Question 2

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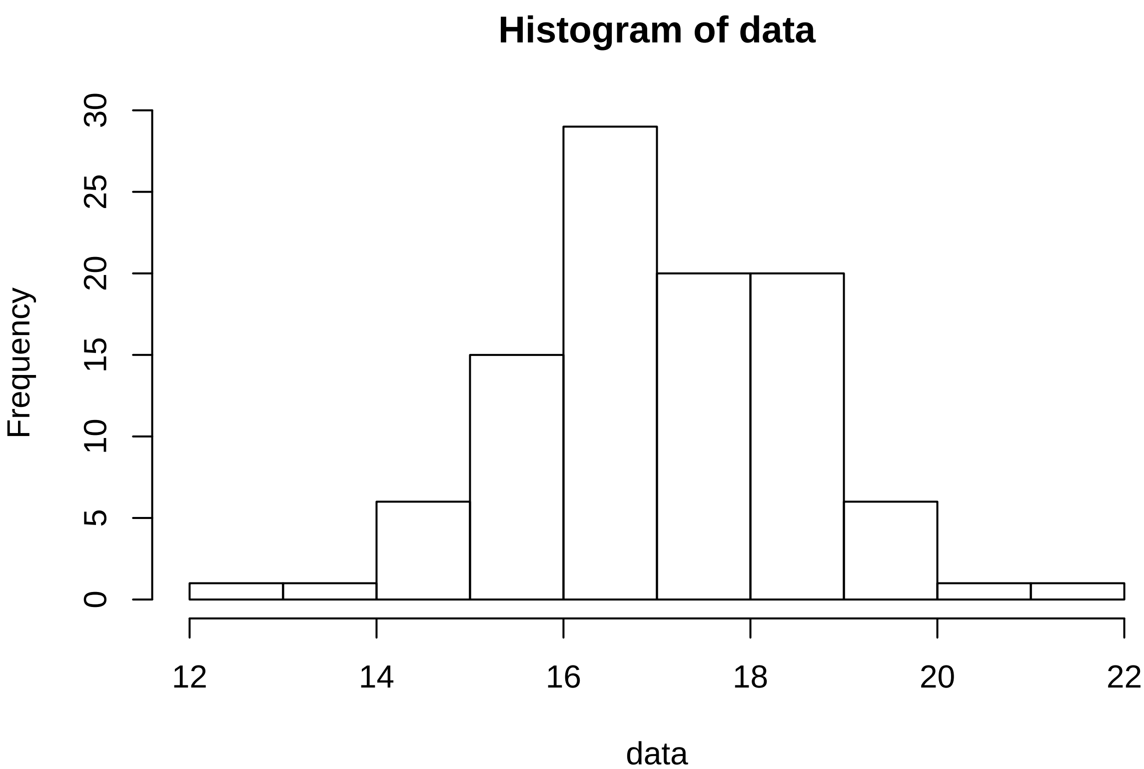
a)

let's first import data and take a look at histogram:




```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   12.70   16.26   16.95   17.09   18.19   21.81
```

now lets define the log-likelihood function and first and second derivatives(for $\gamma = 0$ constatn) :

