ASSIGNMENT FIVE

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Question one

$$\text{Popularity Rating}_{ij} = \left[\beta_0 + b_{0j}\right] + \epsilon_{ij}$$

where,

- $\text{Popularity Rating}_{ij}$ is the average recived populairty rating for student i in classroom j;
- β_0 is the fixed-effect of intercept, b_{0j} is the random-effect of intercept for classroom j; and
- ϵ_{ij} is the error for student i in classroom j.

The full specification of a model consists of a mathematical description of the distributional assumptions. Mixed-effects models have distributional assumptions on the errors (ϵ_{ij}) and on each set of random-effects included in the model (b_{0j} in our model). The assumptions on the errors are:

- Independence;
- Conditional normality;
- Conditional means are 0; and
- Homoskedasticity of the conditional variances σ^2_ϵ.

The assumptions on each set of random-effects are:

- Independence;
- Normality;
- Mean of 0; and
- There is some variance, $\sigma^2_{b_0}$ (often just denoted σ^2_0)

In mathematical notation the assumptions for the unconditional random intercepts model can be written as:

$$\epsilon_{ij} \overset{i.i.d}{\sim} \mathcal{N}\left(0, \sigma^2_\epsilon\right)$$

$$b_{0j} \overset{i.i.d}{\sim} \mathcal{N}\left(0, \sigma^2_0\right)$$

Question two

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: popularity ~ 1 + (1 | class)
##    Data: joined_data
##
##      AIC      BIC   logLik deviance df.resid
##   6333.5   6350.3  -3163.7   6327.5     1997
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5662 -0.6983  0.0021  0.6758  3.3173
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  class    (Intercept) 0.6945   0.8333
##  Residual             1.2218   1.1053
## Number of obs: 2000, groups:  class, 100
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  5.07786    0.08696    58.4
```

The variance estimates are

$\hat{\sigma}_{\epsilon}^2 = 0.6945$

$\hat{\sigma}_{0}^2 = 1.2218$

Since one mathematical property of variances is that they are additive, we can compute the total unexplained variation by summing the variance estimates:

$$\begin{aligned}\sigma^2_{\text{Total Unexplained}} &= \hat{\sigma}_{0}^2 + \hat{\sigma}_{\epsilon}^2 \\ &= 0.6945 + 1.2218 \\ &= 1.9163\end{aligned}$$

The proportion of **unaccounted variation at the classroom-level** is:

$$\frac{0.6945}{1.9163} = 0.3624172$$

The proportion of **unaccounted variation at the student-level** is:

$$\frac{1.2218}{1.9163} = 0.6375828$$

Question three

```
##
## Model selection based on AICc:
##
##         K    AICc Delta_AICc AICcWt Cum.Wt       LL
## Model 3 5 4943.98       0.00      1      1 -2466.98
## Model 2 4 5564.27     620.29      0      1 -2778.13
## Model 1 4 5831.80     887.81      0      1 -2911.89
## Model 0 3 6333.48    1389.50      0      1 -3163.73
```

Question four

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: popularity ~ 1 + extra + female + (1 | class)
##    Data: joined_data
##
##      AIC      BIC   logLik deviance df.resid
##     4944     4972    -2467     4934     1995
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.2113 -0.6578 -0.0048  0.6739  2.9771
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  class    (Intercept) 0.6204   0.7876
##  Residual             0.5915   0.7691
## Number of obs: 2000, groups:  class, 100
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  2.14138    0.11696   18.31
## extra        0.44151    0.01615   27.34
## female       1.25314    0.03741   33.50
##
## Correlation of Fixed Effects:
##        (Intr) extra
## extra  -0.706
## female -0.100 -0.085
```

The fitted equation is

$$\hat{\text{Popularity Rating}}_{ij} = 2.14 + 0.44(\text{Extraversion}_{ij}) + 1.25(\text{Sex}_{ij})$$

Question five

A rule-of-thumb is that *t*-values greater than 2 support inclusion of the predictor. Here the *t*-value associated with extraversion is $t = 27.34$. This is evidence for including extraversion in the model.Here the *t*-value associated with sex is $t = 33.50$.This is evidence for including sex in the model.

Question six

Each one-unit difference in extraversion is associated with a 0.44-point difference in popularity rating, on average, controlling for differences in sex.

Question seven

Female students have a popularity rating that is 1.25-points higher, on average, than male students, controlling for differences in extraversion of the student.

Question eight

```
## Linear mixed model fit by maximum likelihood  ['lmerMod']
## Formula: popularity ~ 1 + extra + female + teacherExp + (1 | class)
##    Data: joined_data
##
##      AIC      BIC   logLik deviance df.resid
##   4874.3   4907.9  -2431.1   4862.3     1994
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.1794 -0.6492 -0.0067  0.6708  3.0103
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  class    (Intercept) 0.2888   0.5374
##  Residual             0.5914   0.7690
## Number of obs: 2000, groups:  class, 100
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept) 0.809326   0.168828   4.794
## extra       0.454484   0.016154  28.134
## female      1.254095   0.037265  33.653
## teacherExp  0.088409   0.008676  10.190
##
## Correlation of Fixed Effects:
##            (Intr) extra  female
## extra      -0.592
## female     -0.040 -0.090
## teacherExp -0.801  0.141 -0.037
```

The fitted equation is

$$\hat{\text{Popularity Rating}}_{ij} = 0.81 + 0.455(\text{Extraversion}_{ij}) + 1.254(\text{Sex}_{ij}) + 0.089(\text{Teacher Experience}_{\bullet j})$$

Question nine

Table of Model Evidence for The five fitted models

Hypothesis	LL	K	AICc	ΔAICc	AIC Wt.	ER
Model4	-2431.15	6	4874.34	0.00	1	1.000000e+00
Model3	-2466.98	5	4943.98	69.64	0	1.327907e+15

Note. LL = Log-Likelihood; K = Model df; AIC Wt. = Model Probability; ER = Evidence Ratio

Hypothesis	LL	K	AICc	△AICc	AIC Wt.	ER
Model2	-2778.13	4	5564.27	689.93	0	6.563808e+149
Model 1	-2911.89	4	5831.80	957.46	0	8.116362e+207
Model 0	-3163.73	3	6333.48	1459.14	0	Inf

Note. LL = Log-Likelihood; K = Model df; AIC Wt. = Model Probability; ER = Evidence Ratio

Question ten

Table one :Fixed-Effects Coefficients and Standard Errors for a Taxonomy of Fitted Models to Predict Popularity-Rating, on average, for 2000 different students from 100 different classrooms.. All Models Included a Random-Effect of Intercept and were Fitted using Maximum Likelihood.

	Outcome: Popularity-Rating, on average				
	Model 0	Model 1	Model 2	Model3	Model4
Extraversion		0.486		0.442	0.454
		(0.020)		(0.016)	(0.016)
Sex			1.350	1.253	1.254
			(0.044)	(0.037)	(0.037)
Teacher Experience					0.088
					(0.009)
Constant	5.078	2.543	4.394	2.141	0.809
	(0.087)	(0.141)	(0.075)	(0.117)	(0.169)
Corrected AIC	6333.5	5831.8	5564.3	4944	4874.3
Log Likelihood	-3,163.734	-2,911.888	-2,778.126	-2,466.976	-2,431.148

Table2: Variance estimates from fitting Model 0, Model 1, Model2, Model3 and Model4

Estimate	Model0	Model1	Mdoel2	Model3	Model4
σ^2_ϵ	1.22	0.93	0.83	0.59	0.59
σ^2_0	0.70	0.83	0.48	0.62	0.29