Canonical Forms

Logic Design

Canonical Forms

- It is useful to specify Boolean functions in a form that:
  - Allows comparison for equality.
  - Has a correspondence to the truth tables
- Canonical Forms in common usage:
- **Sum of Minterms (SOM)**
- **Product of Maxterms (POM)**

Minterms

- Minterms are **AND** terms with every variable present in either true or complemented form.
- There are $\mathbf{2^n}$ minterms for n variables.
- Example: Two variables (X and Y) produce 2 x 2 = 4 combinations:

    XY (both normal)

    XY'(X normal, Y complemented)

    X'Y (X complemented, Y normal)

    X'Y' (both complemented)

- Thus there are four minterms of two variables.

Maxterms

- Maxterms are **OR** terms with every variable in true or complemented form.
- There are $\mathbf{2^n}$ maxterms for n variables.
- Example: Two variables (X and Y) produce 2 x 2 = 4 combinations:

  X+Y (both normal)

  X+Y’ (X normal, Y complemented)

  X’+Y (X complemented, Y normal)

  X’+Y’ (both complemented)

Maxterms and Minterms

- Examples: Two variable minterms and maxterms.

Index	Minterm	Maxterm
0	$\overline{x}\,\overline{y}$	$x + y$
1	$\overline{x}\,y$	$x + \overline{y}$
2	$x\,\overline{y}$	$\overline{x} + y$
3	$x\,y$	$\overline{x} + \overline{y}$

- The index above is important for describing which variables in the terms are true and which are complemented.

Purpose of the Index

- The index is used to determine whether the variable is shown in the true form or complemented form.
- For **Minterms**:
  - "1" means the variable is "Not Complemented" and
  - "0" means the variable is "Complemented".
- For **Maxterms**:
  - "0" means the variable is "Not Complemented" and
  - "1" means the variable is "Complemented".

Index Example

- Example: (for three variables)
  - Assume the variables are called X, Y, and Z.
  - The standard order is X, then Y, then Z.
  - **Minterm** 0, called m_0 is X'Y'Z'
  - **Maxterm** 0, called M_0 is (X + Y + Z).
  - Minterm 6 ?
  - Maxterm 7 ?
- Example: (for four variables)
  - Assume the variables are called A,B,C and D
  - **Minterm** 0, called m_0 is A'B'C'D'
  - **Maxterm** 0, called M_0 is (A+B+C+D)
  - Minterm 7 ?
  - Maxterm 15?

DeMorgan's Theorem

#	Theorem	Dual	Name
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \ldots} =$ $\overline{B_0} + \overline{B_1} + \overline{B_2} \ldots$	$\overline{B_0 + B_1 + B_2 \ldots} =$ $\overline{B_0} \bullet \overline{B_1} \bullet \overline{B_2} \ldots$	DeMorgan's Theorem

The complement of the product
is the
sum of the complements.

Dual: The **complement** of the **sum**
is the
product of the **complements**.

DeMorgan's Theorem Example 1

$Y = \overline{(A+\overline{BD})\overline{C}}$

DeMorgan’s Theorem Example 2

$Y = \overline{(\overline{AC}E+\overline{D}) + B}$

Minterm and Maxterm Relationship

- Review: DeMorgans Theorem

$$(XY)' = X'+Y'$$

$$(X+Y)' = X'Y'$$

- Two–variable example:

$$M_2 = X'+Y \quad \Leftrightarrow \quad m_2 = X.Y'$$

$$\boxed{\mathbf{M_i = \overline{m_i} \qquad m_i = \overline{M_i}}}$$

Thus M_i is the complement of m_i

Canonical Sum of Minterms

- Any Boolean function can be expressed as a **sum of minterms (SOM)**
  - The minterms used are the terms corresponding to the 1's
- Example: Implement $F = X+X'Y'$ as a sum of minterms.
  - First expand terms:

$$F = X(Y+Y') + X'Y' = XY + XY' + X'Y'$$

  - Express as SOM:

$$F= m_3+m_2+m_0$$

$$F(X,Y) = \Sigma_m(0,2,3)$$

SOM Example

- Show that F = A+B'C => $F(A,B,C) = \Sigma_m(1,4,5,6,7)$

Canonical Product of Maxterms

- Any Boolean Function can be expressed as a **Product of Maxterms (POM)**.
  - For the function table, the maxterms used are the terms corresponding to the 0's.

- Example: F(X,Y,Z)=X+X'Y'

$$X+X'Y' = (X+X')(X+Y') = X+Y' = X+Y'+Z.Z' = (X+Y'+Z')(X+Y'+Z)$$

$$= M_2.M_3$$

POM Example

- Convert the following to product of maxterms:

$$F(A,B,C)= AC' + BC + A'B'$$

Function Complements

- Select the minterms **missing** in the sum-of-minterms canonical forms.

$$F(X,Y,Z)=\Sigma_m(1,3,5,7)$$

$$F'(X,Y,Z)=\Sigma_m(0,2,4,6)$$

- Alternatively, the complement of the function is the product of maxterms with the **same indices**.

$$F'(X,Y,Z)=\Pi_M(1,3,5,7)$$

From Logic to Gates

Two-level logic: ANDs followed by ORs

Example: $Y = \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$