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Faculty of Engineering and Architectural Science

Program: Mechanical Engineering

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REPORT:	**Project 1: Figure "8" Mechanism**

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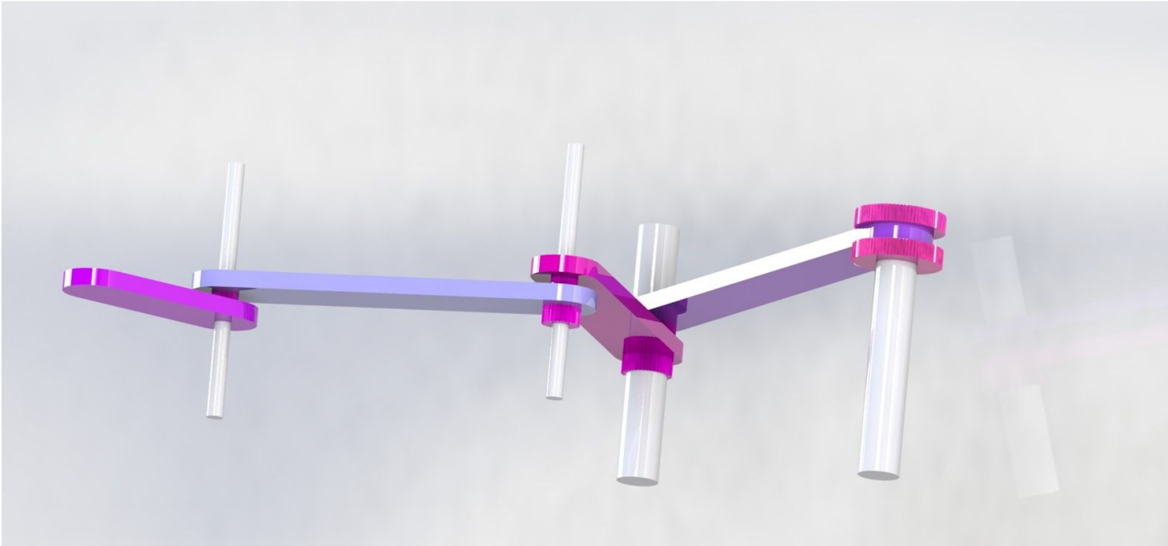
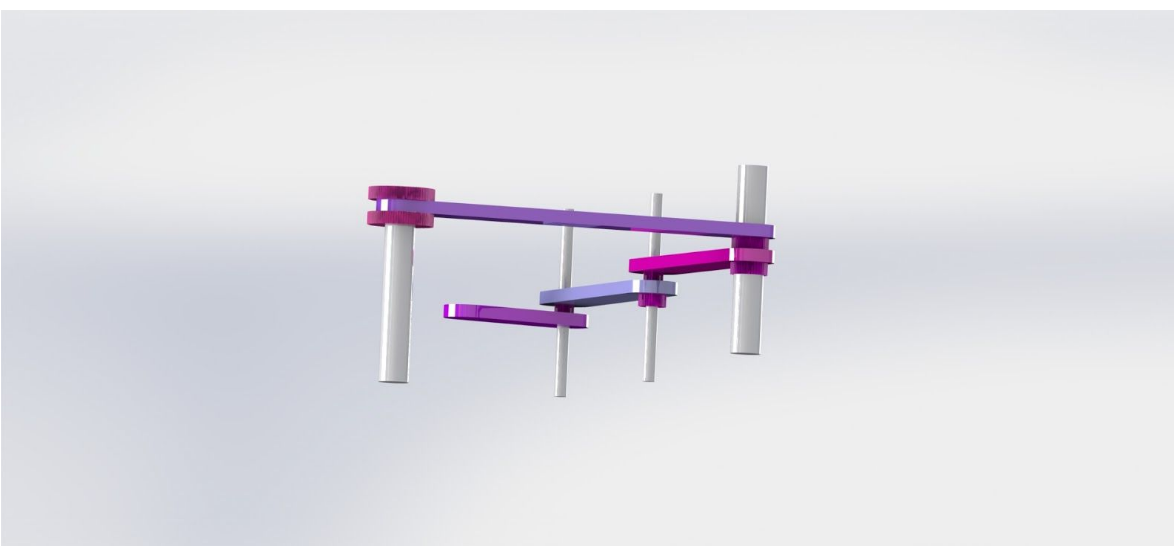


4 BAR SINGLE COUPLED SERIAL CHAIN MECHANISM DRAWING “FIGURE 8”



Executive Summary

The purpose of the project summarized in the following report was to design and test a planar mechanism, a point on which describes a Lemniscate of Bernoulli (figure eight) when the input link is rotated through 360 degrees. Multiple design methods were considered and attempted by the design team, including mathematical, and graphical mechanism synthesis procedures, and references to past designs. The team settled on a mathematical approach to the design, as such a method provided an adequate description of the desired curve without sacrificing the simplicity of the mechanism. Synthesis of Fourier series to describe the desired motion of the mechanism yielded a four-term series. While the stroke of the coupler point of the chosen mechanism is very similar to the ideal Lemniscate of Bernoulli, it is not identical, and thus a vector-based error analysis was conducted, identifying an error of 70.5 percent. Further experimentation with novel design procedures, including physical prototyping and use of CAD/CAM software can be pursued to better approximate the desired coupler path in further experimentation.

