

نحوه محاسبه دترمینان ماتریس 4*4:

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \rightarrow a \times \begin{vmatrix} f & g & h \\ i & k & l \\ m & o & p \end{vmatrix} - b \times \begin{vmatrix} e & g & h \\ i & k & l \\ m & o & p \end{vmatrix} + c \times \begin{vmatrix} e & f & h \\ i & j & l \\ m & n & p \end{vmatrix} - d \times \begin{vmatrix} e & f & g \\ i & j & k \\ m & n & o \end{vmatrix}$$

$$\begin{aligned} & + [a \times f \times (k \times p - o \times l) - a \times g \times (j \times p - l \times n) + a \times h \times (j \times o - k \times n)] \\ & - [b \times e \times (k \times p - l \times o) - b \times g \times (i \times p - l \times m) + b \times h \times (i \times o - k \times m)] \\ & + [c \times e \times (j \times p - l \times n) - c \times f \times (i \times p - l \times m) + c \times h \times (i \times n - j \times m)] \\ & - [d \times e \times (j \times o - k \times n) - d \times f \times (i \times o - k \times m) + d \times g \times (i \times n - j \times m)] \end{aligned}$$

مثال:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \rightarrow 1 \times \begin{vmatrix} 6 & 7 & 8 \\ 10 & 11 & 12 \\ 14 & 15 & 16 \end{vmatrix} - 2 \times \begin{vmatrix} 5 & 7 & 8 \\ 9 & 11 & 12 \\ 13 & 15 & 16 \end{vmatrix} + 3 \times \begin{vmatrix} 5 & 6 & 8 \\ 9 & 10 & 12 \\ 13 & 14 & 16 \end{vmatrix} -$$

$$4 \times \begin{vmatrix} 5 & 6 & 7 \\ 9 & 10 & 11 \\ 13 & 14 & 15 \end{vmatrix}$$

$$\begin{aligned} & + [1 \times 6 \times (11 \times 16 - 15 \times 12) - 1 \times 7 \times (10 \times 16 - 12 \times 14) + 1 \times 8 \times (10 \times 15 - 11 \times 14)] \\ & - [2 \times 5 \times (11 \times 16 - 12 \times 15) - 2 \times 7 \times (9 \times 16 - 12 \times 13) + 2 \times 8 \times (9 \times 15 - 11 \times 13)] \\ & + [3 \times 5 \times (10 \times 16 - 12 \times 14) - 3 \times 6 \times (9 \times 16 - 12 \times 13) + 3 \times 8 \times (9 \times 14 - 10 \times 13)] \\ & - [4 \times 5 \times (10 \times 15 - 11 \times 14) - 4 \times 6 \times (9 \times 15 - 11 \times 13) + 4 \times 7 \times (9 \times 14 - 10 \times 13)] \end{aligned}$$

$$\text{Det}(A) = 0$$

نحوه محاسبه دترمینان ماتریس 4*4 در محیط برنامه نویسی :

```
import numpy as np

A=np.array([[1,2,3,4],[5,6,7,8],[9,10,11,12],[13,14,15,16]])

det=np.linalg.det(A)

print(round(det,20))

0.0
```

نحوه محاسبه دترمینان ماتریس 5*5:

$$\begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \\ p & q & r & s & t \\ u & v & w & x & y \end{bmatrix} \Rightarrow a \times \begin{vmatrix} g & h & i & j \\ l & m & n & o \\ q & r & s & t \\ v & w & x & y \end{vmatrix} - b \times \begin{vmatrix} f & h & i & j \\ k & m & n & o \\ p & r & s & t \\ u & w & x & y \end{vmatrix} + c \times \begin{vmatrix} f & g & i & j \\ k & l & n & o \\ p & q & s & t \\ u & v & x & y \end{vmatrix}$$

$$- d \times \begin{vmatrix} f & g & h & j \\ k & l & m & o \\ p & q & r & t \\ u & v & w & y \end{vmatrix} + e \times \begin{vmatrix} f & g & h & i \\ k & l & m & n \\ p & q & r & s \\ u & v & w & x \end{vmatrix}$$

$$\begin{bmatrix} g & h & i & j \\ l & m & n & o \\ q & r & s & t \\ v & w & x & y \end{bmatrix} \Rightarrow g \times \begin{vmatrix} m & n & o \\ r & s & t \\ w & x & y \end{vmatrix} - h \times \begin{vmatrix} l & n & o \\ q & s & t \\ v & x & y \end{vmatrix} + i \times \begin{vmatrix} l & m & o \\ q & r & t \\ v & w & y \end{vmatrix} - j \times \begin{vmatrix} l & m & n \\ q & r & s \\ v & w & x \end{vmatrix}$$

