

Intelligent Systems

Dijkstra & Prim

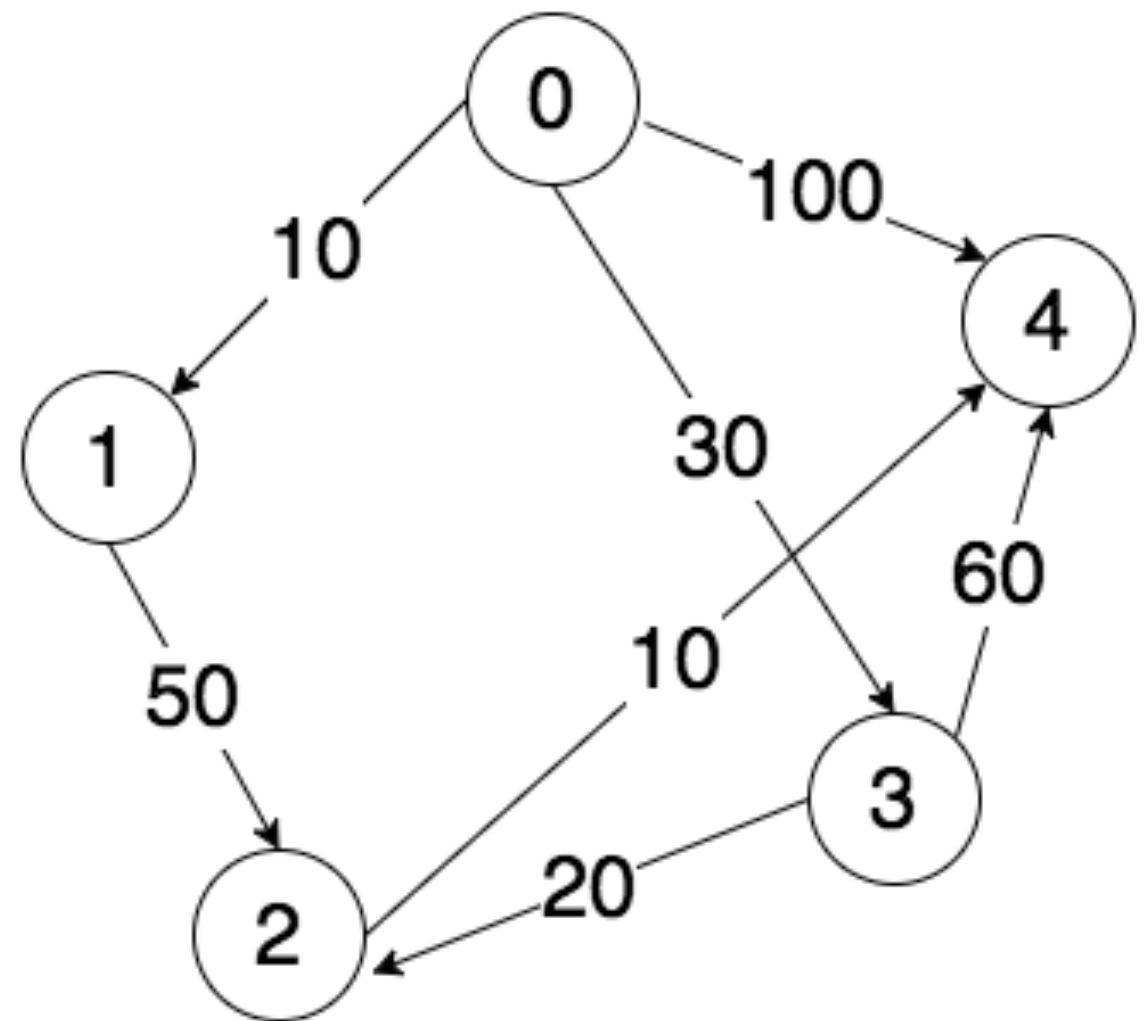
Sinan NAR

Content

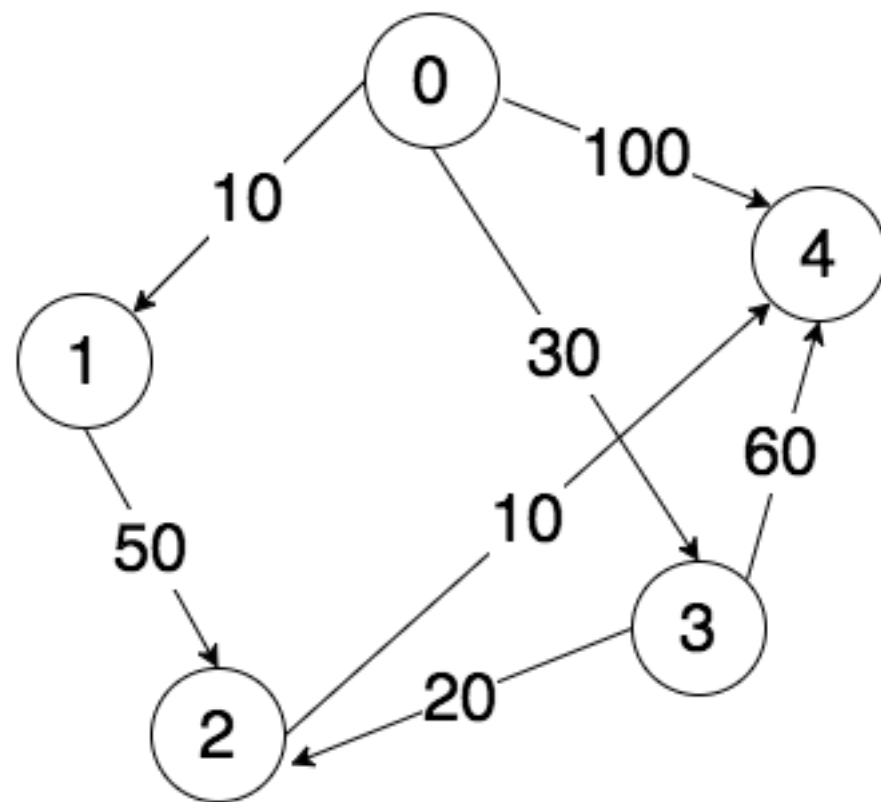
- Dijkstra - How to pronounce & Shortest Path
- Prim - Minimum Spanning Tree
- Implementation (I will not explain code)
- Algorithm Details

DIJKSTRA

- Shortest path from one vertex to other vertices
- Need two sets, S and V-S
- Will be using tables $p[v]$ and $d[v]$