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1. Introduction

1.1 Background Info

In 1876 , Alfred B. Kempe came up with a proof known as the “Kempe ‘s University Theorem” that demonstrates a method to build a planar mechanism with all the dimensions and configurations based on the algebraic equation of a desired closed curve which the mechanism is going to draw by its coupler point by a crank input [1].Kempe ‘s theorem was proven to be remarkably effective as it generates the close-curves by very limited number of links and the mechanism is interestingly intuitive to work with compared to other similar methodologies such as “Kobel” ‘s and “Artobolevskii” ‘s algorithm that constructs too many bars to count[1].This comparison between the two methods put forward above is shown in the following figure .

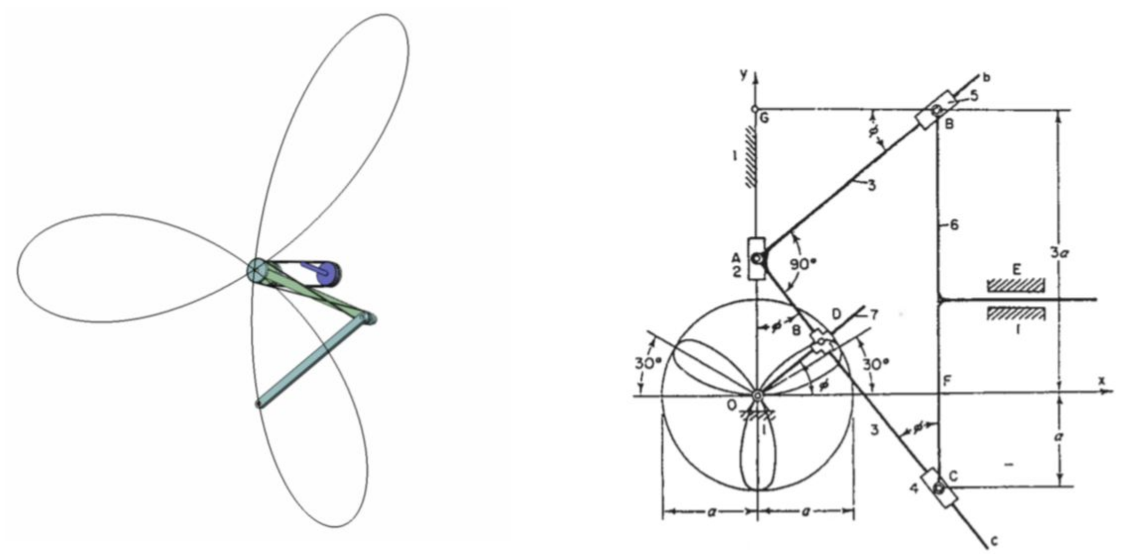

Figure 1.The Trifolium curve drawn according to Kempe ‘s theorem vs Artobolevskii ‘s method

As shown in the figure above for the example of trifolium curve , based on Kempe ‘s theorem the mechanism synthesised is only a 2 bar single couple serial chain (same mechanism as the one used in our final design) .However , to draw the same curve using Artobolevskii ‘s method we need 8 bars and 10 joints known as the “conograph“ .Therefore , despite the fact that Kempe ‘s theorem to synthesise the mechanism is a complex process and requires series of calculations , the ultimate design is a practical one.Further ,in the Artobolevskii ‘s method to guide the coupler to trace the curve we need numerous accurate points (e.g. for 6-bar linkage 15 precision points required) which is beyond our computing capabilities[1] .

1.2 Purpose

The main goal of this project was to generate a complex closed-curve of figure 8 represented by polar equation (1) to be drawn by a point on the coupler of a planar mechanism :

$$r^2 = a^2 \cos 2\theta \sec^4\theta$$

(EQ1)

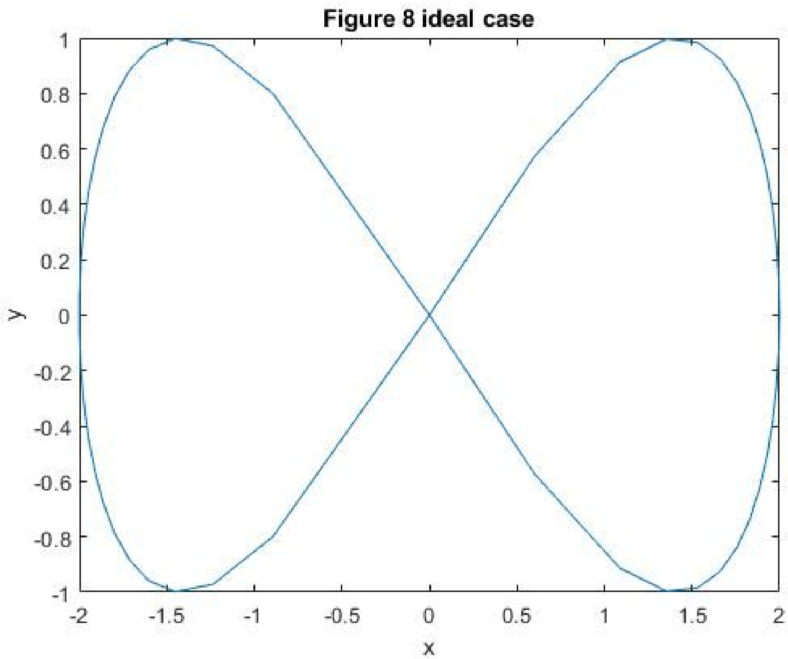

Figure 2 .Ideal “figure 8”

It is proven that any closed curve (more importantly trigonometric curves) can be accurately approximated by taking advantage of fourier series representation .This is done by utilizing fourier series expansion of X and Y components of the above mentioned polar equation on matlab which are both a function of crank position angle [2].

