MACM 316 – Homework 7

- Problems are not to be submitted. The related quiz will be given in class.
- Feel free to use Canvas discussions but please keep in mind that these forums are open.

A. Polynomial bases

1. a) Recall from lecture that the Legendre polynomials are defined as the unique polynomials satisfying:

$$\int_{-1}^{1} P_n(x)P_k(x)dx = \begin{cases} 0 & k \neq n \\ 1 & k = n \end{cases} \tag{1}$$

where $P_n(x)$ is a polynomial of degree n. Find the first four (ie. $n \leq 3$) Legendre polynomials.

b) Decompose the monomials x^2 and x^3 into linear combinations of the Legendre polynomials (ie. write them as the sum of multiples of the Legendre polynomials).

B. Least squares

Section 8.1 Ex 3

Section 8.1 Ex 5

1. Consider the data $f(1) = 2$, $f(0.5) = 0$, $f(0) = 1$, $f(-0.5) = -1$, and $f(-1) = -2$.

a) Use the first three Legendre polynomials to find a least-squares fit to the data. If you are unsure of your answer to A1, you can confirm it with the lecture notes or textbook.

b) The Legendre polynomials are not the only set of basis polynomials we can use. For example, the Chebyshev polynomials are defined as:

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x). \tag{2}$$

Use the first three Chebyshev polynomials to find a least-squares fit to the data. Compare your answer to (a).

2. Repeat question 1 using the data $f(1) = 1.4687$, $f(0.5) = 1.4469$, $f(0) = 1$, $f(-0.5) = 0.5323$, and $f(-1) = 0.1988$. Re-use as much of your work from question 1 as you can.

3. a) Show that $QR = A$ where:

$$Q = \begin{bmatrix} -1/3 & 2/3 \\ -2/3 & 1/3 \\ 2/3 & 2/3 \end{bmatrix}, \quad R = \begin{bmatrix} 3/4 & 9 \\ 0 & 3 \end{bmatrix}, \quad A = \begin{bmatrix} -1/4 & -1 \\ -1/2 & -5 \\ 1/2 & 8 \end{bmatrix}. \tag{3}$$

Include a confirmation that Q is orthogonal.

b) Use a QR factorization to solve the least-squares problem

$$A\mathbf{z} = \begin{bmatrix} 1/4 \\ 3/2 \\ -5/2 \end{bmatrix} \tag{4}$$

where A is defined in 3a).

C. Numerical differentiation

Section 4.1 Ex 1

Section 4.1 Ex 3

Section 4.1 Ex 5