$$\begin{aligned} & + [a \times g \times m \times (s \times y - t \times x) - a \times g \times n \times (r \times y - t \times w) + a \times g \times o \times (r \times x - s \times w)] \\ & - [a \times h \times l \times (s \times y - t \times x) - a \times h \times n \times (q \times y - t \times v) + a \times h \times o \times (q \times x - s \times v)] \\ & + [a \times i \times l \times (r \times y - t \times w) - a \times i \times m \times (q \times y - t \times v) + a \times i \times o \times (q \times w - r \times v)] \\ & - [a \times j \times l \times (r \times x - s \times w) - a \times j \times m \times (q \times x - s \times v) + a \times j \times n \times (q \times w - r \times v)] \end{aligned}$$

$$\begin{bmatrix} f & h & i & j \\ k & m & n & o \\ p & r & s & t \\ u & w & x & y \end{bmatrix} \Rightarrow f \times \begin{vmatrix} m & n & o \\ r & s & t \\ w & x & y \end{vmatrix} - h \times \begin{vmatrix} k & n & o \\ p & s & t \\ u & x & y \end{vmatrix} + i \times \begin{vmatrix} k & m & o \\ p & r & t \\ u & w & y \end{vmatrix} - j \times \begin{vmatrix} k & m & n \\ p & r & s \\ u & w & x \end{vmatrix}$$

$$\begin{aligned} & - [b \times f \times m \times (s \times y - t \times x) - b \times f \times n \times (r \times y - t \times w) + b \times f \times o \times (r \times x - s \times w)] \\ & + [b \times h \times k \times (s \times y - t \times x) - b \times h \times n \times (p \times y - t \times u) + b \times h \times o \times (p \times x - s \times u)] \\ & - [b \times i \times k \times (r \times y - t \times w) - b \times i \times m \times (p \times y - t \times u) + b \times i \times o \times (p \times w - r \times u)] \\ & + [b \times j \times k \times (r \times x - s \times w) - b \times j \times m \times (p \times x - s \times u) + b \times j \times n \times (p \times w - r \times u)] \end{aligned}$$

$$\begin{bmatrix} f & g & i & j \\ k & l & n & o \\ p & q & s & t \\ u & v & x & y \end{bmatrix} \Rightarrow f \times \begin{vmatrix} l & n & o \\ q & s & t \\ v & x & y \end{vmatrix} - g \times \begin{vmatrix} k & n & o \\ p & s & t \\ u & x & y \end{vmatrix} + i \times \begin{vmatrix} k & l & o \\ p & q & t \\ u & v & y \end{vmatrix} - j \times \begin{vmatrix} k & l & n \\ p & q & s \\ u & v & x \end{vmatrix}$$

$$\begin{aligned} & + [c \times f \times l \times (s \times y - t \times x) - c \times f \times n \times (q \times y - t \times v) + c \times f \times o \times (q \times x - s \times v)] \\ & - [c \times g \times k \times (s \times y - t \times x) - c \times g \times n \times (p \times y - t \times u) + c \times g \times o \times (p \times x - s \times u)] \\ & + [c \times i \times k \times (q \times y - t \times u) - c \times i \times l \times (p \times y - t \times u) + c \times i \times o \times (p \times v - q \times u)] \\ & - [c \times j \times k \times (q \times x - s \times v) - c \times j \times l \times (p \times x - s \times u) + c \times j \times n \times (p \times v - q \times u)] \end{aligned}$$

$$\begin{bmatrix} f & g & h & j \\ k & l & m & o \\ p & q & r & t \\ u & v & w & y \end{bmatrix} \Rightarrow f \times \begin{vmatrix} l & m & o \\ q & r & t \\ v & w & y \end{vmatrix} - g \times \begin{vmatrix} k & m & o \\ p & r & t \\ u & w & y \end{vmatrix} + h \times \begin{vmatrix} k & l & o \\ p & q & t \\ u & v & y \end{vmatrix} - j \times \begin{vmatrix} k & l & m \\ p & q & r \\ u & v & w \end{vmatrix}$$