1.3 Requirements

In this project, it was desired to design a planar mechanism, one point on which would describe a lemniscate of Bernoulli (or as close an approximation of such as possible) as the input link completed a complete rotation of 360 degrees. Should the mechanism be unable to accurately describe the desired path, then an error analysis would be necessary to review and quantify the inaccuracy in path.

2. Calculations and Results

2.1 Method

Concept Design 1: **Loop closure equation**

A loop is a path that a point on the mechanism traces and at the end reaches the starting point again. In this method, vectors in a complex plane are used to represent the movement of each link and generate the loop. After writing the loop closure equation and deriving the equations for the real components and the imaginary components, it is possible to find the position, velocity or acceleration of any point with respect to the given values and inputs [3]. Using loop closure equation, the equation for the position of the coupler was found to be:

xC(i) = `r2*cos(theta21(i))+rCB*cos(theta31(i)+betaC) (EQ2)`
yC(i) =`r2*sin(theta21(i))+rCB*sin(theta31(i)+betaC) (EQ3)`

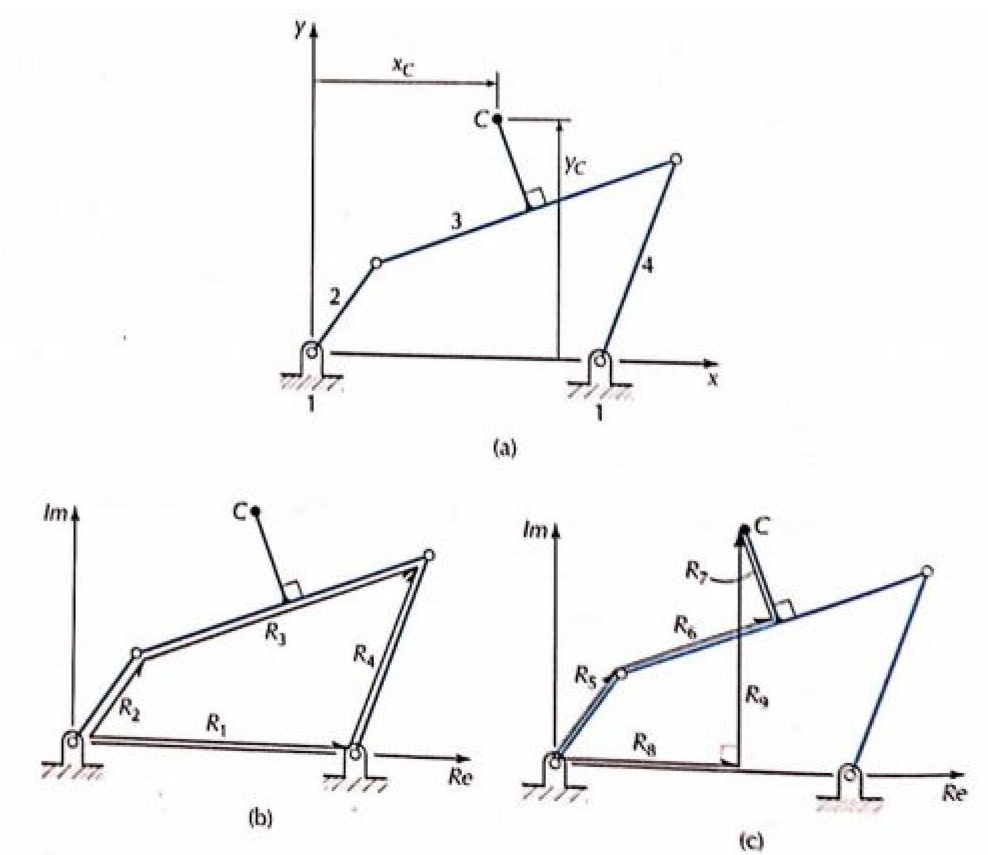

Figure3: representation the position of the coupler and the vector loops [3].

Also, the formula for achieving a typical “8 Figure” in a polar system was given as:

$r^2 = a^2 cos2\Theta sec^4\Theta$ (EQ4)
To convert it into cartesian system:
$x = rcos\Theta$ (EQ5)
$y = rsin\Theta$ (EQ6)

Using this concept, (EQ2) was set equal to (EQ5) and (EQ3) equal to (EQ6). As theta31 is dependent on other variables, we ended up with 2 equations and 6 unknowns. Reasonable values were assigned to four of the variables and r4 and betaC were kept as unknowns to have two equations two unknowns, but after writing a code in MATLAB to solve the equations, MATLAB failed to give any values for r4 or betaC.