STEP 1



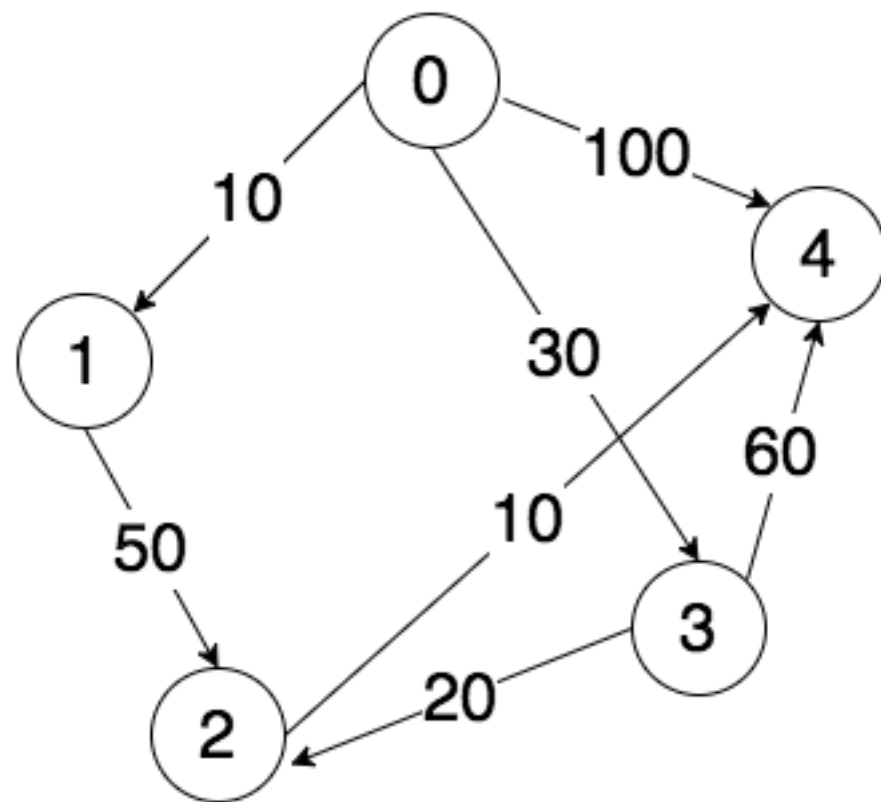
$$S = \{0\}$$

$$V-S = \{1,2,3,4\}$$

v	d[v]	p[v]
1	10	0
2	inf	0
3	30	0
4	100	0



STEP 2



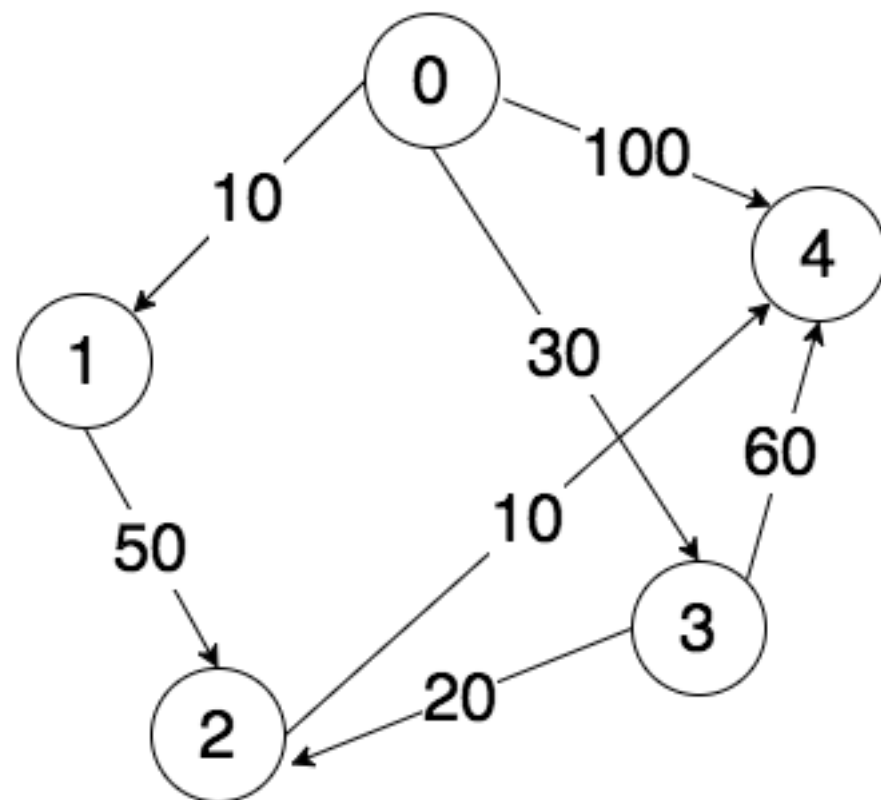
$S = \{0, 1\}$

$V-S = \{2, 3, 4\}$

v	d[v]	p[v]
1	10	0
2	60	1
3	30	0
4	100	0



STEP 3



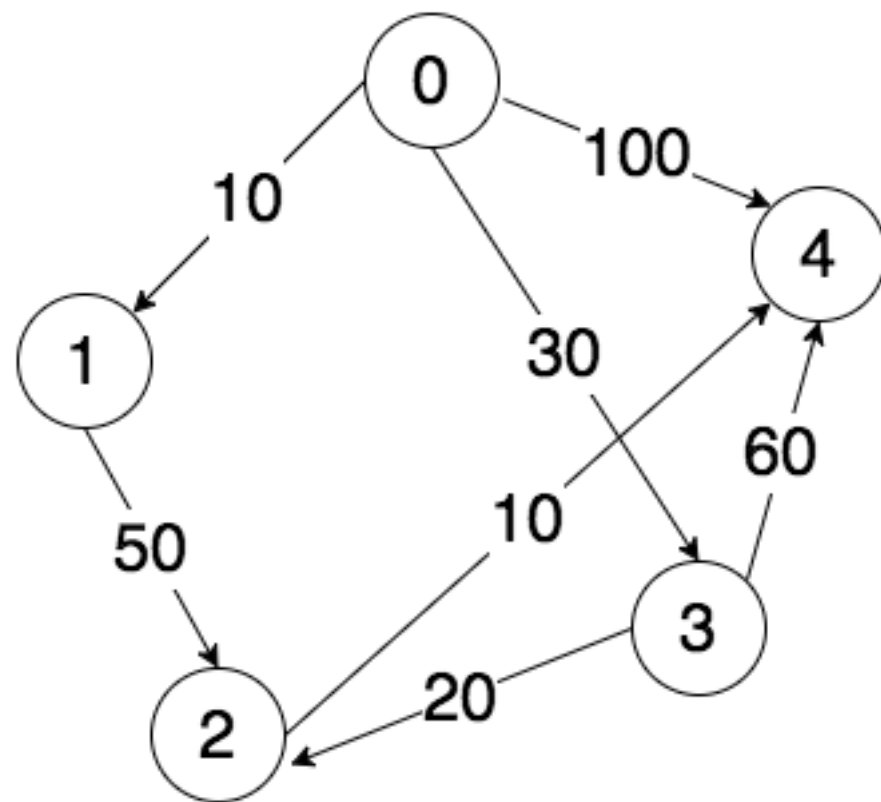
$S = \{0, 1, 3\}$

$V-S = \{2, 4\}$

v	d[v]	p[v]
1	10	0
2	50	3
3	30	0
4	90	3



STEP 4



$S = \{0, 1, 3, 2\}$

$V-S = \{4\}$

v	d[v]	p[v]
1	10	0
2	50	3
3	30	0
4	60	2

```

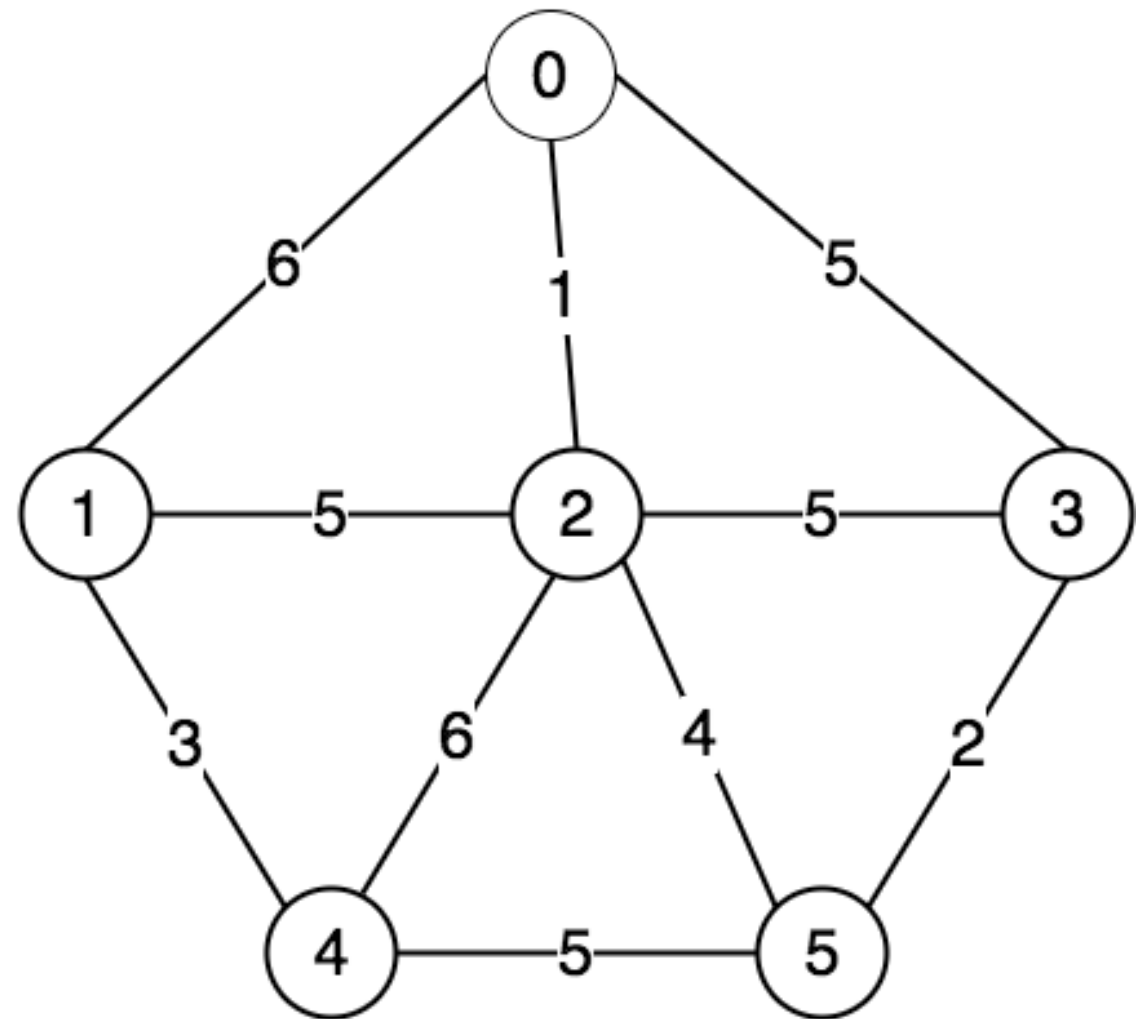
    /**
     * Dijkstra's Shortest Path algorithm
     * pre: graph to be searched is a weighted directed graph with only positive weights
     *     pred and dist are arrays of size V
     * @param graph The weighted graph to be searched
     * @param start The start vertex
     * @param pred Output array to contain the predecessors in the shortest path
     * @param dist Output array to contain the distance in the shortest path
     */
    public static void dijkstrasAlgorithm(Graph graph,
                                         int start,
                                         int[] pred,
                                         double[] dist){

        int numV = graph.getNumV();
        HashSet<Integer> vMinusS = new HashSet<>(numV);
        //Initialize V - S
        for(int i = 0; i < numV; i++){
            if(i != start)
                vMinusS.add(i);
        }
        // Initialize pred and dist
        for(int v : vMinusS){
            pred[v] = start;
            dist[v] = graph.getEdge(start, v).getWeight();
        }
        //Main loop
        while(vMinusS.size() != 0){
            //Find the value u in V - S with the smallest dist[u]
            double minDist = Double.POSITIVE_INFINITY;
            int u = -1;
            for(int v : vMinusS){
                if(dist[v] < minDist){
                    minDist = dist[v];
                    u = v;
                }
            }
            // Remove u from vMinusS
            vMinusS.remove(u);
            //Update the distances
            Iterator<Edge> edgeIter = graph.edgeIterator(u);
            while(edgeIter.hasNext()){
                Edge edge = edgeIter.next();
                int v = edge.getDest();
                if(vMinusS.contains(new Integer(v))){
                    double weight = edge.getWeight();
                    if(dist[u] + weight < dist[v]){
                        dist[v] = dist[u] + weight;
                        pred[v] = u;
                    }
                }
            }
        }
    }
}