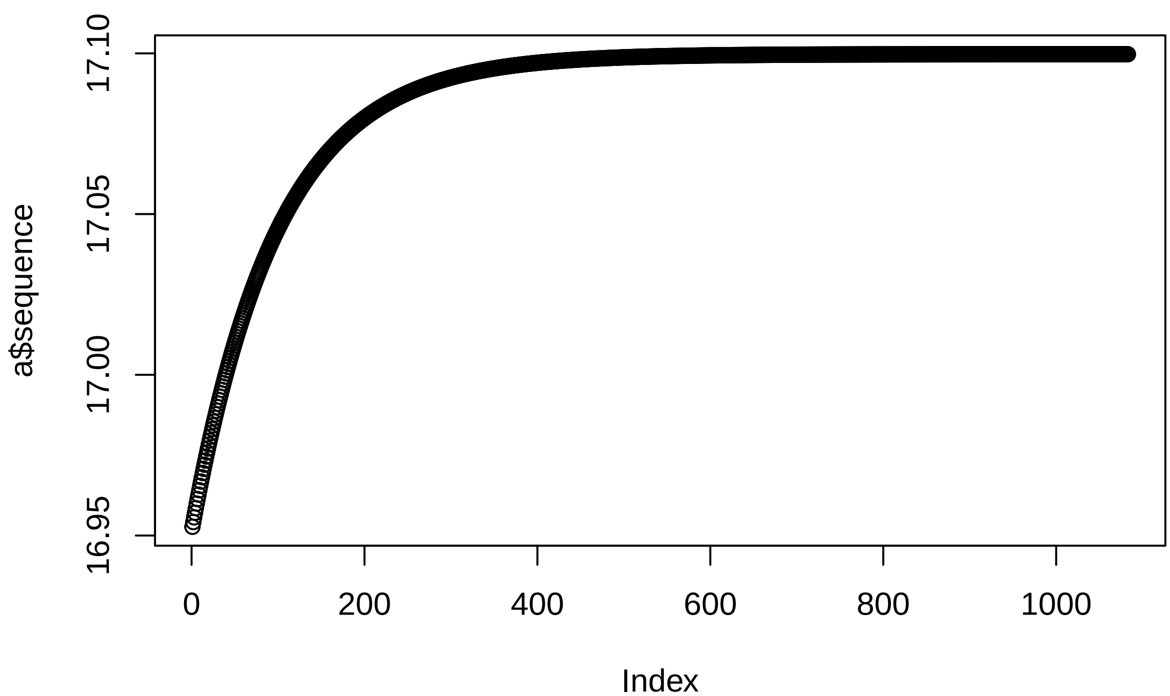
```
x = data
n = length(x)
density = function(x, mu){
  f = exp(-x+mu)/((1+exp(-x+mu))^2)
  return(f)}

loglik = function( x, mu,n){
  ll = mu*n - sum(x) - 2*sum(log(1+exp(-x+mu)))
  return(ll)}
```

now begin newton-raphson algorithm :

```
x = data
mle_nr=function(xvec,stop_crit,sRate){
  startvalue=median(xvec);
```

```r
  n=length(xvec);
  mu_curr=startvalue;
  nn=0
  mu_seq = NULL
  ####compute first derivative of log-likelihood #####
  first_derivll=n-2*sum(exp(-xvec+mu_curr)/(1+exp(-xvec+mu_curr)));
  ### Continue algorithm until the first derivative ###
  ### of the log-likelihood is within stop criterion ##
  while(abs(first_derivll)>stop_crit){
    ####compute second derivative of log-likelihood #####
    second_derivll=-2*sum(exp(-xvec+mu_curr)/(1+exp(-xvec+mu_curr))^2);
    #### Newton-Raphson's  update of estimate of mu ####
    mu_new=mu_curr-sRate *(first_derivll/second_derivll);
    mu_seq = c(mu_seq, mu_new);
    mu_curr=mu_new;
    ####compute first derivative of log-likelihood #####
    first_derivll=n-2*sum(exp(-xvec+mu_curr)/(1+exp(-xvec+mu_curr)));
    nn=nn+1
  }
  return (list(thetahat=mu_curr, iterations =nn ,sequence = mu_seq))
}
```

```r
## starting value = meadian(x)  , step size= 0.01 ,stop criterion =0.0001
a= mle_nr( x, 0.0001, 0.01)
plot(a$sequence)
```