Concept Design 2 : **Mechanism for drawing polyzonal type curves**

The following 10 bar mechanism can be utilized to draw the leminscate of Gerono which is very similar to the “figure 8” that our team was trying to achieve .To the links 1 to 4 which is a simple 4 link mechanism with link number 4 as the slider along link 3 is assembled link 5 which is joint with links 6 , 7 and 8.links 9 and 10 (cruciform slide-block) are attached to slider link 4 and 7[5]. Link 10 is a cruciform slide-block with mutually perpendicular slides which is the link that draws the leminscate of Gerono.The condition that should be taken into account for the mechanism to accurately draw the curve is :
OE=EC=FG=AB=OB=a/2 and EF=OG

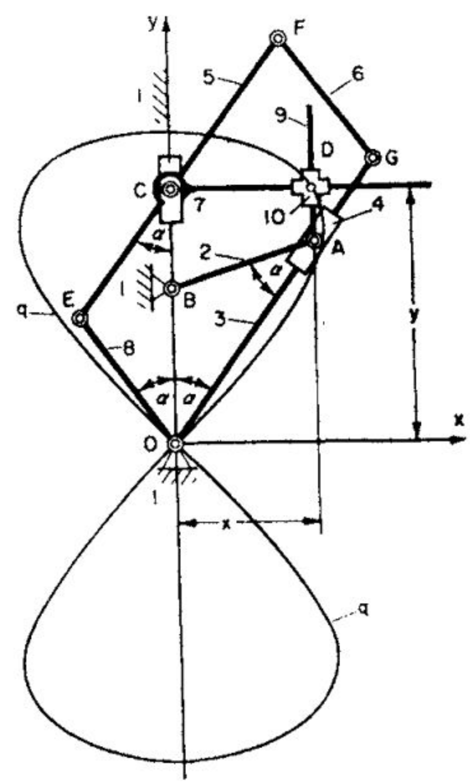

Figure4 . Drawing lemniscate of Gerono with the shown 10 bar mechanism[5]

The x and y component of point that draws the curve are as follows :

$$y = OE\cos\alpha + EC\cos\alpha$$

$$x = OA\sin\alpha = (OB\cos\alpha + AB\cos\alpha)\sin\alpha\,.$$

(EQ 7)

Solving for y :

$$y = \sqrt{\left(\frac{d^2}{4} + \frac{dx}{2}\right)} + \sqrt{\left(\frac{d^2}{4} - \frac{dx}{2}\right)}\ .$$

(EQ 8)

(d is an arbitrary parameter)

Analysis of the concept : Kempe ‘s theorem for drawing trigonometric curves

This concept design was investigated to see whether it satisfies the requirements stated in the previous sections .Despite the fact that the curve drawn by this mechanism (leminscate of Gerono) seems to be very similar to that of “figure 8” , there are 10 links necessary to assemble such mechanism and evaluating such linkage wouldn’t be reasonable for our design .

Final concept Design : 4 bar single coupled serial chain mechanism

In this method, first the x and y coordinate functions are given as follows:

$$P_T = \begin{Bmatrix} \rho\cos\theta \\ \rho\sin\theta \end{Bmatrix} = \begin{Bmatrix} x(\theta) \\ y(\theta) \end{Bmatrix} = \begin{Bmatrix} \sum_{k=0}^{m} a_k\cos k\theta + b_k\sin k\theta \\ \sum_{k=0}^{m} c_k\cos k\theta + d_k\sin k\theta \end{Bmatrix} \quad \text{(EQ9)}$$

Each x and y components are converted into fourier series. We tried 2 different series. One with 4 terms and the other with 8 terms to determine which one is more accurate.

The following procedures helps to find the dimension of the links:

Introduce the complex form of the curve as:

$$P(\theta) = x(\theta) + iy(\theta) = \sum_{k=0}^{m}(a_k\cos k\theta + b_k\sin k\theta) + \sum_{k=0}^{m} i(c_k\cos k\theta + d_k\sin k\theta) \quad \text{(EQ10)}$$

Magnitudes of Lk and Mk are obtained by:

$$L_k = \frac{1}{2}\sqrt{(a_k + d_k)^2 + (c_k - b_k)^2}. \quad \text{(EQ11)}$$

$$M_k = \frac{1}{2}\sqrt{(a_k - d_k)^2 + (c_k + b_k)^2}. \quad \text{(EQ12)}$$

The relative phase angles are given by:

$$\psi_k = \arctan\frac{c_k - b_k}{a_k + d_k}; \quad \text{(EQ13)}$$

$$\eta_k = \arctan\frac{c_k + b_k}{a_k - d_k} \quad \text{(EQ14)}$$

The mechanism consists of m links rotating clockwise and m links rotating counter-clockwise. Each counter-clockwise link followed by a clockwise link.

L_k is the length of the links rotating counter-clockwise with the speed frequency of k, and initial phase angle of ψ_k .

M_k is the length of the links rotating clockwise with the speed frequency of k, and initial phase angle of η_k .

Finally, calculating all the values mentioned above, the end point of the mechanism which is the coupler point will draw the desired pattern [4].

2.2 Link Dimensions

Table 2.2.1 - Manufacturing Specifications

Manufacturing specifications of the mechanism	
k	speed ratio
Bi	link number
Sj	link length
sigma	inital angle
TD,j	Gear size on link j
TG,j+1	Gear size on link j+1, attached to TD,j by belt

Table 2.2.2 - The single coupled serial chain Mechanism Link Lengths , Initial Crank position and Gear Ratios

	k	name	Bi	Sj	sigma	TD,j	TG,j+1
NA	0	NA	NA	NA	NA	NA	NA
1	1	L1	0.85	30.00	-0.14	80	40
2	-1	M1	0.56	19.96	0.25	40	20
3	3	L3	0.57	20.06	-0.21	30	20
4	-3	M3	0.26	9.08	1.39	21.00	18
5	4	L4	0.06	2.20	-1.03	32.00	28
6	-4	M4	0.08	2.84	0.52	15	20
7	2	L2	0.03	0.92	-0.71	20.00	30
8	-2	M2	0.05	1.92	0.21	10	20

Link ratios L4 , M4 , L2 , M2 can be disregarded as the ratios are negligible compared to the rest of the clockwise and counterclockwise links .It is concluded that even if the mentioned links are omitted from the design of our mechanism the motion path of the “figure 8” wouldn’t significantly change .

