Toronto Home Price Index

A Time Series Analysis

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AISC Time Series Discussion Group, 2021

Outline

1. Introduction
2. Data: Home Price Index (HPI)
3. Problem Statement: Forecast the HPI
4. Validation Framework
5. Model Framework: Vector AutoRegressive (VAR)
6. Forecasts & Validation
7. Conclusions & Future Work

Repo

- Purpose: Forecast Home Price Index (HPI) in Toronto by neighborhood.
- Validation: MSE calculated for each time period & neighborhood.
- Benchmark: Moving average return.
- Code: https://github.com/sinan-ozel/toronto-hpi

Home Price Index (HPI)

Home Price Index

- Based on the sold prices of residential real estate.
- Corrects for the estate's features to make it less volatile, compared to averages.
- https://trreb.ca/index.php/market-news/mls-home-price-index/

	Area	Date / Type	2020-06-01	2020-07-01	2020-08-01	2020-09-01	2020-10-01	2020-11-01	2020-12-01	2021-01-01	2021-02-01	2021-03-01	2021-04-01
HPI	Milton	Apartment	287.6	289.8	290.9	287.8	288.9	288.7	NaN	292.4	298.8	306.6	320.2
		Composite	289.5	285.5	289.1	294.5	297.8	301.7	NaN	325.5	341.9	346.0	347.6
		Single-Family Attached	303.2	297.4	303.4	308.9	314.2	320.5	NaN	349.1	367.8	368.1	369.0
		Single-Family Detached	283.2	284.7	287.9	294.2	297.5	301.8	NaN	329.3	346.5	349.4	348.7
		Townhouse	291.9	302.8	309.3	319.3	317.3	319.9	NaN	324.5	344.2	362.5	363.2
	Mississauga	Apartment	307.2	308.5	309.4	310.6	308.0	307.1	303.0	302.6	309.2	321.3	329.5
		Composite	287.4	291.1	294.3	296.0	295.8	296.2	296.9	301.5	313.8	327.0	332.6
		Single-Family Attached	275.3	280.0	285.2	288.0	290.1	292.4	295.7	302.7	320.1	333.3	336.5
		Single-Family Detached	273.5	278.5	282.9	285.8	287.3	288.8	292.3	300.0	316.3	330.2	334.9
		Townhouse	288.2	294.7	298.8	298.8	298.0	297.9	299.2	301.8	312.7	326.7	332.2
	Toronto C10	Apartment	315.5	307.4	306.7	305.6	302.1	305.0	303.0	298.9	305.6	313.8	326.2
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		Single-Family Detached	265.6	283.1	282.1	277.2	275.0	278.2	277.1	287.3	295.6	295.3	301.8
		Townhouse	279.9	279.1	281.8	277.0	282.5	282.2	279.0	266.5	272.5	273.2	284.8

Home Price Index (HPI)

Granularity

- Area: Defined by the Toronto Region Real Estate Board (TRREB). (Toronto C10, Mississauga, Milton, etc...)
- Type: Housing type
- Date: Monthly, first of every month

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Home Price Index (HPI)

Panel Data

- Each panel is an Area x Type pair, i.e. each row in the below table.
- Each panel has either 103 or 69 data points (months)
- We can run multivariate analysis or multiple univariate analyses.

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Goal

Forecast HPI

- Come up with the best forecast of HPI possible.
- Refresh knowledge of time series models in the process.
- Present results to spark discussion.

Research Questions

- Can we beat a basic benchmark?
- Which approach will work best?
- Can we develop a better approach?

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Validation: Moving Window, Next Month Forecast MSE

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Moving Window

4-month Training Sample

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Validation: Moving Window, Next Month Forecast MSE

Moving Window

4-month Training Sample

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Validation: Moving Window, Next Month Forecast MSE

Moving Window

4-month Training Sample

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Moving Window

4-month Training Sample

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Validation: Moving Window, Next Month Forecast MSE

Moving Window

4-month Training Sample Validation 1 Month After

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		Single-Family Attached	254.0	271.8	272.2	266.5	262.9	269.3	269.4	279.0	287.9	288.8	292.2
		Single-Family Detached	265.6	283.1	282.1	277.2	275.0	278.2	277.1	287.3	295.6	295.3	301.8
		Townhouse	279.9	279.1	281.8	277.0	282.5	282.2	279.0	266.5	272.5	273.2	284.8