```


PRIM

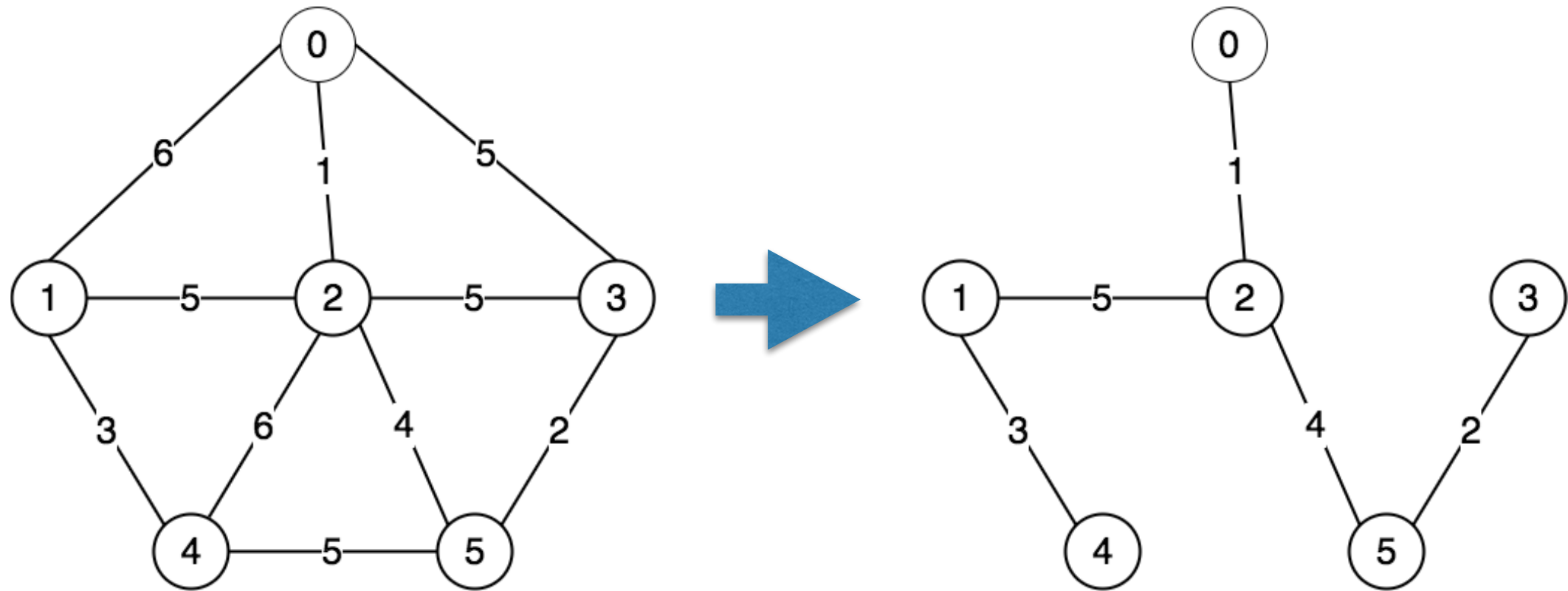
- Find minimum spanning tree in a graph
- Need two sets, S and V-S



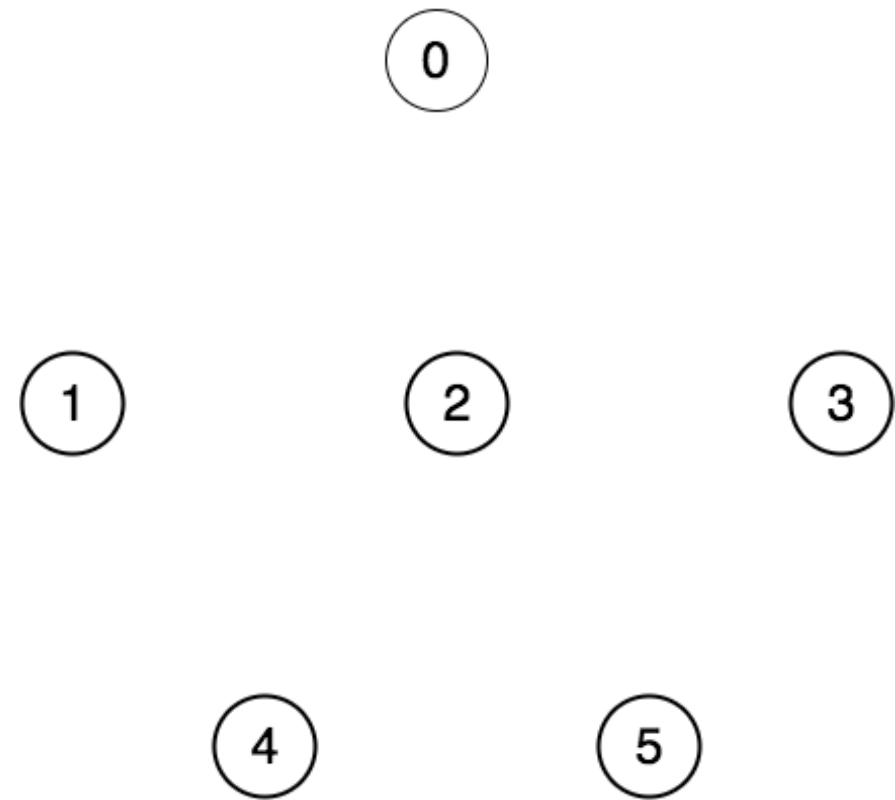
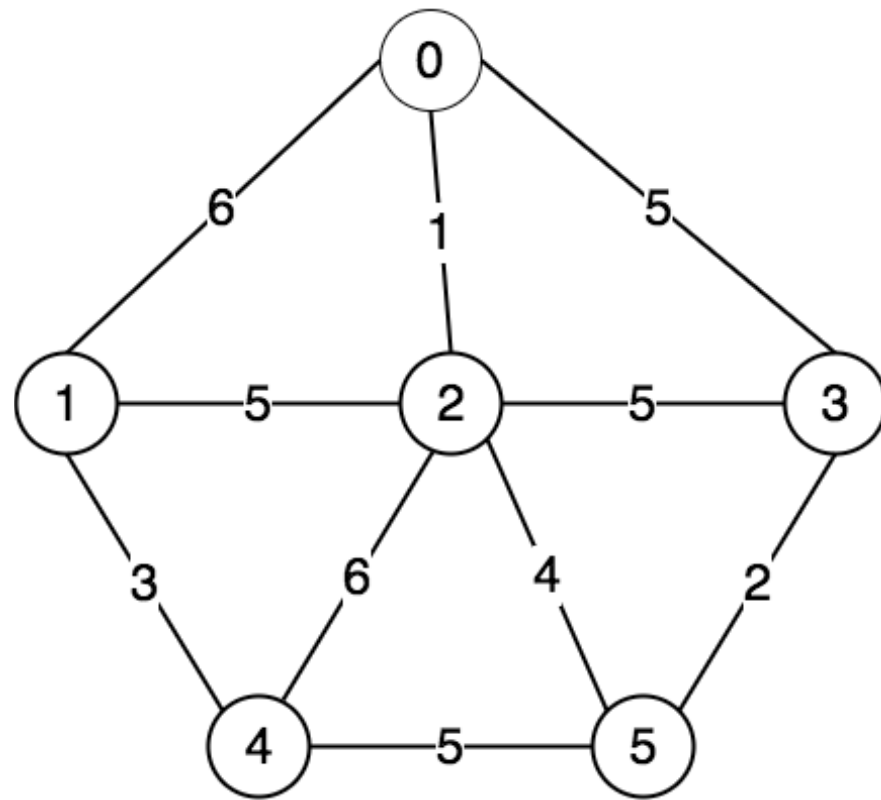
“For example, if we want to start up our own long-distance phone company and need to connect the cities shown in the given figure, finding minimum spanning tree would allow us to build the cheapest network.”

