n):
        for j in range(0, n, 2):
            two_pixels = img[i, j:j+2]
            new_array[i, j // 2] = np.dot(h0, two_pixels)
            detail_array[i, j // 2] = np.dot(h1, two_pixels)
    return new_array, detail_array
```

In [17]:
```
img_with_spesific_range_of_n_px = np.random.randint(24, 27, (16, 16))
img_with_spesific_range_of_m_px = np.random.randint(201, 205, (16, 16)
J32_spesific_px = np.concatenate((img_with_spesific_range_of_n_px, img
random_image = np.random.randint(0, 255, (32, 32))
I, detail_array1 = haar_wavelet_transform(random_image)
sum_detailI = np.sum(np.abs(detail_array1) > 1/100) / (32 * 32)

Ir, detail_array2 = haar_wavelet_transform(J32_spesific_px)
sum_detailIr = np.sum(np.abs(detail_array2) > 1/100) / (32 * 32)

print(sum_detailI, sum_detailIr)
```

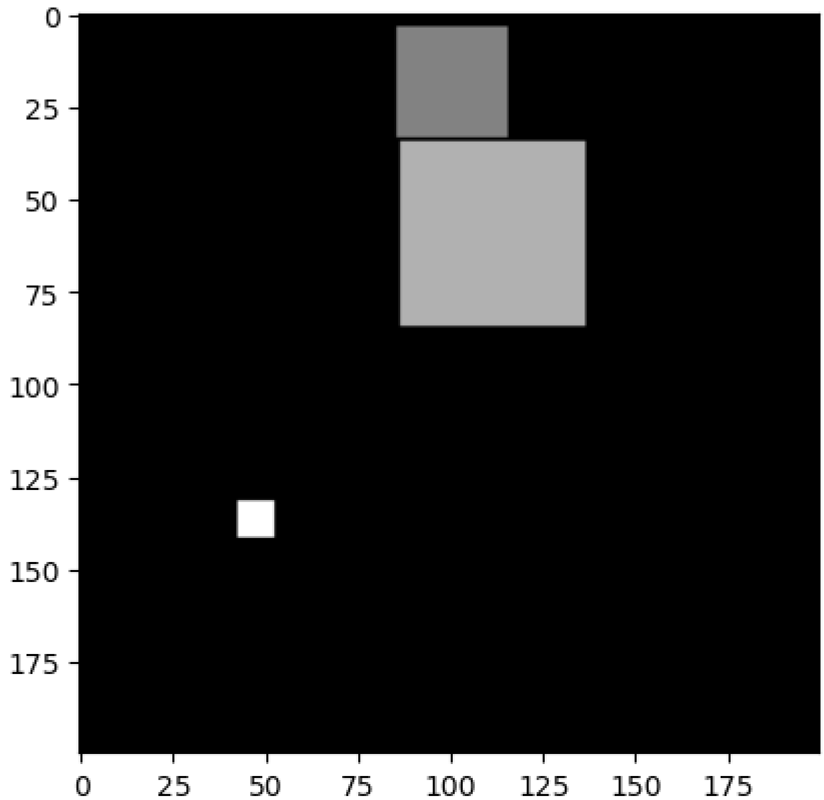
```
0.4951171875 0.0234375
```

Sum detail is used as a measure of the level of detail in an image. If the fraction is high, the image contains a lot of detail information, while if the fraction is low, the image contains less detail information.

Image Proccessing

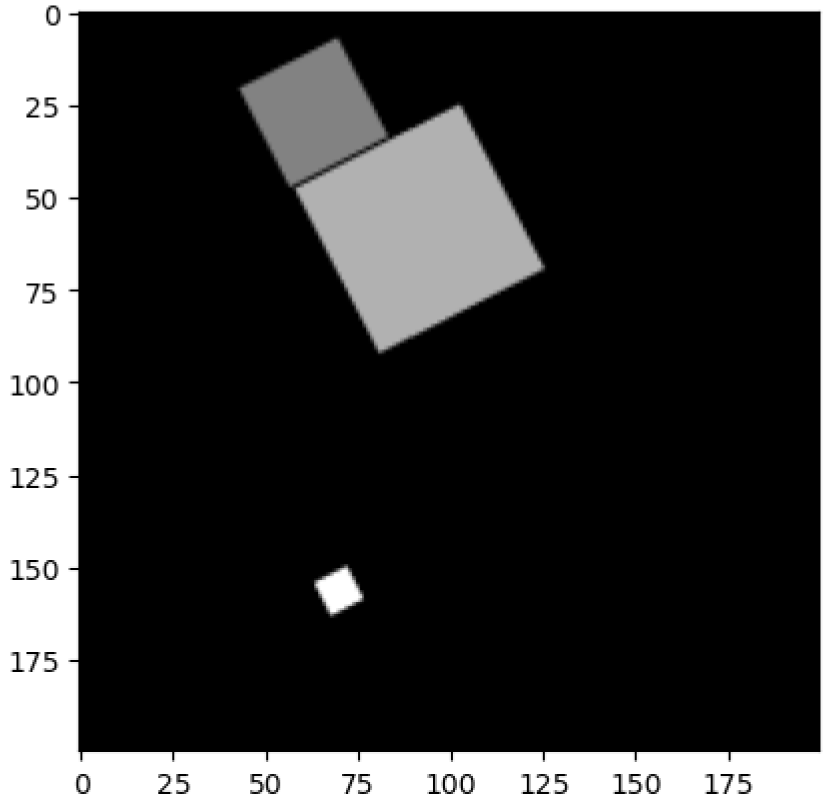
In [20]:
```
pixel_image = np.zeros((200, 200), dtype=np.uint8)