Validation: Moving Window, Next Month Forecast MSE

Moving Window

4-month Training Sample Validation 1 Month After

	Area	Date / Type	2020-06-01	2020-07-01	2020-08-01	2020-09-01	2020-10-01	2020-11-01	2020-12-01	2021-01-01	2021-02-01	2021-03-01	2021-04-01
HPI	Milton	Apartment	287.6	289.8	290.9	287.8	288.9	288.7	NaN	292.4	298.8	306.6	320.2
		Composite	289.5	285.5	289.1	294.5	297.8	301.7	NaN	325.5	341.9	346.0	347.6
		Single-Family Attached	303.2	297.4	303.4	308.9	314.2	320.5	NaN	349.1	367.8	368.1	369.0
		Single-Family Detached	283.2	284.7	287.9	294.2	297.5	301.8	NaN	329.3	346.5	349.4	348.7
		Townhouse	291.9	302.8	309.3	319.3	317.3	319.9	NaN	324.5	344.2	362.5	363.2
	Mississauga	Apartment	307.2	308.5	309.4	310.6	308.0	307.1	303.0	302.6	309.2	321.3	329.5
		Composite	287.4	291.1	294.3	296.0	295.8	296.2	296.9	301.5	313.8	327.0	332.6
		Single-Family Attached	275.3	280.0	285.2	288.0	290.1	292.4	295.7	302.7	320.1	333.3	336.5
		Single-Family Detached	273.5	278.5	282.9	285.8	287.3	288.8	292.3	300.0	316.3	330.2	334.9
		Townhouse	288.2	294.7	298.8	298.8	298.0	297.9	299.2	301.8	312.7	326.7	332.2
	Toronto C10	Apartment	315.5	307.4	306.7	305.6	302.1	305.0	303.0	298.9	305.6	313.8	326.2
		Composite	299.4	298.5	297.8	295.6	292.7	295.7	294.0	293.3	300.5	306.1	317.0
		Single-Family Attached	254.0	271.8	272.2	266.5	262.9	269.3	269.4	279.0	287.9	288.8	292.2
		Single-Family Detached	265.6	283.1	282.1	277.2	275.0	278.2	277.1	287.3	295.6	295.3	301.8
		Townhouse	279.9	279.1	281.8	277.0	282.5	282.2	279.0	266.5	272.5	273.2	284.8

Validation: Moving Window, Next Month Forecast MSE

Moving Window

4-month Training Sample Validation 1 Month After

	Area	Date / Type	2020-06-01	2020-07-01	2020-08-01	2020-09-01	2020-10-01	2020-11-01	2020-12-01	2021-01-01	2021-02-01	2021-03-01	2021-04-01
HPI	Milton	Apartment	287.6	289.8	290.9	287.8	288.9	288.7	NaN	292.4	298.8	306.6	320.2
		Composite	289.5	285.5	289.1	294.5	297.8	301.7	NaN	325.5	341.9	346.0	347.6
		Single-Family Attached	303.2	297.4	303.4	308.9	314.2	320.5	NaN	349.1	367.8	368.1	369.0
		Single-Family Detached	283.2	284.7	287.9	294.2	297.5	301.8	NaN	329.3	346.5	349.4	348.7
		Townhouse	291.9	302.8	309.3	319.3	317.3	319.9	NaN	324.5	344.2	362.5	363.2
	Mississauga	Apartment	307.2	308.5	309.4	310.6	308.0	307.1	303.0	302.6	309.2	321.3	329.5
		Composite	287.4	291.1	294.3	296.0	295.8	296.2	296.9	301.5	313.8	327.0	332.6
		Single-Family Attached	275.3	280.0	285.2	288.0	290.1	292.4	295.7	302.7	320.1	333.3	336.5
		Single-Family Detached	273.5	278.5	282.9	285.8	287.3	288.8	292.3	300.0	316.3	330.2	334.9
		Townhouse	288.2	294.7	298.8	298.8	298.0	297.9	299.2	301.8	312.7	326.7	332.2
	Toronto C10	Apartment	315.5	307.4	306.7	305.6	302.1	305.0	303.0	298.9	305.6	313.8	326.2
		Composite	299.4	298.5	297.8	295.6	292.7	295.7	294.0	293.3	300.5	306.1	317.0
		Single-Family Attached	254.0	271.8	272.2	266.5	262.9	269.3	269.4	279.0	287.9	288.8	292.2
		Single-Family Detached	265.6	283.1	282.1	277.2	275.0	278.2	277.1	287.3	295.6	295.3	301.8
		Townhouse	279.9	279.1	281.8	277.0	282.5	282.2	279.0	266.5	272.5	273.2	284.8