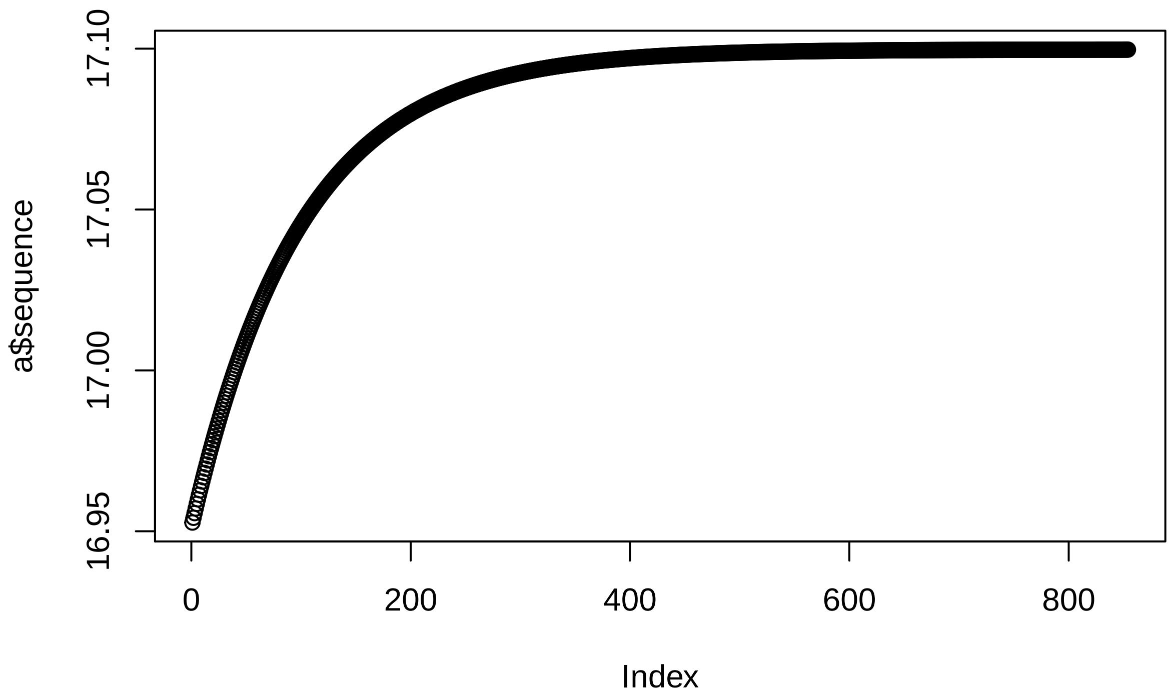

```r
a$iterations #number of iterations
```

```
## [1] 1083
```

```r
a$thetahat
```

```
## [1] 17.09973
```

```
## starting value = meadian(x) , step size= 0.01 ,stop criterion =0.001
a= mle_nr( x, 0.001, 0.01)
plot(a$sequence)
```



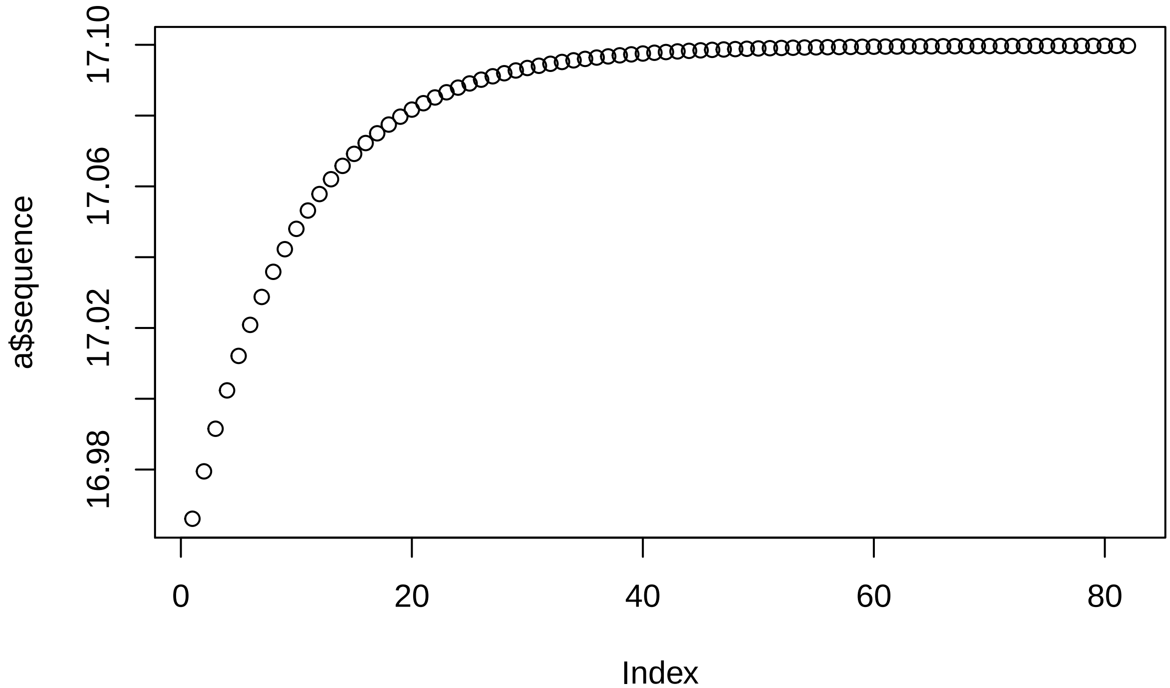

```
a$iterations #number of iterations
```

```
## [1] 854
```

```
a$thetahat
```

```
## [1] 17.0997
```

```
## starting value = meadian(x) , step size= 0.1 ,stop criterion =0.001
a= mle_nr( x, 0.001, 0.1)
plot(a$sequence)
```



