Quantum Entanglement Entropy [1]

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[1] a detailed extension of the notes of Dr. Buot

This notes introduces the basic idea of quantum entanglement characterized by entanglement entropy. The tools of density operator are discuss in detail. The maximally entangled pure state are discuss as well, including the criterion that distinguishes a pure state from a mixed state.

Quantum entanglement is a physical phenomenon which occurs when pairs or groups of particles are generated or interact in ways such that the quantum state of each particle cannot be described independently of the state of the other(s), even when the particles are separated by a large distance - instead, a quantum state must be described for the system as a whole.[2]

[2] copied from wikipedia

Bipartite Systems

Consider the subsystem's Hilbert space $\mathcal{H}_{\mathcal{A}}$ and $\mathcal{H}_{\mathcal{B}}$. The bipartite system is given to be the tensor product of the space.

$$\mathcal{H}_{\mathcal{AB}} = \mathcal{H}_{\mathcal{A}} \otimes \mathcal{H}_{\mathcal{B}}$$

Entanglement entropy

The reduced entropy S_A is the von Neumann entropy of the reduced density matrix, ρ_A,

$$\rho_A = Tr_B(\rho) \tag{1}$$

which is read as partial trace over system B.

Bell States - 2-qubit subsystem

A Bell state is given as,

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{2}$$

This has a density operator of,

$$\begin{aligned}
\rho &= |\psi\rangle\langle\psi| \\
&= \frac{1}{\sqrt{2}}\Big(|00\rangle + |11\rangle\Big)\frac{1}{\sqrt{2}}\Big(\langle 00| + \langle 11|\Big) \\
&= \frac{1}{2}\Big(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\Big)
\end{aligned}$$

4-qubit subsystem

Generalization

References