Validation: Moving Window, Next Month Forecast MSE

Moving Window

4-month Training Sample Validation 1 Month After

	Area	Date / Type	2020-06-01	2020-07-01	2020-08-01	2020-09-01	2020-10-01	2020-11-01	2020-12-01	2021-01-01	2021-02-01	2021-03-01	2021-04-01
HPI	Milton	Apartment	287.6	289.8	290.9	287.8	288.9	288.7	NaN	292.4	298.8	306.6	320.2
		Composite	289.5	285.5	289.1	294.5	297.8	301.7	NaN	325.5	341.9	346.0	347.6
		Single-Family Attached	303.2	297.4	303.4	308.9	314.2	320.5	NaN	349.1	367.8	368.1	369.0
		Single-Family Detached	283.2	284.7	287.9	294.2	297.5	301.8	NaN	329.3	346.5	349.4	348.7
		Townhouse	291.9	302.8	309.3	319.3	317.3	319.9	NaN	324.5	344.2	362.5	363.2
	Mississauga	Apartment	307.2	308.5	309.4	310.6	308.0	307.1	303.0	302.6	309.2	321.3	329.5
		Composite	287.4	291.1	294.3	296.0	295.8	296.2	296.9	301.5	313.8	327.0	332.6
		Single-Family Attached	275.3	280.0	285.2	288.0	290.1	292.4	295.7	302.7	320.1	333.3	336.5
		Single-Family Detached	273.5	278.5	282.9	285.8	287.3	288.8	292.3	300.0	316.3	330.2	334.9
		Townhouse	288.2	294.7	298.8	298.8	298.0	297.9	299.2	301.8	312.7	326.7	332.2
	Toronto C10	Apartment	315.5	307.4	306.7	305.6	302.1	305.0	303.0	298.9	305.6	313.8	326.2
		Composite	299.4	298.5	297.8	295.6	292.7	295.7	294.0	293.3	300.5	306.1	317.0
		Single-Family Attached	254.0	271.8	272.2	266.5	262.9	269.3	269.4	279.0	287.9	288.8	292.2
		Single-Family Detached	265.6	283.1	282.1	277.2	275.0	278.2	277.1	287.3	295.6	295.3	301.8
		Townhouse	279.9	279.1	281.8	277.0	282.5	282.2	279.0	266.5	272.5	273.2	284.8