As you may have noticed some of the links are very short. For example links 4 and 2 are 30 times shorter than link one, therefore their contribution is rather limited. In the final design we will then utilize this fact and not manufacture those links.

2.3 Figures and Results

The end-point of the shortest link is what will draw the figure 8, several crank positions have been given below. All the figures are to scale. For dimensions refer to table 2.2.2.

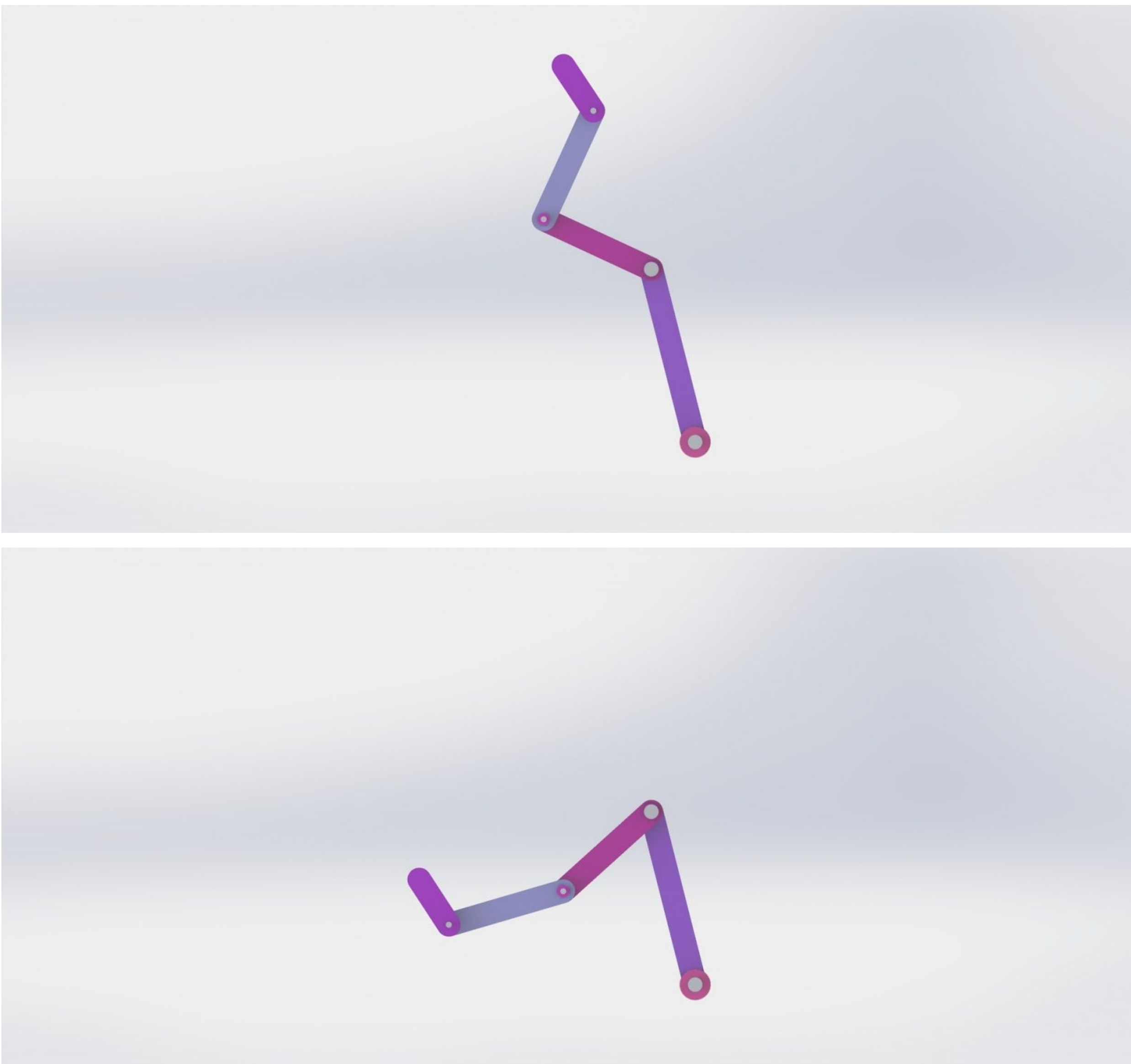

Figure 5. Shows 2 crank positions to illustrate the motion of the system.

3. Analysis and Conclusion

3.1 Error Analysis

For the error analysis we employ a measurement of distance from point to origin. A set of vectors at equal angles theta will calculate the distance from the origin. One vector is on the ideal graph, while the other is on the mechanism drawn graph. The ratio between the two will be our error.

This measurement seemed suitable as we only care about relative error between our model versus a more accurate fourier series model (with 8 terms), therefor as long as the error measurement is the same with both we should get the idea of what we are working with.

It should be noted that this method has a downfall; where our graph has a gap at the far right end, therefore we skip that section from the error calculation.

When using this method we find that the mechanism graph passes the origin several times and this results in an error value of inf or zero depending on what angle the calculation was made, for this reason we have also omitted all values of zero and Inf from the calculation results as well.

The code to find the error is done as seen in Appendix under error analysis. The program calculates the norm (we didn't use the norm function) of each graph. The program will then display the error.

Figure.6 & Figure.7 Show the 4 term fourier generation and the more accurate 8 term fourier generation respectively. Deciding between the two is rather difficult as both have their problems.