–Elliot & Koffman (the book I stole this example :)

We want this in another term



STEP 1



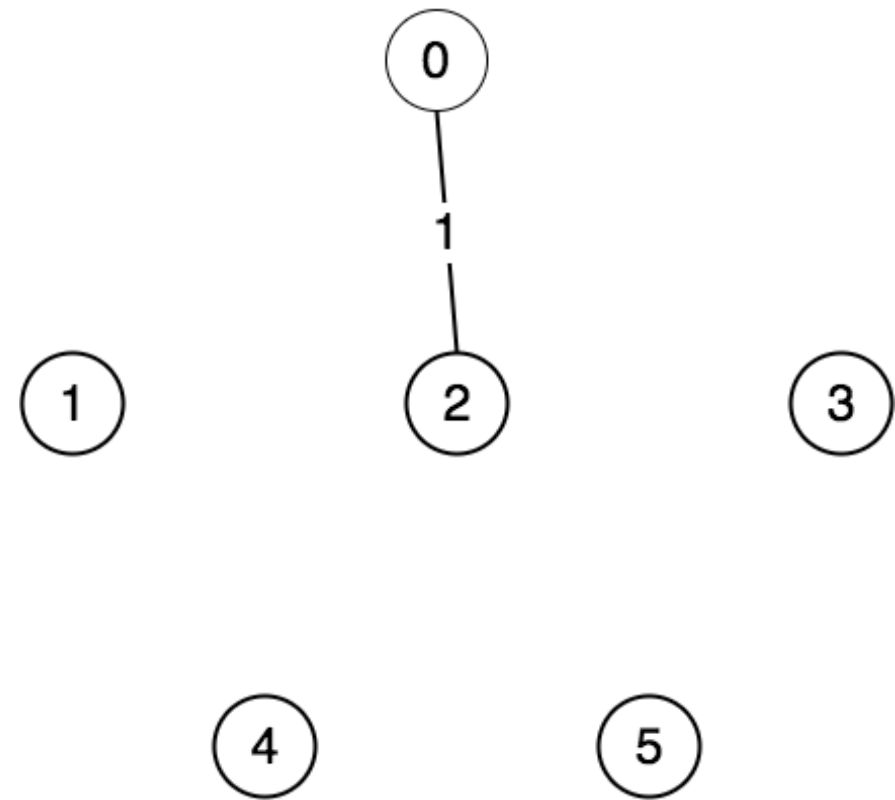
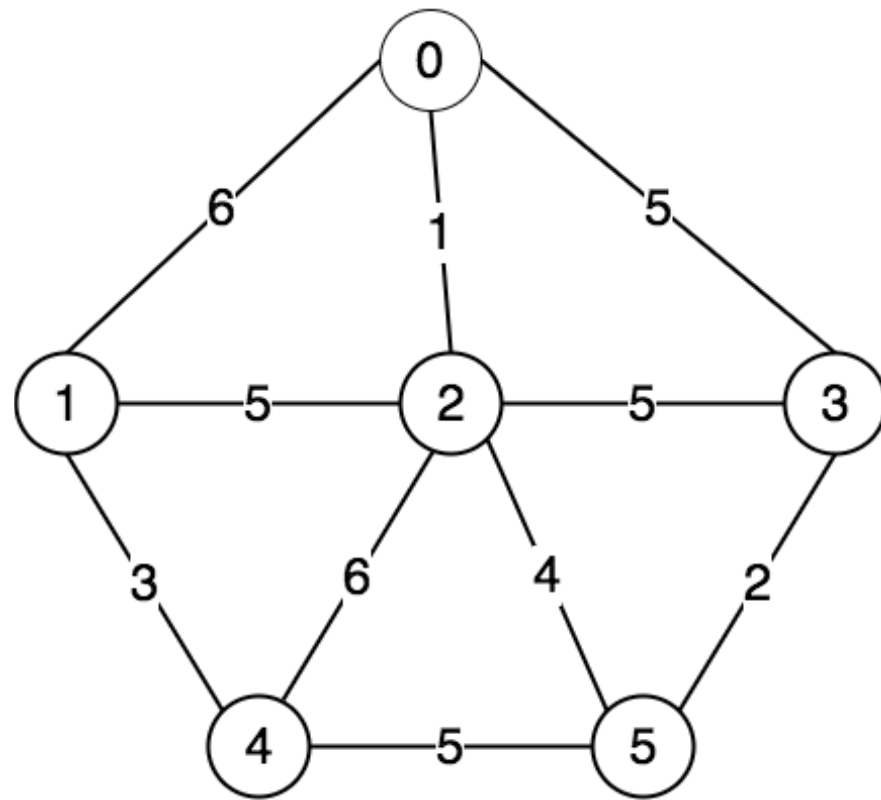
$$S = \{0\}$$