Validation: Moving Window, Next Month Forecast MSE

Moving Window

4-month Training Sample Validation 1 Quarter After

	Area	Date / Type	2020-06-01	2020-07-01	2020-08-01	2020-09-01	2020-10-01	2020-11-01	2020-12-01	2021-01-01	2021-02-01	2021-03-01	2021-04-01
HPI	Milton	Apartment	287.6	289.8	290.9	287.8	288.9	288.7	NaN	292.4	298.8	306.6	320.2
		Composite	289.5	285.5	289.1	294.5	297.8	301.7	NaN	325.5	341.9	346.0	347.6
		Single-Family Attached	303.2	297.4	303.4	308.9	314.2	320.5	NaN	349.1	367.8	368.1	369.0
		Single-Family Detached	283.2	284.7	287.9	294.2	297.5	301.8	NaN	329.3	346.5	349.4	348.7
		Townhouse	291.9	302.8	309.3	319.3	317.3	319.9	NaN	324.5	344.2	362.5	363.2
	Mississauga	Apartment	307.2	308.5	309.4	310.6	308.0	307.1	303.0	302.6	309.2	321.3	329.5
		Composite	287.4	291.1	294.3	296.0	295.8	296.2	296.9	301.5	313.8	327.0	332.6
		Single-Family Attached	275.3	280.0	285.2	288.0	290.1	292.4	295.7	302.7	320.1	333.3	336.5
		Single-Family Detached	273.5	278.5	282.9	285.8	287.3	288.8	292.3	300.0	316.3	330.2	334.9
		Townhouse	288.2	294.7	298.8	298.8	298.0	297.9	299.2	301.8	312.7	326.7	332.2
	Toronto C10	Apartment	315.5	307.4	306.7	305.6	302.1	305.0	303.0	298.9	305.6	313.8	326.2
		Composite	299.4	298.5	297.8	295.6	292.7	295.7	294.0	293.3	300.5	306.1	317.0
		Single-Family Attached	254.0	271.8	272.2	266.5	262.9	269.3	269.4	279.0	287.9	288.8	292.2
		Single-Family Detached	265.6	283.1	282.1	277.2	275.0	278.2	277.1	287.3	295.6	295.3	301.8
		Townhouse	279.9	279.1	281.8	277.0	282.5	282.2	279.0	266.5	272.5	273.2	284.8

Validation: Moving Window, Next Month Forecast MSE

Moving Window

4-month Training Sample Validation 1 Quarter After

	Area	Date / Type	2020-06-01	2020-07-01	2020-08-01	2020-09-01	2020-10-01	2020-11-01	2020-12-01	2021-01-01	2021-02-01	2021-03-01	2021-04-01
HPI	Milton	Apartment	287.6	289.8	290.9	287.8	288.9	288.7	NaN	292.4	298.8	306.6	320.2
		Composite	289.5	285.5	289.1	294.5	297.8	301.7	NaN	325.5	341.9	346.0	347.6
		Single-Family Attached	303.2	297.4	303.4	308.9	314.2	320.5	NaN	349.1	367.8	368.1	369.0
		Single-Family Detached	283.2	284.7	287.9	294.2	297.5	301.8	NaN	329.3	346.5	349.4	348.7
		Townhouse	291.9	302.8	309.3	319.3	317.3	319.9	NaN	324.5	344.2	362.5	363.2
	Mississauga	Apartment	307.2	308.5	309.4	310.6	308.0	307.1	303.0	302.6	309.2	321.3	329.5
		Composite	287.4	291.1	294.3	296.0	295.8	296.2	296.9	301.5	313.8	327.0	332.6
		Single-Family Attached	275.3	280.0	285.2	288.0	290.1	292.4	295.7	302.7	320.1	333.3	336.5
		Single-Family Detached	273.5	278.5	282.9	285.8	287.3	288.8	292.3	300.0	316.3	330.2	334.9
		Townhouse	288.2	294.7	298.8	298.8	298.0	297.9	299.2	301.8	312.7	326.7	332.2
	Toronto C10	Apartment	315.5	307.4	306.7	305.6	302.1	305.0	303.0	298.9	305.6	313.8	326.2
		Composite	299.4	298.5	297.8	295.6	292.7	295.7	294.0	293.3	300.5	306.1	317.0
		Single-Family Attached	254.0	271.8	272.2	266.5	262.9	269.3	269.4	279.0	287.9	288.8	292.2
		Single-Family Detached	265.6	283.1	282.1	277.2	275.0	278.2	277.1	287.3	295.6	295.3	301.8
		Townhouse	279.9	279.1	281.8	277.0	282.5	282.2	279.0	266.5	272.5	273.2	284.8

Validation: Moving Window, Next Month Forecast MSE

Moving Window

4-month Training Sample Validation 1 Quarter After

	Area	Date / Type	2020-06-01	2020-07-01	2020-08-01	2020-09-01	2020-10-01	2020-11-01	2020-12-01	2021-01-01	2021-02-01	2021-03-01	2021-04-01
HPI	Milton	Apartment	287.6	289.8	290.9	287.8	288.9	288.7	NaN	292.4	298.8	306.6	320.2
		Composite	289.5	285.5	289.1	294.5	297.8	301.7	NaN	325.5	341.9	346.0	347.6
		Single-Family Attached	303.2	297.4	303.4	308.9	314.2	320.5	NaN	349.1	367.8	368.1	369.0
		Single-Family Detached	283.2	284.7	287.9	294.2	297.5	301.8	NaN	329.3	346.5	349.4	348.7
		Townhouse	291.9	302.8	309.3	319.3	317.3	319.9	NaN	324.5	344.2	362.5	363.2
	Mississauga	Apartment	307.2	308.5	309.4	310.6	308.0	307.1	303.0	302.6	309.2	321.3	329.5
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	Toronto C10	Apartment	315.5	307.4	306.7	305.6	302.1	305.0	303.0	298.9	305.6	313.8	326.2
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Validation: Moving Window, Next Month Forecast MSE