The first has a gap on the right side while the latter has a very non uniform shape.

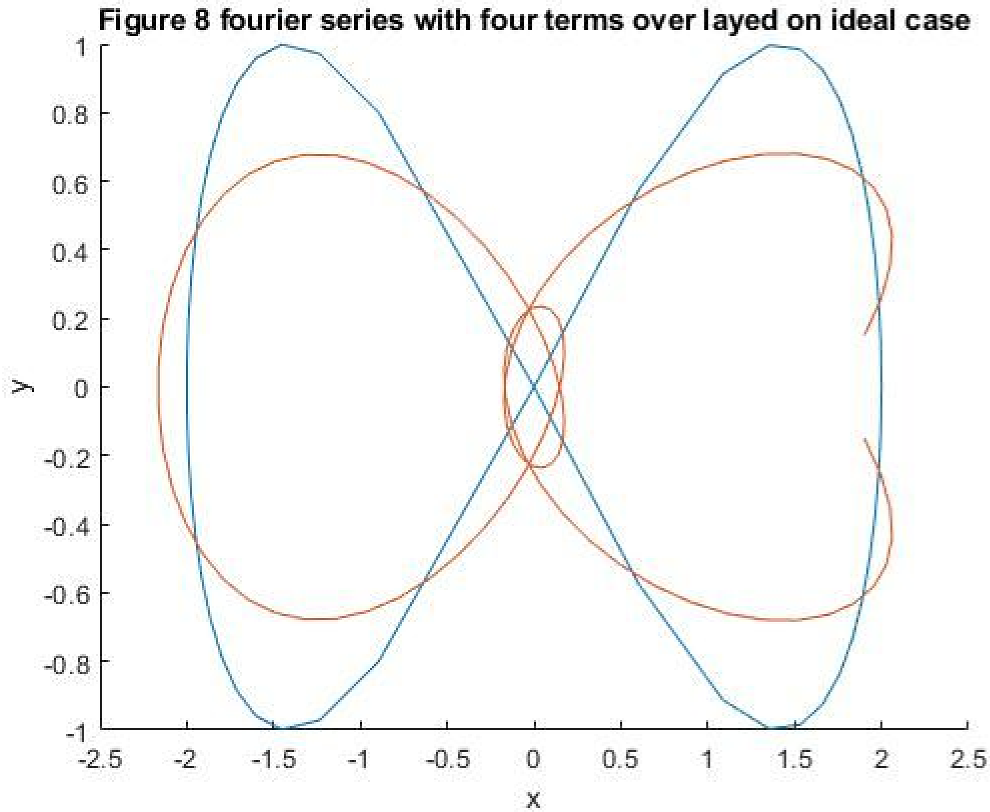

Figure 6 . “Figure 8” generated by 4 term fourier series vs the ideal case

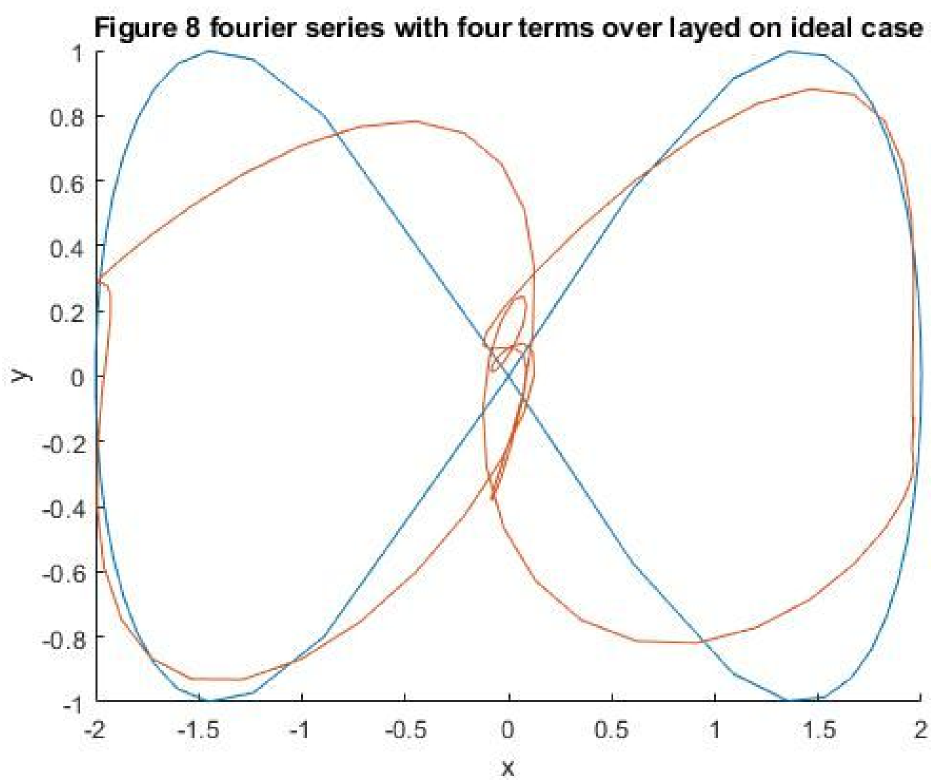

Figure 7 . “Figure 8” generated by 8 term fourier series vs the ideal case

The error obtained from both mechanisms is given below:

Figure	Error	Visual Figure
8 term	70.5%	Figure.6
4 term	70.6%	Figure.7

Table. Shows the errors obtained from the results.
As can be seen the error does not change much as the addition of more terms only slightly brings us closer to the main figure in terms of error, however visually there is a large difference.

