

Assigned: 19 September 2019

## Project #4 – Integrals and Intervals

EE 511: Fall 2019

**Due: Thursday, 26 September 2019 at 16:00.** Late penalty: 15% per day before 28 September at 16:00.

1. Approximate the following integrals using a Monte Carlo simulation. Compare your estimates with the exact values (if known):
  - a.  $\int_{-2}^2 e^{x+x^2} dx$ .
  - b.  $\int_{-\infty}^{\infty} e^{-x^2} dx$ .
  - c.  $\int_0^1 \int_0^1 e^{-(x+y)^2} dy dx$ .
2. Define the random variable  $X = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2$  where  $Z_k \sim N(0,1)$ . Then  $X \sim \chi^2(4)$ . Generate 10 samples from  $X$  by first sampling  $Z_i$  for  $i = 1, 2, 3, 4$  and then computing  $X$ . Plot the empirical distribution  $F_{10}^*(x)$  for your samples and overlay the theoretical distribution  $F(x)$ . Estimate a lower bound for  $\|F_{10}^*(x) - F(x)\|_{\infty}$  by computing the maximum difference at each of your samples:  $\max_{x_i} |F_{10}^*(x_i) - F(x_i)|$ . Then find the 25th, 50th, and 90<sup>th</sup> percentiles using your empirical distribution and compare the value to the theoretical percentile values for  $\chi^2(4)$ . Repeat the above using 100 and 1000 samples from  $X$ .
3. A geyser is a hot spring characterized by an intermittent discharge of water and steam. Old Faithful is a famous cone geyser in Yellowstone National Park, Wyoming. It has a predictable geothermal discharge and since 2000 it has erupted every 44 to 125 minutes. Refer to the addendum data file that contains waiting times and the durations for 272 eruptions. Compute a 95% statistical confidence interval for the mean waiting time using data from only the first 15 eruptions. Compare this to a 95% bootstrap confidence interval using the same 15 data samples. Repeat these calculations using all the data samples. Comment on the relative width of the confidence intervals when using only 15 samples vs using all samples.