$$\begin{aligned} & - [d \times f \times l \times (r \times y - t \times w) - d \times f \times m \times (q \times y - t \times v) + d \times f \times o \times (q \times w - r \times v)] \\ & + [d \times g \times k \times (r \times y - t \times w) - d \times g \times m \times (p \times y - t \times u) + d \times g \times o \times (p \times w - r \times u)] \\ & - [d \times h \times k \times (q \times y - t \times v) - d \times h \times l \times (p \times y - t \times u) + d \times h \times o \times (p \times v - q \times u)] \\ & + [d \times j \times k \times (q \times w - r \times v) - d \times j \times l \times (p \times w - r \times u) + d \times j \times m \times (p \times v - q \times u)] \end{aligned}$$

$$\begin{bmatrix} f & g & h & i \\ k & l & m & n \\ p & q & r & s \\ u & v & w & x \end{bmatrix} \Rightarrow f \times \begin{vmatrix} l & m & n \\ q & r & s \\ v & w & x \end{vmatrix} - g \times \begin{vmatrix} k & m & n \\ p & r & s \\ u & w & x \end{vmatrix} + h \times \begin{vmatrix} k & l & n \\ p & q & s \\ u & v & x \end{vmatrix} - i \times \begin{vmatrix} k & l & m \\ p & q & r \\ u & v & w \end{vmatrix}$$

$$\begin{aligned} & + [e \times f \times l \times (r \times x - s \times w) - e \times f \times m \times (q \times x - s \times v) + e \times f \times n \times (q \times w - r \times v)] \\ & - [e \times g \times k \times (r \times x - s \times w) - e \times g \times m \times (p \times x - s \times u) + e \times g \times n \times (p \times w - r \times u)] \\ & + [e \times h \times k \times (q \times x - s \times v) - e \times h \times l \times (p \times x - s \times u) + e \times h \times n \times (p \times v - q \times u)] \\ & - [e \times i \times k \times (q \times w - r \times v) - e \times i \times l \times (p \times w - r \times u) + e \times i \times m \times (p \times v - q \times u)] \end{aligned}$$

مثال:

$$B = \begin{bmatrix} 01 & 02 & 03 & 04 & 05 \\ 06 & 07 & 08 & 09 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{bmatrix} \rightarrow 01 \times \begin{bmatrix} 07 & 08 & 09 & 10 \\ 12 & 13 & 14 & 15 \\ 17 & 18 & 19 & 20 \\ 22 & 23 & 24 & 25 \end{bmatrix} - 02 \times \begin{bmatrix} 06 & 08 & 09 & 10 \\ 11 & 13 & 14 & 15 \\ 16 & 18 & 19 & 20 \\ 21 & 23 & 24 & 25 \end{bmatrix} + 03 \times \begin{bmatrix} 06 & 07 & 09 & 10 \\ 11 & 12 & 14 & 15 \\ 16 & 17 & 19 & 20 \\ 21 & 22 & 24 & 25 \end{bmatrix}$$

$$-04 \times \begin{bmatrix} 06 & 07 & 08 & 10 \\ 11 & 12 & 13 & 15 \\ 16 & 17 & 18 & 20 \\ 21 & 22 & 23 & 25 \end{bmatrix} + 05 \times \begin{bmatrix} 06 & 07 & 08 & 09 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \\ 21 & 22 & 23 & 24 \end{bmatrix}$$

$$\begin{bmatrix} 07 & 08 & 09 & 10 \\ 12 & 13 & 14 & 15 \\ 17 & 18 & 19 & 20 \\ 22 & 23 & 24 & 25 \end{bmatrix} \rightarrow 07 \times \begin{bmatrix} 13 & 14 & 15 \\ 18 & 19 & 20 \\ 23 & 24 & 25 \end{bmatrix} - 08 \times \begin{bmatrix} 12 & 14 & 15 \\ 17 & 19 & 20 \\ 22 & 24 & 25 \end{bmatrix} + 09 \times \begin{bmatrix} 12 & 13 & 15 \\ 17 & 18 & 20 \\ 22 & 23 & 25 \end{bmatrix} - 10 \times \begin{bmatrix} 12 & 13 & 14 \\ 17 & 18 & 19 \\ 22 & 23 & 24 \end{bmatrix}$$

