Because the integral $I = \int_0^1 \frac{\cos(x^{-1}\log(x))}{x}\,\mathrm{d}x.$ have infinitely-many oscillation and it is singular at x= 0. The integral can't be found using Trapezoidal or Simpson's.

As suggested by the instruction, we could find the root of f(x) at the points of 1> a_1 > a_2>…>0 by using a_1 = exp (- b_i) and b = fzero(@(x) x*exp(x) - (i - 1/2)*pi,0). Then I implemented a function to calculate I_i by using the integral function. The value of Q_n is the sum of I_i and Q_n should be implemented as a function taken n as input value.

	200	400	600	800	1000
Q_N	0.322044195024863	0.322694365078523	0.322914722317801	0.323025915866245	0.323093049732836
Q_hat_N	0.323367423794285	0.323367430663115	0.323367431372944	0.323367431548050	0.323093049732836
Q_N	1200	1400	1600	1800	2000
	0.323138015587490	0.323170253559917	0.323194506152915	0.323213418224194	0.323228581795080
Q_hat_N	0.323367431638916	0.323367431653211	0.323367431661267	0.323367431666150	0.323367431669281

For the same reason above, Aitken's method should be a function that take n as an input. And Q_hat_N = Q(n) - (((Q(n+1) - Q(n)).^2)/(Q(n+2)-2*Q(n +1) + Q(n))). The integral are presented in the figure above.

For the function $I(p) = \int_0^1 \frac{\cos(x^{-p}\log(x))}{x}\,\mathrm{d}x$, we can redefine the function

by change -1 to -p. But I don't know how to to find the root of f(x) in this case. So I can't analyze this situation.