Moving Window

4-month Training Sample Validation 1 Quarter After

	Area	Date / Type	2020-06-01	2020-07-01	2020-08-01	2020-09-01	2020-10-01	2020-11-01	2020-12-01	2021-01-01	2021-02-01	2021-03-01	2021-04-01
HPI	Milton	Apartment	287.6	289.8	290.9	287.8	288.9	288.7	NaN	292.4	298.8	306.6	320.2
		Composite	289.5	285.5	289.1	294.5	297.8	301.7	NaN	325.5	341.9	346.0	347.6
		Single-Family Attached	303.2	297.4	303.4	308.9	314.2	320.5	NaN	349.1	367.8	368.1	369.0
		Single-Family Detached	283.2	284.7	287.9	294.2	297.5	301.8	NaN	329.3	346.5	349.4	348.7
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AR(2) $\longrightarrow$ 2-panel VAR(2)

AR(2) Model

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \epsilon_t$$

AR(2) $\longrightarrow$ 2-panel VAR(2)

AR(2) Model on One Panel

I added the subscript 1 to show that this is panel 1 among many.

$$Y_{1,t} = \beta_0 + \beta_{1,1} Y_{1,t-1} + \beta_{1,2} Y_{1,t-2} + \epsilon_t$$

AR(2) $\longrightarrow$ 2-panel VAR(2)

AR(2) Model on One Panel

Now I added two more terms from another panel.

$$Y_{1,t} = \beta_0 + \beta_{1,1} Y_{1,t-1} + \beta_{1,2} Y_{1,t-2} + \beta_{2,1} Y_{2,t-1} + \beta_{2,2} Y_{2,t-2} + \epsilon_t$$

AR(2) $\longrightarrow$ 2-panel VAR(2)

VAR(2) Model on Two Panels

Finally, I do the same for Panel 2.

$$Y_{1,t}=\beta_0+\beta_{11,1}Y_{1,t-1}+\beta_{11,2}Y_{1,t-2}+\beta_{12,1}Y_{2,t-1}+\beta_{12,2}Y_{2,t-2}+\epsilon_{1,t}$$

$$Y_{2,t}=\beta_0+\beta_{21,1}Y_{1,t-1}+\beta_{21,2}Y_{1,t-2}+\beta_{22,1}Y_{2,t-1}+\beta_{22,2}Y_{2,t-2}+\epsilon_{2,t}$$

AR(2) $\longrightarrow$ 2-panel VAR(2)

Why VAR Models?

- Why not: we have additional data, so why not use it?
- Explanatory power: Impulse Response Functions (IRF)
- Granger Causality