size = [10, 30, 50]
for i in range(len(size)):
    x = np.random.randint(0, 140)
    y = np.random.randint(0, 140)
    brightness = np.random.randint(50, 256)
    pixel_image[y: y + size[i], x: x + size[i]] = brightness


plt.imshow(pixel_image, cmap='gray')
```

Out[20]: <matplotlib.image.AxesImage at 0x7fe864bd32e0>



In [21]:
```
random = np.random.uniform(10, 80)
M = cv2.getRotationMatrix2D((100,100), random, 1)
pixel_image_rotate = cv2.warpAffine(pixel_image, M, (200, 200))

plt.imshow(pixel_image_rotate, cmap='gray')
```

Out[21]: <matplotlib.image.AxesImage at 0x7fe864ade6a0>

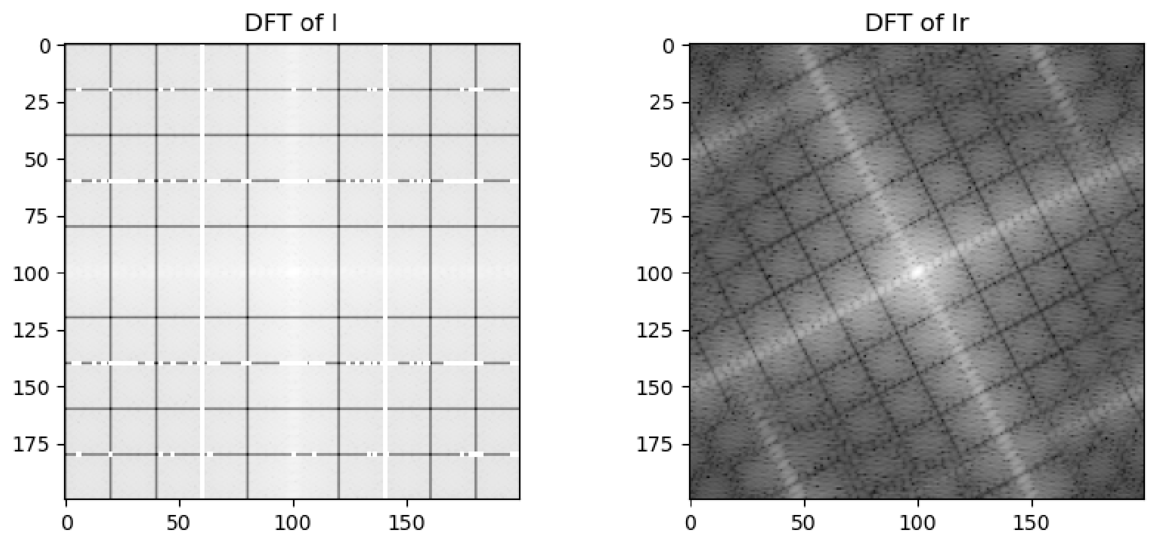


2D DFT

In [22]:
```
I_dft = np.fft.fftshift(np.fft.fft2(pixel_image))
Ir_dft = np.fft.fftshift(np.fft.fft2(pixel_image_rotate))