$$V-S = \{1,2,3,4,5\}$$

choose smallest u to v , u from S , v from $V-S$

smallest choice is $(0,2)$

STEP 2



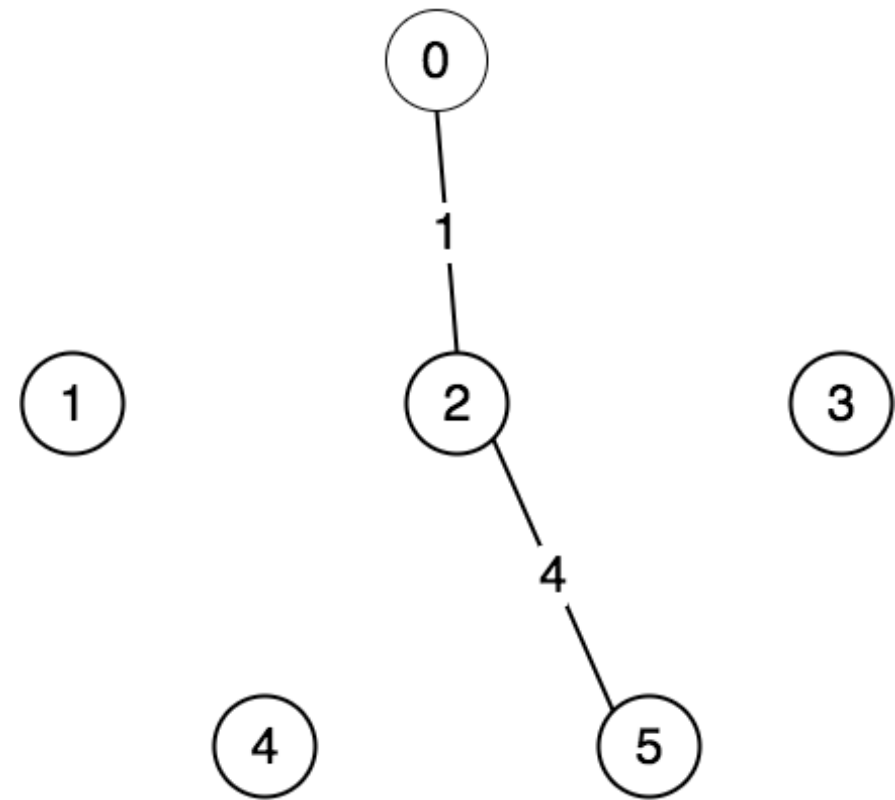
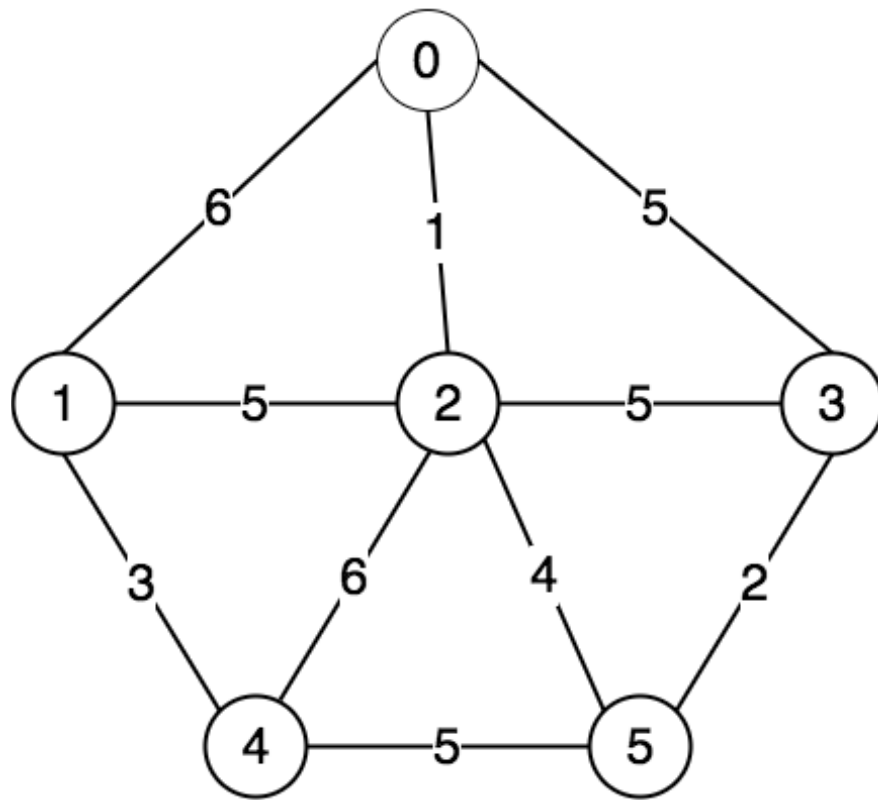
$$S = \{0, 2\}$$