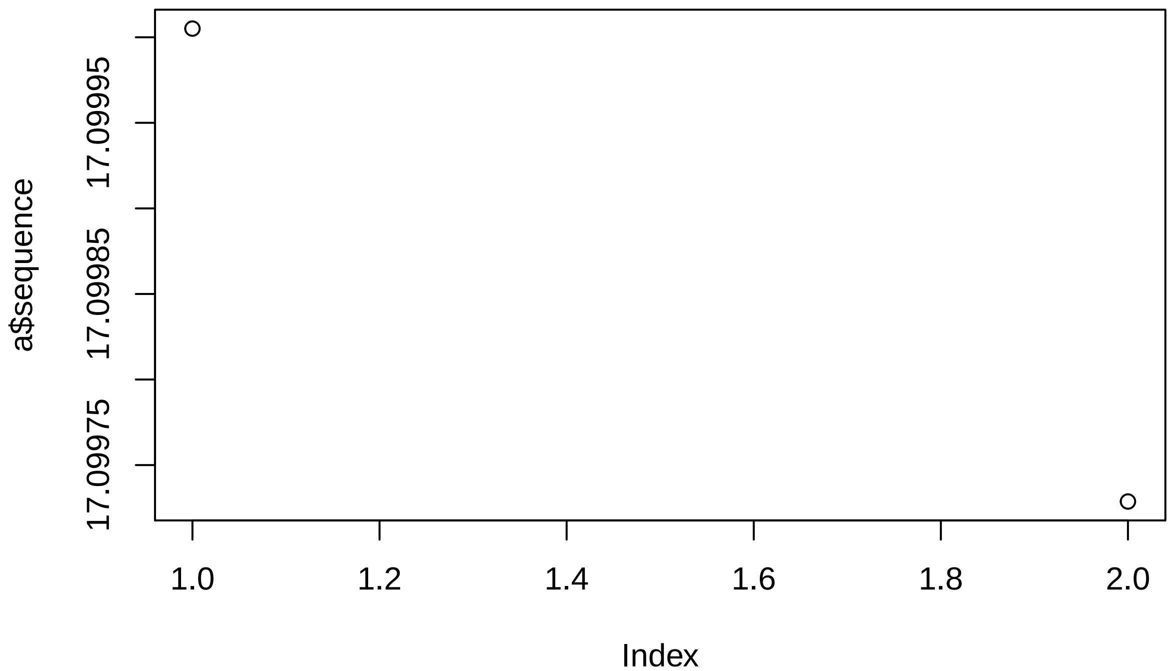

```
a$iterations #number of iterations
```

```
## [1] 82
```

```
a$thetahat
```

```
## [1] 17.0997
```

```
## starting value = meadian(x) , step size= 1 ,stop criterion =0.001
a= mle_nr( x, 0.001, 1)
plot(a$sequence)
```



```r
a$iterations #number of iterations
```

```
## [1] 2
```

```r
a$thetahat
```

```
## [1] 17.09973
```

for step size of larger than 1 algorithm diverges,

iii)

using the observed fisher information as an estimate for expected fisher information , and pluging in the estimated μ:

```r
#second_derivll=-2*sum(exp(-xvec+mu_curr)/(1+exp(-xvec+mu_curr))^2)
second_derivll=-2*sum(exp(-x+17.09973)/(1+exp(-x+17.09973))^2)
```

hence the 95% confidence interval will be :

```r
lb =17.09973 - qnorm(0.975)*1/ sqrt(-1*second_derivll)
ub=17.09973 + qnorm(0.975)*1/ sqrt(-1*second_derivll)
c(lb,ub)
```

```
## [1] 16.77221 17.42725
```