$$\begin{aligned} & + [1 \times 7 \times 13 \times (19 \times 25 - 20 \times 24) - 1 \times 7 \times 14 \times (18 \times 25 - 20 \times 23) + 1 \times 7 \times 15 \times (18 \times 24 - 19 \times 23)] \\ & - [1 \times 8 \times 12 \times (19 \times 25 - 20 \times 24) - 1 \times 8 \times 14 \times (17 \times 25 - 20 \times 22) + 1 \times 8 \times 15 \times (17 \times 24 - 19 \times 22)] \\ & + [1 \times 9 \times 12 \times (18 \times 25 - 20 \times 23) - 1 \times 9 \times 13 \times (17 \times 25 - 20 \times 22) + 1 \times 9 \times 15 \times (17 \times 23 - 18 \times 22)] \\ & - [1 \times 10 \times 12 \times (18 \times 24 - 19 \times 23) - 1 \times 10 \times 13 \times (17 \times 24 - 19 \times 22) + 1 \times 10 \times 14 \times (17 \times 23 - 18 \times 22)] \end{aligned}$$

$$\begin{bmatrix} 06 & 08 & 09 & 10 \\ 11 & 13 & 14 & 15 \\ 16 & 18 & 19 & 20 \\ 21 & 23 & 24 & 25 \end{bmatrix} \rightarrow 06 \times \begin{bmatrix} 13 & 14 & 15 \\ 18 & 19 & 20 \\ 23 & 24 & 25 \end{bmatrix} - 08 \times \begin{bmatrix} 11 & 14 & 15 \\ 16 & 19 & 20 \\ 21 & 24 & 25 \end{bmatrix} + 09 \times \begin{bmatrix} 11 & 13 & 15 \\ 16 & 18 & 20 \\ 21 & 23 & 25 \end{bmatrix} - 10 \times \begin{bmatrix} 11 & 13 & 14 \\ 16 & 18 & 19 \\ 21 & 23 & 24 \end{bmatrix}$$

$$\begin{aligned} & - [2 \times 6 \times 13 \times (19 \times 25 - 20 \times 24) - 2 \times 6 \times 14 \times (18 \times 25 - 20 \times 23) + 2 \times 6 \times 15 \times (18 \times 24 - 19 \times 23)] \\ & + [2 \times 8 \times 11 \times (19 \times 25 - 20 \times 24) - 2 \times 8 \times 14 \times (16 \times 25 - 20 \times 21) + 2 \times 8 \times 15 \times (16 \times 24 - 19 \times 21)] \\ & - [2 \times 9 \times 11 \times (18 \times 25 - 20 \times 23) - 2 \times 9 \times 13 \times (16 \times 25 - 20 \times 21) + 2 \times 9 \times 15 \times (16 \times 23 - 18 \times 21)] \\ & + [2 \times 10 \times 11 \times (18 \times 24 - 19 \times 23) - 2 \times 10 \times 13 \times (16 \times 24 - 19 \times 21) + 2 \times 10 \times 14 \times (16 \times 23 - 18 \times 21)] \end{aligned}$$

$$\begin{bmatrix} 06 & 07 & 09 & 10 \\ 11 & 12 & 14 & 15 \\ 16 & 17 & 19 & 20 \\ 21 & 22 & 24 & 25 \end{bmatrix} \rightarrow 06 \times \begin{vmatrix} 12 & 14 & 15 \\ 17 & 19 & 20 \\ 22 & 24 & 25 \end{vmatrix} - 07 \times \begin{vmatrix} 11 & 14 & 15 \\ 16 & 19 & 20 \\ 21 & 24 & 25 \end{vmatrix} + 09 \times \begin{vmatrix} 11 & 12 & 15 \\ 16 & 17 & 20 \\ 21 & 22 & 25 \end{vmatrix} - 10 \times \begin{vmatrix} 11 & 12 & 14 \\ 16 & 17 & 19 \\ 21 & 22 & 24 \end{vmatrix}$$

$$\begin{aligned} & + [3 \times 6 \times 12 \times (19 \times 25 - 20 \times 24) - 3 \times 6 \times 14 \times (17 \times 25 - 20 \times 22) + 3 \times 6 \times 15 \times (17 \times 24 - 19 \times 22)] \\ & - [3 \times 7 \times 11 \times (19 \times 25 - 20 \times 24) - 3 \times 7 \times 14 \times (16 \times 25 - 20 \times 21) + 3 \times 7 \times 15 \times (16 \times 24 - 19 \times 21)] \\ & + [3 \times 9 \times 11 \times (19 \times 25 - 20 \times 24) - 3 \times 9 \times 14 \times (16 \times 25 - 20 \times 21) + 3 \times 9 \times 15 \times (16 \times 24 - 19 \times 21)] \\ & - [3 \times 10 \times 11 \times (17 \times 24 - 19 \times 22) - 3 \times 10 \times 12 \times (16 \times 24 - 19 \times 21) + 3 \times 10 \times 14 \times (16 \times 22 - 17 \times 21)] \end{aligned}$$

