Assigned: 05 February 2020

Project #3 – Samples and statistics

EE 511: Spring 2020

Due: Wednesday, 12 February 2020 at 14:00. Late penalty: 15% per day before 14 February at 14:00.

1. A components manufacturer delivers a batch of 125 microchips to a parts distributor. The distributor checks for lot conformance by counting the number of defective chips in a random sampling (without replacement) of the lot. If the distributor finds any defective chips in the sample they reject the entire lot. Suppose that there are six defective units in the lot of 125 microchips. Simulate the lot sampling to estimate the probability that the distributor will reject the lot if it tests five microchips. What is the fewest number of microchips that the distributor should test to reject this lot 95% of the time?

2. Suppose that 120 cars arrive at a freeway onramp per hour on average. Simulate one hour of arrivals to the freeway onramp: (1) subdivide the hour into small time intervals (< 1 second) and then (2) perform a Bernoulli trial to indicate a car arrival within each small time-interval. Generate a histogram for the number of arrivals per hour. Repeat the counting experiment by sampling directly from an equivalent Poisson distribution by using the inverse transform method (described in class). Generate a histogram for the number of arrivals per hour using this method. Overlay the theoretical p.m.f. on both histograms. Comment on the results.

3. Define the random variable $N = \min\{n: \sum_{i=1}^{n} X_i > 4\}$ as the smallest number of uniform random samples whose sum is greater than four. Generate a histogram using 100, 1000, and 10000 samples for N. Comment on $E[N]$.

4. Produce a sequence $\{X_k\}$ where $p_j = \frac{p}{j}$ for $j = 1, 2, \cdots, 60$ where p is a constant for you to determine. [This is equivalent to spinning the minute hand on a clock and observing the stopping position if $P[\text{stop on minute } j] = \frac{p}{j}$]. Generate a histogram. Define the random variable $N_j = \min\{k: X_k = j\}$. Simulate sampling from N_{60}. Estimate $E[N_{60}]$ and $Var[N_{60}]$. Compare you estimates with the theoretical values.

5. Use the accept-reject method to sample from the following distribution p_j by sampling from the (non-optimal) uniform auxiliary distribution ($q_j = 0.05$ for $j = 1, \ldots, 20$):

$$p_1 = p_2 = p_3 = p_4 = p_5 = 0.06, p_6 = p_9 = 0.15, p_7 = p_{10} = 0.13, p_8 = 0.14.$$

   Generate a histogram and overlay the target distribution p_j. Compute the sample mean and sample variance and compare these values to the theoretical values. Estimate the efficiency of your sampler with the following ratio:

$$\text{Efficiency} = \frac{\#\, accepted}{\#\, accepted + \#\, rejected}$$

   Compare your estimate of the efficiency to the theoretical efficiency given your choice for the constant c.