$$V-S = \{1, 3, 4, 5\}$$

choose smallest u to v , u from S , v from $V-S$

smallest choice is $(2, 5)$

STEP 3



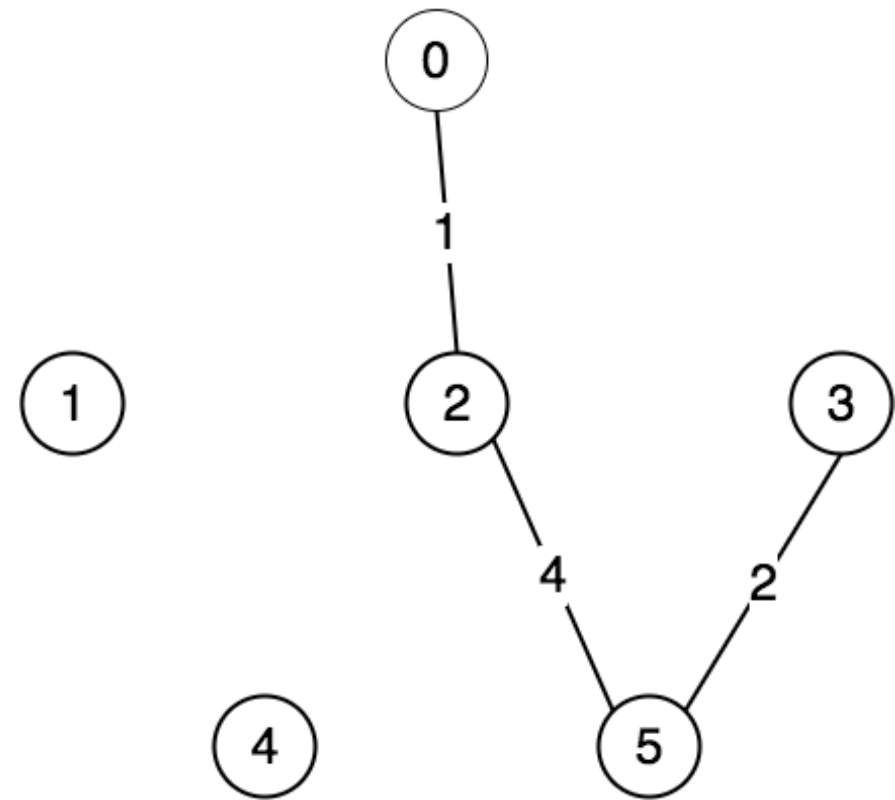
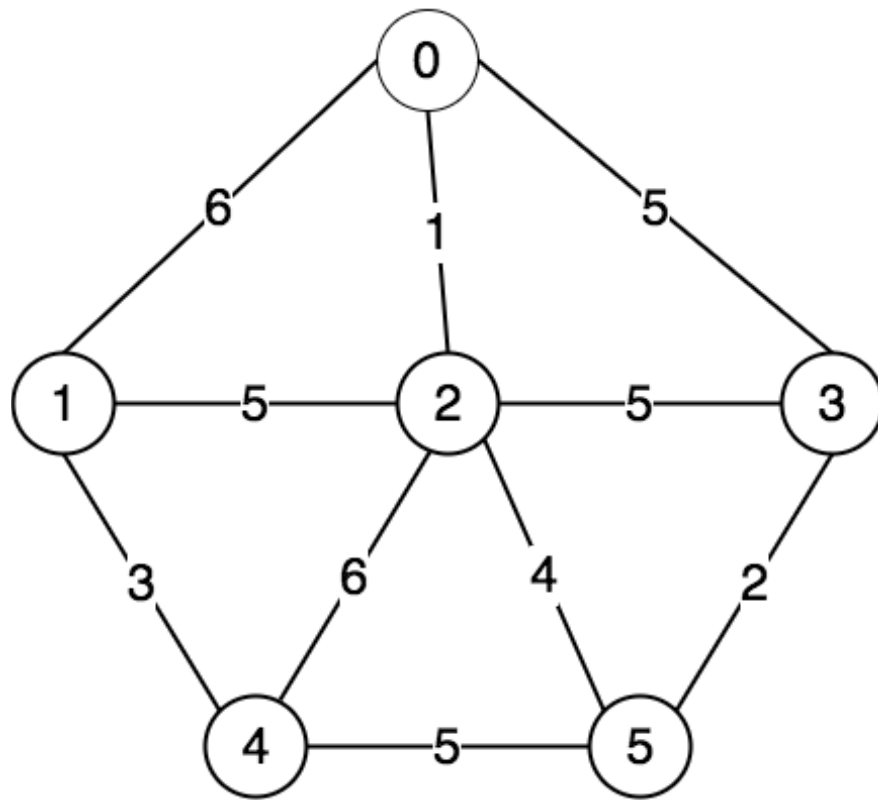
$$S = \{0, 2, 5\}$$

$$V-S = \{1, 3, 4\}$$

choose smallest u to v , u from S , v from $V-S$

smallest choice is $(5, 3)$

STEP 4



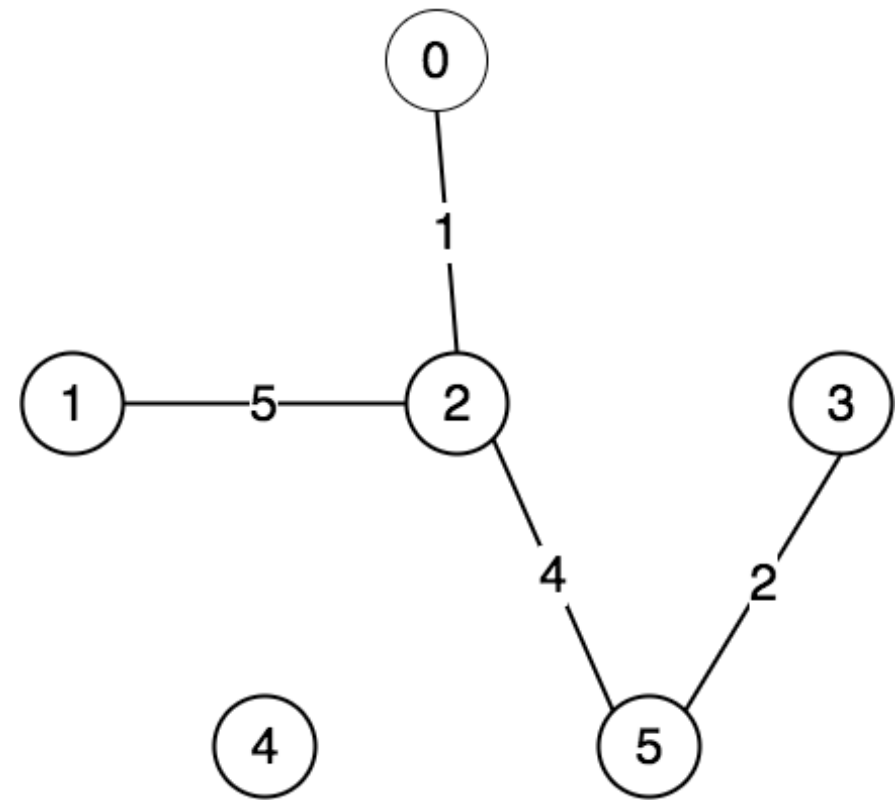
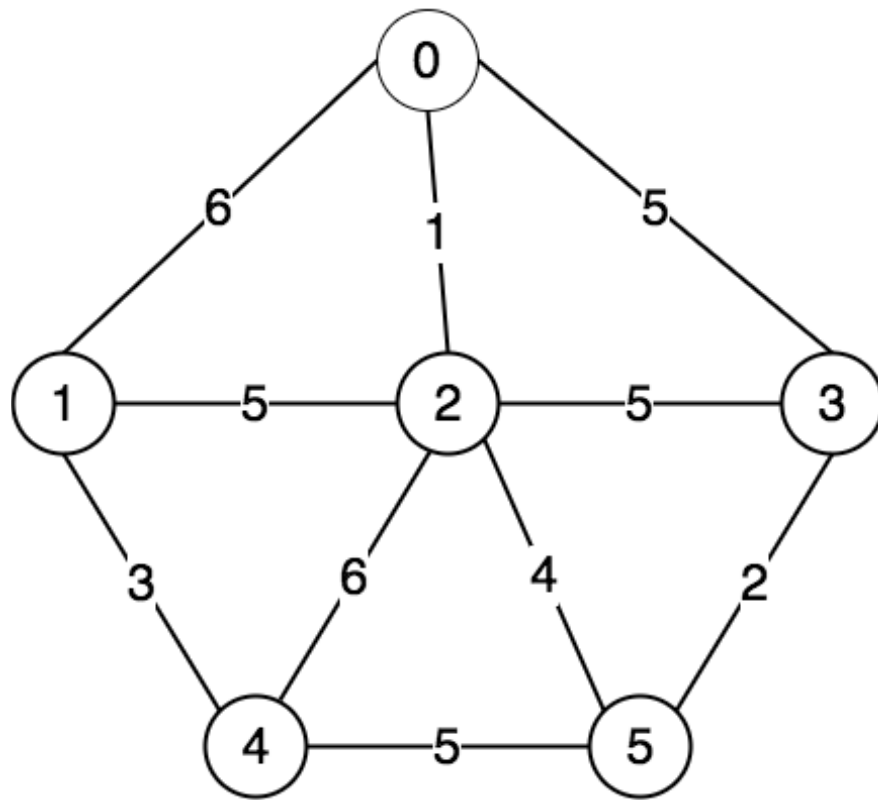
$S = \{0, 2, 3, 5\}$