plt.figure(figsize=(10, 4))
plt.subplot(121)
plt.imshow(np.log(np.abs(I_dft)), cmap='gray')
plt.title('DFT of I')

plt.subplot(122)
plt.imshow(np.log(np.abs(Ir_dft)), cmap='gray')
plt.title('DFT of Ir')
```

```
/tmp/ipykernel_6002/1051944270.py:6: RuntimeWarning: divide by zero e
ncountered in log
  plt.imshow(np.log(np.abs(I_dft)), cmap='gray')
```

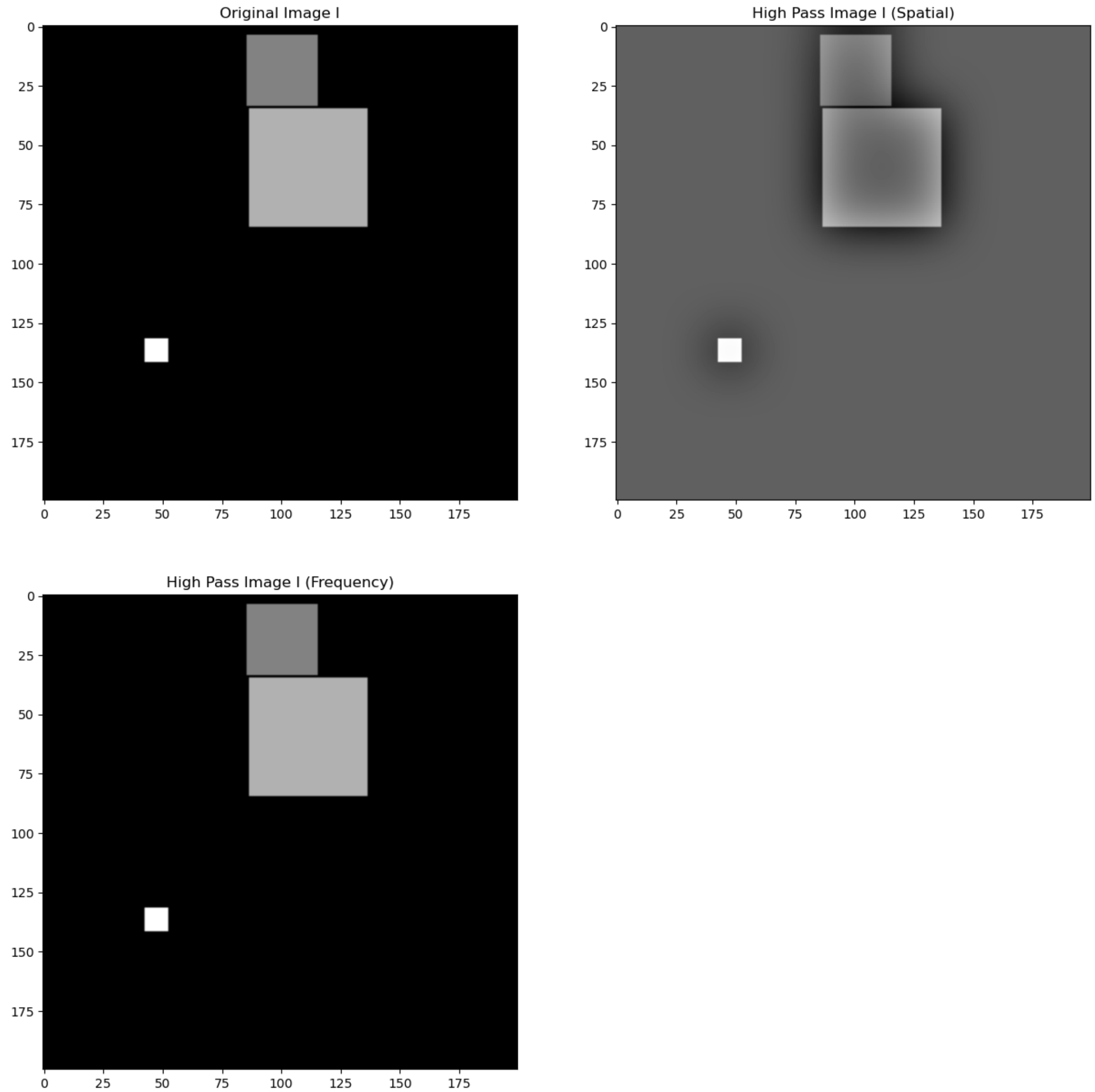
Out[22]:
```
Text(0.5, 1.0, 'DFT of Ir')
```

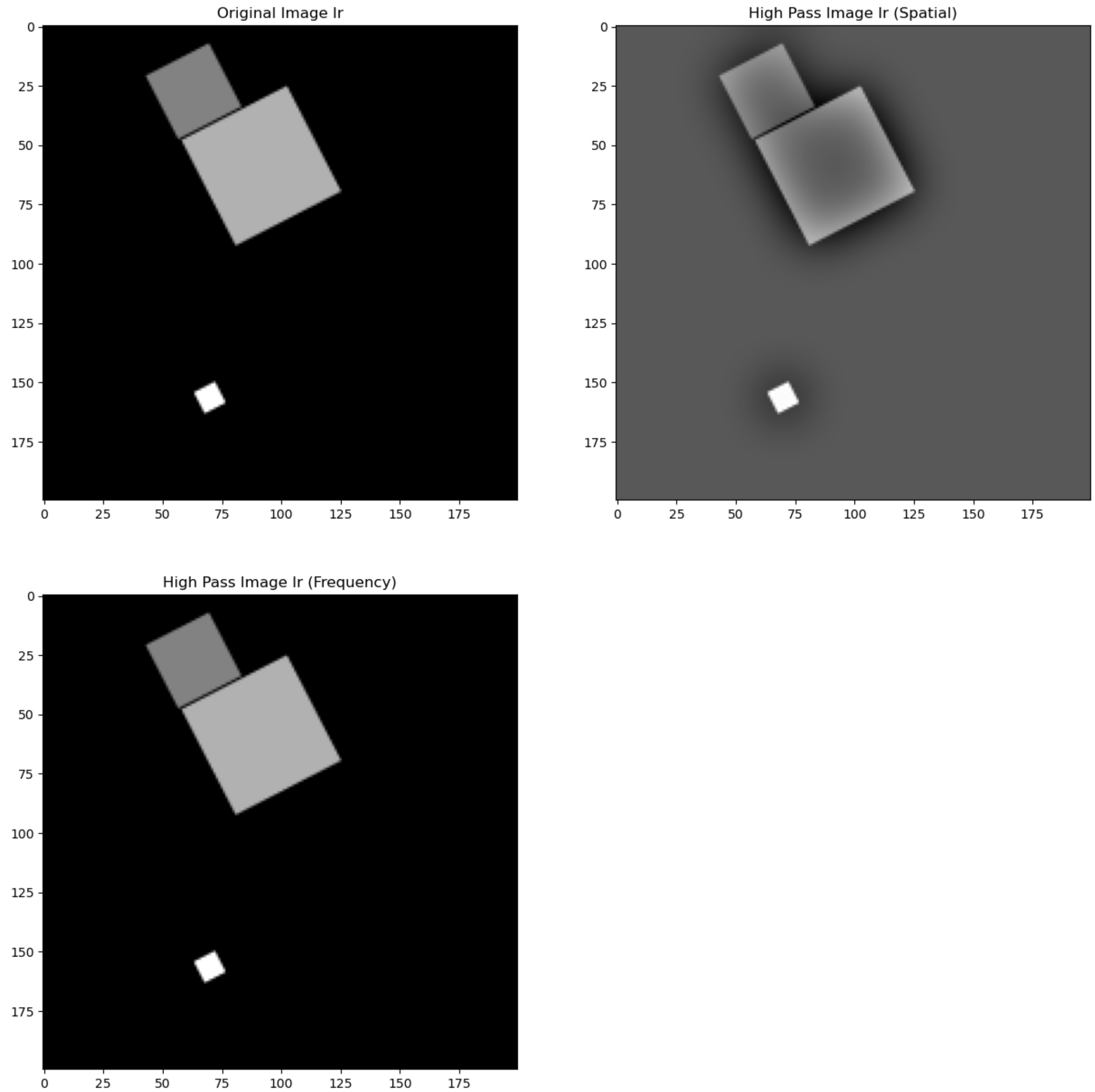


By visualizing the DFTs of the two images, we can observe their frequency content and analyze the differences between them. In the center we have zero frequency.

Image filtering

In [23]:
```
I = pixel_image
Ir = pixel_image_rotate