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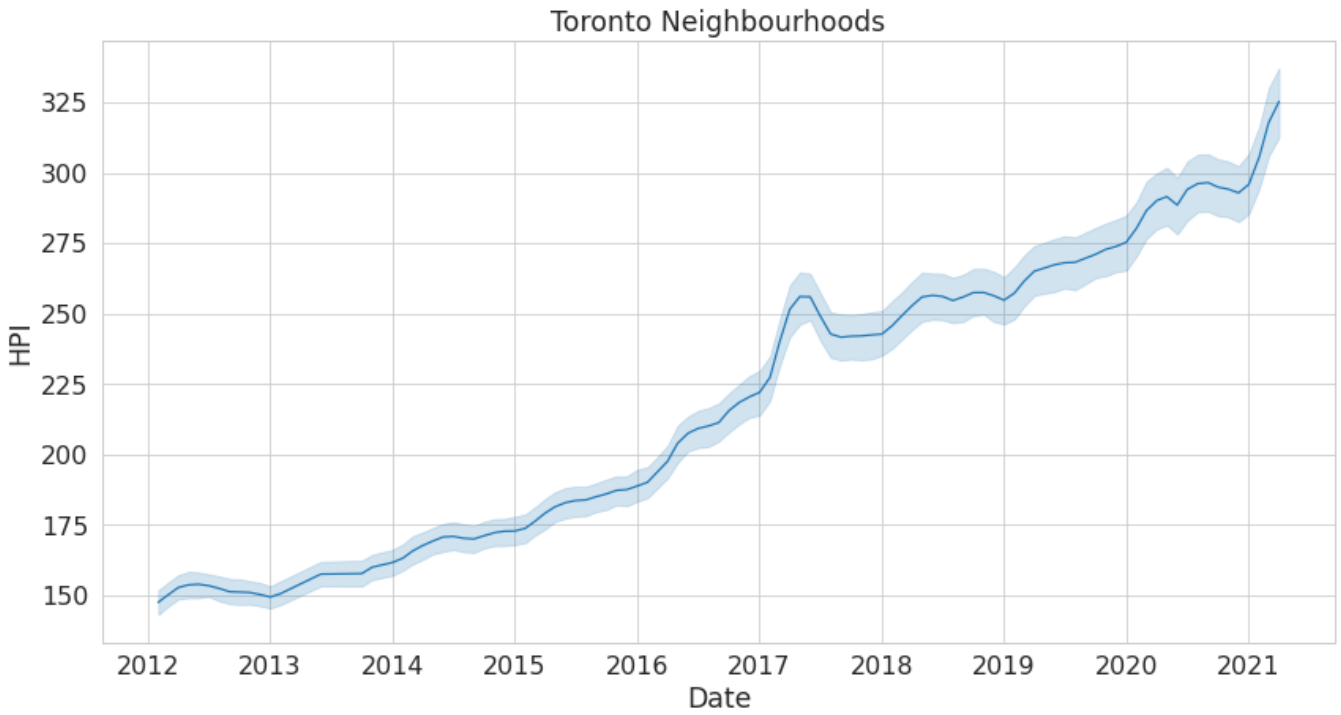
Stationarity: Dickey-Fuller & Differencing



Stationarity: Dickey-Fuller & Differencing

Are the panels stationary?

- Visually, it is clear that the panels are not stationary.
- For each panel, I run the Augmented Dickey-Fuller test
- I reject non-stationarity for over 99% of the panels.

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Differencing & Percent Changes

Differencing

$$\Delta Y_t = Y_t - Y_{t-1}$$

Percent Changes, i.e. "Returns"

$$r_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} \approx \log Y_t - \log Y_{t-1}$$

Justification for Percent Changes

- Easier to interpret
- Easier to discuss with stakeholders and laypeople
- Normalizes the amount of change to each period.