$V-S = \{1, 4\}$

choose smallest u to v , u from S , v from $V-S$

smallest choice is $(2, 1)$

STEP 5



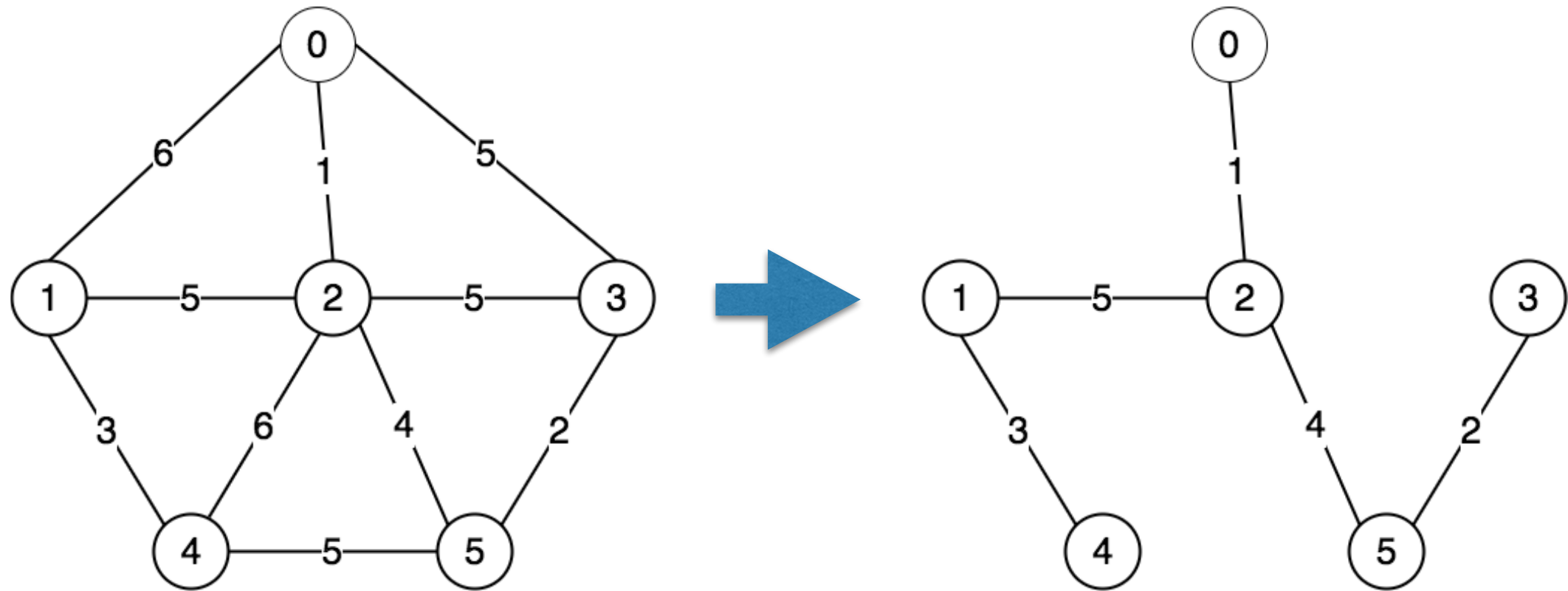
$S = \{0, 1, 2, 3, 5\}$

$V-S = \{4\}$

choose smallest u to v , u from S , v from $V-S$

smallest choice is $(1, 4)$

STEP 6



```

/**
 * Prim's Minimum Spanning Tree
 * @param graph
 * @param start
 * @return
 */
public static void primsAlgorithm(Graph graph,
                                  int start,
                                  int[] pred,
                                  double[] dist){

    int numV = graph.getNumV();
    HashSet<Integer> vMinusS = new HashSet<>(numV);
    //Initialize V - S
    for(int i = 0; i < numV; i++){
        if(i != start)
            vMinusS.add(i);
    }
    // Initialize pred and dist
    for(int v : vMinusS){
        pred[v] = start;
        dist[v] = graph.getEdge(start, v).getWeight();
    }
    //Main loop
    while(vMinusS.size() != 0){
        //Find the value u in V - S with the smallest dist[u]
        double minDist = Double.POSITIVE_INFINITY;
        int u = -1;
        for(int v : vMinusS){
            if(dist[v] < minDist){
                minDist = dist[v];
                u = v;
            }
        }
        // Remove u from vMinusS
        vMinusS.remove(u);
        //Update the distances
        Iterator<Edge> edgeIter = graph.edgeIterator(u);
        while(edgeIter.hasNext()){
            Edge edge = edgeIter.next();
            int v = edge.getDest();
            if(vMinusS.contains(new Integer(v))){
                double weight = edge.getWeight();
                if(weight < dist[v]){
                    dist[v] = weight;
                    pred[v] = u;
                }
            }
        }
    }
}

```

[illegible]

References

- https://en.wikipedia.org/wiki/Dijkstras_algorithm
- https://en.wikipedia.org/wiki/Prim%27s_algorithm
- <https://github.com/jimlay14/Data-Structures>
- <http://www.vogella.com/tutorials/JavaAlgorithmsDijkstra/article.html>
- Koffman & Wolfgang; Objects, Abstraction, Data Structures and Design using JAVA [book]