3.2 Conclusion and Recommendation

Given the overall favourable shape of the output motion, it was decided to use the 4 term fourier series in assembling the mechanism. This choice was informed by the more desirable form of the coupler point curve, as the two options are negligibly different in terms of error. As an exercise in planar mechanism construction, the synthesis mechanism is acceptable; would the Lemniscate be required in the construction of a machine however, considerable work would be required to bring the coupler's path closer in line with the desired motion. Further refinements to the design are clearly attainable, and the investment of further time and development resources would likely yield a truer representation of the desired function. In fact the best move forward from here would be to utilize more terms on the order of 15-20 in our fourier series.

The smoother path of the 4-term mechanism compared to that of the 8-term would yield a gentler, more consistent motion in a mechanism attached to the coupler point, eliminating many of the spikes and jerking motions present in the 8-term mechanism. This choice was made in the interest of ease of manufacture for any theoretical machine utilising the designed mechanism, as well as to ensure greater part life and to resist sudden, premature material failures that erratic motions and sudden application of force can cause.

Further developmental steps would include further refinement and expansion of the mathematical model used to create the attained link dimensions, as well as the use of CAD/CAM software to simulate the coupler motion. Such a model would also be invaluable in efforts to create physical prototypes; a 3D printed model would allow for the physical properties of the mechanism to be tested, opening the way for further design improvements. We have provided an animation of the motion of the system rendered in solidworks, however this does not include a motion path as we were not able to place this in our version of solidworks. We have email you the video.

4. References

[1] Y. Liu and J. M. Mccarthy, “Design of Mechanisms to Draw Trigonometric Plane Curves,” *Journal of Mechanisms and Robotics*, vol. 9, no. 2, p. 024503, 2017.

[3] W.L. Cleghorn and N. Dechev, “Mechanics of Machines, 2nd edition”, Oxford University Press, 2015.

[4] N. Yurtoğlu, “http://www.historystudies.net/dergi//birinci-dunya-savasinda-bir-asayis-sorunu-sebinkarahisar-ermeni-isyani20181092a4a8f.pdf,” History Studies International Journal of History, vol. 10, no. 7, pp. 241–264, 2018.

[5] I. I. ARTOBOLEVSKII, “Mechanisms for the Generation of Plane Curves,” pp. 186–188, 1964.

5. Appendix

Single coupled serial chain mechanism with 4 term fourier series results

Table 1 . OUTPUT from Matlab code

TABLE 1	X	Y	
0	-0.1123	0	a0
1	1.3844	0.0251	a1
2	0.2587	0.2934	b1
3	-0.0727	0.0057	a2
4	-0.0281	0.0331	b2
5	0.6001	0.1329	a3
6	0.3715	0.5074	b3
7	0.1018	-0.0133	a4
8	0.0928	-0.0374	b4
9	1.0588	0.9728	w

Table 2 . Use to find L M omega n, shows the coefficients of fourier series (eqn 2.25,2.26)

TABLE 2	a	b	c	d
0	-0.1123	NA	0	NA
1	1.3844	0.2587	0.0251	0.2934
2	-0.0727	-0.0281	0.0057	0.0331
3	0.6001	0.3715	0.1329	0.5074
4	0.1018	0.0928	-0.0133	-0.0374

Table 3 . Length and Initial angles of the mechanism

TABLE 3	Lk	Mk	Omega	Nu
1	0.85	0.56	-0.14	0.25
2	0.03	0.05	-0.71	0.21
3	0.57	0.26	-0.21	1.39
4	0.06	0.08	-1.03	0.52

Table 4.Shows the Gear ratios (eqn 7.11)

TABLE 4	k	TD,j	TG,j+1
NA	0	NA	NA
1	1	2	1
2	-1	2	1
3	3	1.5	1
4	-3	1.17	1
5	4	1.14	1
6	-4	0.75	1
7	2	0.67	1
8	-2	0.5	1

Single coupled serial chain mechanism with 4 term fourier series Matlab codes

```
clc
clear
%% fourier_init
%this section will first plot our actual ideal function
%The PTx & PTy will hold the cartesian coordinates of the ideal function
%creatFit : will calculate the fourier series with four terms for the X and
%Y values

alpha = 2;
th = linspace(0,2*pi);
rho = real(sqrt(alpha.^2.*cos(2.*th).*sec(th).^4));
PTx = rho.*cos(th);
PTy = rho.*sin(th);


fx = createFit(th,PTx);
fy = createFit(th,PTy);

%% plot
%this section plot what the mechanisms final drawing.
%utilize coeffname if you are confused about the order of coefficients
PT = [coeffvalues(fx);coeffvalues(fy)]';
sprintf("X coef.  Y coef.")
disp(PT)
coeffnames(fx)
a0= PT(1,1);
a1= PT(2,1);
b1=PT(3,1);
a2=PT(4,1);
b2=PT(5,1);
a3=PT(6,1);
b3=PT(7,1);
a4=PT(8,1);
b4=PT(9,1);
```

```
w=PT(10,1);

xcom=@(x)a0 + a1*cos(x*w) + b1*sin(x*w) + a2*cos(2*x*w) + b2*sin(2*x*w) + a3*cos(3*x*w) +
b3*sin(3*x*w) +  a4*cos(4*x*w) + b4*sin(4*x*w);

a0= PT(1,2);
a1= PT(2,2);
b1=PT(3,2);
a2=PT(4,2);
b2=PT(5,2);
a3=PT(6,2);
b3=PT(7,2);
a4=PT(8,2);
b4=PT(9,2);
w=PT(10,2);
ycom=@(x)a0 + a1*cos(x*w) + b1*sin(x*w) + a2*cos(2*x*w) + b2*sin(2*x*w) + a3*cos(3*x*w) +
b3*sin(3*x*w) +  a4*cos(4*x*w) + b4*sin(4*x*w);
%this will graph the overlay
hold on

plot(PTx,PTy)
xlabel('x')
ylabel('y')
title('Figure 8 fourier series with four terms over layed on ideal case')
plot(xcom(th),ycom(th))
hold off
%% seperate_plot
figure

plot(PTx,PTy)
xlabel('x')
ylabel('y')
title('Figure 8 ideal case')

figure

plot(xcom(th),ycom(th))
xlabel('x')
ylabel('y')
title('Figure 8 fourier series with four terms')
```

