https://en.wikipedia.org/wiki/Otsu%27s_method
https://www.youtube.com/watch?v=ojapO75FV38

Thresholding

Let us consider an image I of dimension (x, y).

- Compute histogram and probabilities of each intensity level

$\rightarrow$ $Let\ \psi\ the\ pixel\ (the\ index\ in\ the\ matrix\ representing\ the\ image),$
$and\ i(\psi)\ its\ corresponding\ intensity, i \in [\![0, 256[\![$
$(with\ i(\psi)\ the\ value\ at\ the\ index\ \psi\ in\ the\ said\ matrix)$

The probability P of finding a pixel intensity $i(\psi)$ is

$$P(i(\psi)) = \frac{ni(\psi)}{xy}, with\ n\ the\ number\ of\ occurences\ of\ i(\psi)$$

- Global mean and global variance

$\mu_G = \sum_{k=0}^{xy-1} i\,(k).P(i(k))$

$\sigma_G^2 = \sum_{k=0}^{xy-1} (i_k - m_G)^2 P(i_k)$

- Let us suppose we select a threshold T, with a value of t.

Let $C_0 = \{\psi \mid \psi \leq t\}$ and $C_1 = \{\psi \mid \psi > t\}$ the two classes of pixels.
The probabilities $\omega_0(t)\ and\ \omega_1(t)\ are\ such\ that$:
$\omega_0(t) = \sum_{k=0}^{t} P(i_k)$

$\omega_1(t) = \sum_{k=t+1}^{xy-1} P(i_k) = 1 - \omega_0(t)$

The class (conditional) means are:

$\mu_0(t) = \frac{\sum_{k=0}^{t} i_k P(i_k)}{\omega_0(t)}$

$\mu_1(t) = \frac{\sum_{k=t+1}^{xy-1} i_k P(i_k)}{\omega_1(t)}$

$\mu_T = \sum_{k=0}^{xy-1} i_k P(i_k)$

- Otsu's criteria: maximize the between-class variance σ_B^2

$\sigma_B^2 = \omega_0(t)(\mu_0(t) - \mu_T)^2 + \omega_1(t)(\mu_1(t) - \mu_T)^2 = \omega_0(t)\omega_1(t)[\mu_0(t) - \mu_1(t)]^2$

Remark: To verify the expressions:
$\begin{cases} \omega_0\mu_0 + \omega_1\mu_1 = \mu_T \\ \omega_0 + \omega_1 = 1 \end{cases}$

- Process:

- Go through $t \in \left[1, \max\left(i(\psi)\right)\right]$
- Compute σ_B^2, and get the maximum $maximum_V$ (through each iteration, compare current variance with the previous one)
- The desired threshold is $maximum_V$