I = I.astype(np.float32) / 255
Ir = Ir.astype(np.float32) / 255

kernel_size = 51
sigma = 10
gaussian = cv2.getGaussianKernel(kernel_size, sigma)
gaussian = gaussian * gaussian.T

# remove high freq
I_high_pass = I - cv2.filter2D(I, -1, gaussian)
Ir_high_pass = Ir - cv2.filter2D(Ir, -1, gaussian)

I_f = np.fft.fft2(I)
Ir_f = np.fft.fft2(Ir)

gaussian_f = np.fft.fft2(gaussian, (200, 200))

I_high_pass_f = I_f - gaussian_f
Ir_high_pass_f = Ir_f - gaussian_f
I_high_pass_f = np.fft.ifft2(I_high_pass_f)
Ir_high_pass_f = np.fft.ifft2(Ir_high_pass_f)

plt.figure(figsize=(15, 15))
plt.subplot(221)
plt.imshow(I, cmap='gray')
plt.title('Original Image I')
plt.subplot(222)
plt.imshow(I_high_pass, cmap='gray')
plt.title('High Pass Image I (Spatial)')
plt.subplot(223)
plt.imshow(np.abs(I_high_pass_f), cmap='gray')
plt.title('High Pass Image I (Frequency)')
plt.figure(figsize=(15, 15))
plt.subplot(221)
plt.imshow(Ir, cmap='gray')
plt.title('Original Image Ir')
plt.subplot(222)
plt.imshow(Ir_high_pass, cmap='gray')
plt.title('High Pass Image Ir (Spatial)')
plt.subplot(223)
plt.imshow(np.abs(Ir_high_pass_f), cmap='gray')
plt.title('High Pass Image Ir (Frequency)')
plt.show()
```


Original Image I
High Pass Image I (Spatial)
High Pass Image I (Frequency)
0
25
50
75
100
125
150
175



when we do kernels like blur or gray filter, it's easier to convert. The rate is so important because it can convert important pixels to be lost.