Error Analysis codes

```
%% error
%This section will calculate the True error between the Graphs
%We will take vectors to point on each graph which are at the same angle
%theta from the graph. We then find the distance of each to the origin, and
```

```
%divide the result. This will give us error.

%PTx , PTy ideal figure 8 variables.
%xcom(th) , ycom(th) for mechanism drawn figure 8.
%origin = [0 0] is the origin to which we are comparing the points.
idx = 0;
origin = [0 0];
th = linspace(pi/3,5*pi/3);
PTx = rho.*cos(th);
PTy = rho.*sin(th);
error_point = sqrt((xcom(th) - origin(1)).^2  - (ycom(th) -
origin(2)).^2)./sqrt((PTx - origin(1)).^2  - (PTy - origin(2)).^2);
temp = zeros(100,1);
for i = 1:length(error_point)
    if abs(error_point(i)) ~= 0 && abs(error_point(i)) ~= Inf
        idx = idx + 1;
        temp(idx) = real(error_point(i));
    end
end
idx = 0;

 for i = 1:length(temp)
     if temp(i) ~= 0
         idx = idx + 1;
         temp2(idx,1) = temp(i,1);
     end
 end

 error = mean(temp2)
```

Single coupled serial chain mechanism with 8 term fourier series Matlab codes

```
clc
clear
%% POlAR to CARTESIAN & FOURIER CALCULATION
alpha = 2;
th = linspace(0,2*pi);
rho = real(sqrt(alpha.^2.*cos(2.*th).*sec(th).^4));
PTx = rho.*cos(th);
PTy = rho.*sin(th);

 fx = createFit8(th,PTx);
 fy = createFit8(th,PTy);

%% coeff _ init
%utilize coeffname if you are confused about the order of coefficients
```

```
PT = [coeffvalues(fx);coeffvalues(fy)]';
sprintf("X coef.  Y coef.")
disp(PT)
coeffnames(fx);
a0= PT(1,1);
a1= PT(2,1);
b1=PT(3,1);
a2=PT(4,1);
b2=PT(5,1);
a3=PT(6,1);
b3=PT(7,1);
a4=PT(8,1);
b4=PT(9,1);
a5= PT(10,1);
b5=PT(11,1);
a6=PT(12,1);
b6=PT(13,1);
a7=PT(14,1);
b7=PT(15,1);
a8=PT(16,1);
b8=PT(17,1);
w=PT(18,1);


 xcom=@(x)a0 + a1*cos(x*w) + b1*sin(x*w) + a2*cos(2*x*w) + b2*sin(2*x*w) +
a3*cos(3*x*w) + b3*sin(3*x*w) +  a4*cos(4*x*w) + b4*sin(4*x*w)+ a5*cos(5*x*w) +
b5*sin(5*x*w) + a6*cos(6*x*w) + b6*sin(6*x*w) + a7*cos(7*x*w) + b7*sin(7*x*w) +
a8*cos(8*x*w) + b8*sin(8*x*w);

a0= PT(1,2);
a1= PT(2,2);
b1=PT(3,2);
a2=PT(4,2);
b2=PT(5,2);
a3=PT(6,2);
b3=PT(7,2);
a4=PT(8,2);
b4=PT(9,2);
a5= PT(10,1);
b5=PT(11,1);
a6=PT(12,1);
b6=PT(13,1);
a7=PT(14,1);
b7=PT(15,1);
a8=PT(16,1);
b8=PT(17,1);
w=PT(18,2);
ycom=@(x)a0 + a1*cos(x*w) + b1*sin(x*w) + a2*cos(2*x*w) + b2*sin(2*x*w) +
a3*cos(3*x*w) + b3*sin(3*x*w) +  a4*cos(4*x*w) + b4*sin(4*x*w)+ a5*cos(5*x*w) +
```

```
b5*sin(5*x*w) +  a6*cos(6*x*w) + b6*sin(6*x*w) + a7*cos(7*x*w) + b7*sin(7*x*w)
+  a8*cos(8*x*w) + b8*sin(8*x*w);
%this will graph the overlay
hold on

plot(PTx,PTy)
xlabel('x')
ylabel('y')
title('Figure 8 fourier series with four terms over layed on ideal case')
plot(xcom(th),ycom(th))

hold off
%% seperate_plot

figure

plot(PTx,PTy)
xlabel('x')
ylabel('y')
title('Figure 8 ideal case')

figure

plot(xcom(th),ycom(th))
xlabel('x')
ylabel('y')
title('Figure 8 fourier series with four terms')
```