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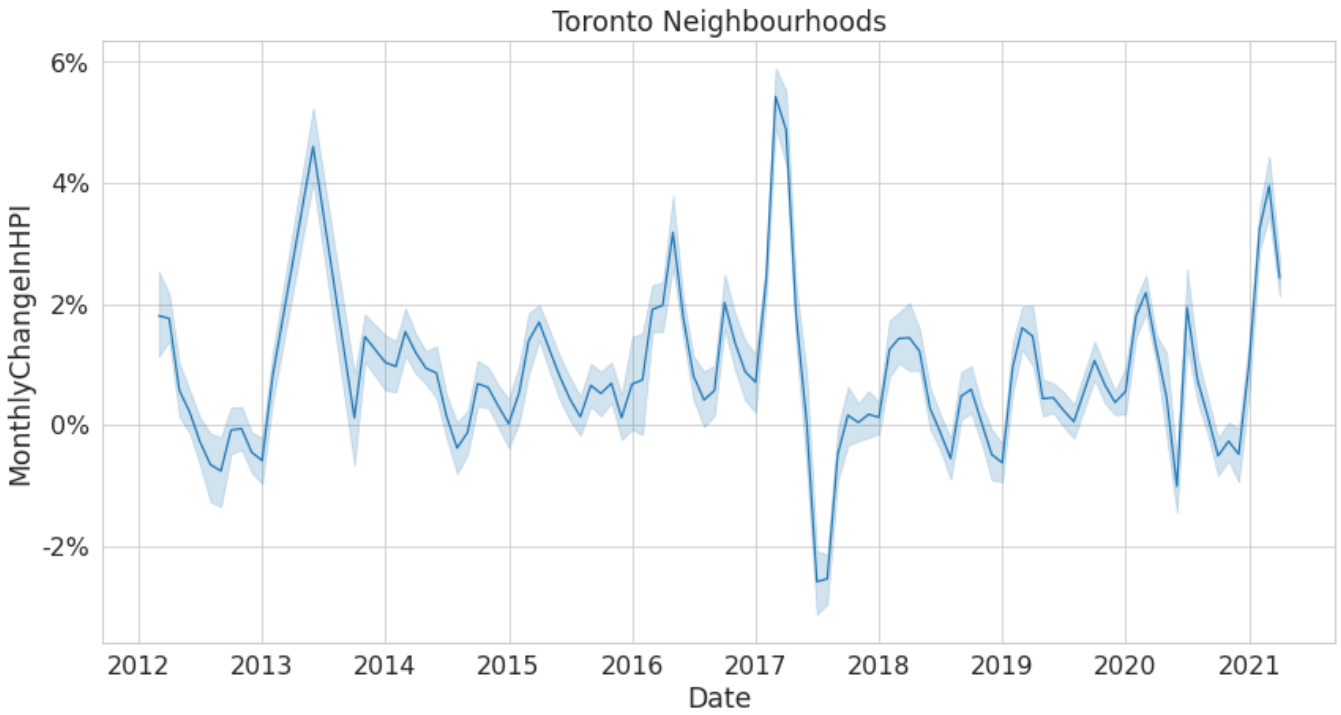
Monthly Percent Changes: i.e "Returns"



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- I fail to reject non-stationarity less than 5% of the panels.
- KPSS test could have been a better option.

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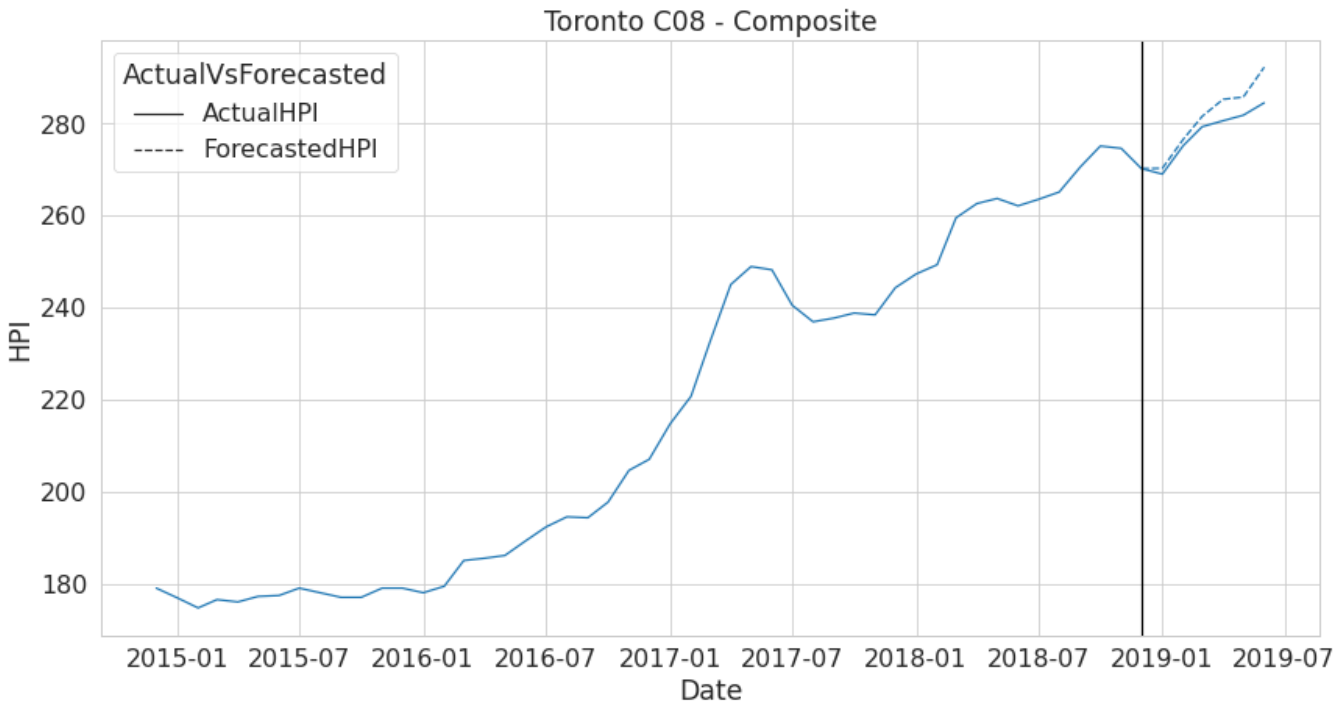