$$\begin{bmatrix} 06 & 07 & 08 & 10 \\ 11 & 12 & 13 & 15 \\ 16 & 17 & 18 & 20 \\ 21 & 22 & 23 & 25 \end{bmatrix} \rightarrow 06 \times \begin{vmatrix} 12 & 13 & 15 \\ 17 & 18 & 20 \\ 22 & 23 & 25 \end{vmatrix} - 07 \times \begin{vmatrix} 11 & 13 & 15 \\ 16 & 18 & 20 \\ 21 & 23 & 25 \end{vmatrix} + 08 \times \begin{vmatrix} 11 & 12 & 15 \\ 16 & 17 & 20 \\ 21 & 22 & 25 \end{vmatrix} - 10 \times \begin{vmatrix} 11 & 12 & 13 \\ 16 & 17 & 18 \\ 21 & 22 & 23 \end{vmatrix}$$

$$\begin{aligned} & - [4 \times 6 \times 12 \times (18 \times 25 - 20 \times 23) - 4 \times 6 \times 13 \times (17 \times 25 - 20 \times 22) + 4 \times 6 \times 15 \times (17 \times 23 - 18 \times 22)] \\ & + [4 \times 7 \times 11 \times (18 \times 25 - 20 \times 23) - 4 \times 7 \times 13 \times (16 \times 25 - 20 \times 21) + 4 \times 7 \times 15 \times (16 \times 23 - 18 \times 21)] \\ & - [4 \times 8 \times 11 \times (17 \times 25 - 20 \times 22) - 4 \times 8 \times 12 \times (16 \times 25 - 20 \times 21) + 4 \times 8 \times 15 \times (16 \times 22 - 17 \times 21)] \\ & + [4 \times 10 \times 11 \times (17 \times 23 - 18 \times 22) - 4 \times 10 \times 12 \times (16 \times 23 - 18 \times 21) + 4 \times 10 \times 13 \times (16 \times 22 - 17 \times 21)] \end{aligned}$$

$$\begin{bmatrix} 06 & 07 & 08 & 09 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \\ 21 & 22 & 23 & 24 \end{bmatrix} \rightarrow 06 \times \begin{vmatrix} 12 & 13 & 14 \\ 17 & 18 & 19 \\ 22 & 23 & 24 \end{vmatrix} - 07 \times \begin{vmatrix} 11 & 13 & 14 \\ 16 & 18 & 19 \\ 21 & 23 & 24 \end{vmatrix} + 08 \times \begin{vmatrix} 11 & 12 & 14 \\ 16 & 17 & 19 \\ 21 & 22 & 24 \end{vmatrix} - 09 \times \begin{vmatrix} 11 & 12 & 13 \\ 16 & 17 & 18 \\ 21 & 22 & 23 \end{vmatrix}$$

$$\begin{aligned} & + [5 \times 6 \times 12 \times (18 \times 24 - 19 \times 23) - 5 \times 6 \times 13 \times (17 \times 24 - 19 \times 22) + 5 \times 6 \times 14 \times (17 \times 23 - 18 \times 22)] \\ & - [5 \times 7 \times 11 \times (18 \times 24 - 19 \times 23) - 5 \times 7 \times 13 \times (16 \times 24 - 19 \times 21) + 5 \times 7 \times 14 \times (16 \times 23 - 18 \times 21)] \\ & + [5 \times 8 \times 11 \times (17 \times 24 - 19 \times 22) - 5 \times 8 \times 12 \times (16 \times 24 - 19 \times 21) + 5 \times 8 \times 14 \times (16 \times 22 - 17 \times 21)] \\ & - [5 \times 9 \times 11 \times (17 \times 23 - 18 \times 22) - 5 \times 9 \times 12 \times (16 \times 23 - 18 \times 21) + 5 \times 9 \times 13 \times (16 \times 22 - 17 \times 21)] \end{aligned}$$

$$\text{Det (B)} = 0$$

نحوه محاسبه دترمینان ماتریس 5*5 در محیط برنامه نویسی :

```
import numpy as np

B=np.array([[1,2,3,4,5],[6,7,8,9,10],[11,12,13,14,15],[16,17,18,19,20],[21,22,23,24,25]])

det=np.linalg.det(B)

print(round(det,20))

0.0
```

نحوه محاسبه دترمینان ماتریس n*n:

$$\det A = \sum (-1)^{i+j} a_{ij} \det A_{ij}$$