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"Return" Moving Average

- Obtain the return.
- Calculate the moving average over the rolling window.
- Use this return to forecast the next month.
- Use exponential growth formula to obtain a forecast for n months into the future.

Average "Return"

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1-month forecast

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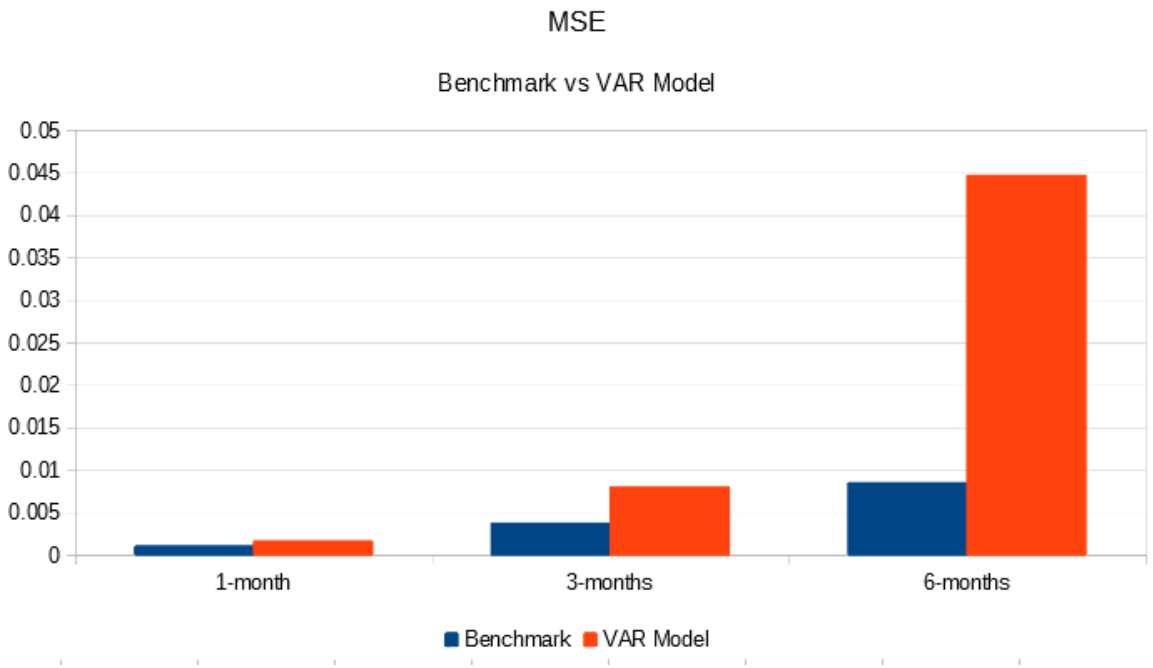
MSE: Benchmark vs VAR model



	1-month	3-months	6-months
Benchmark	0.001016761439712	0.003663453901 35	0.008432082243063
VAR Model	0.001587714029696	0.007939166535112	0.044613766619586

Conclusions & Final Thoughts

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Future Work

I would like to repeat the work with the following sets of models:

- ARMA and derivative models (ARIMA, ARIMAX, etc...)
- VECM
- ARCH & GARCH models
- LSTM networks
- Idea: A bootstrapping of VAR models with different sets of neighbourhoods and years.