

On FX Rates and a Multivariate GARCH model

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1 Notation

We study the covariance matrix of a p -dimensional sequence with each component modeled by a GARCH(1, 1) process:

$$\begin{aligned} X_{i,t} &= \sigma_{i,t} Z_{i,t} \\ (Z_{1,t}, Z_{2,t}, \dots, Z_{p,t}) &\sim N(0, \Sigma) \\ \sigma_{i,t}^2 &= \alpha_{i,0} + \alpha_{i,1} X_{i,t-1}^2 + \beta_{i,1} \sigma_{i,t-1}^2 \end{aligned}$$

2 Spectra of S&P sectors

2.1 Utilities

Fitting the daily returns' data of 25 companies that have been included in the S&P 500 index since 2010-01-01 and categorized in the sector "Utilities", we obtained 23 GARCH(1, 1) models that satisfy the condition

$$\alpha_1 + \beta_1 < 1 \tag{1}$$

Then we simulate the GARCH(1, 1) processes with innovations having the same covariance structure as those of the real data. Figure 1 and 2, 3 compare the eigenvalues and eigenvectors, respectively, of the covariance matrices of the real and the simulated data.

2.2 Materials

A similar analysis on the "Materials" sector produces the following spectra. In this sector, 21 out of 25 return sequences are successfully fitted to GARCH(1, 1) models that satisfy condition (1).

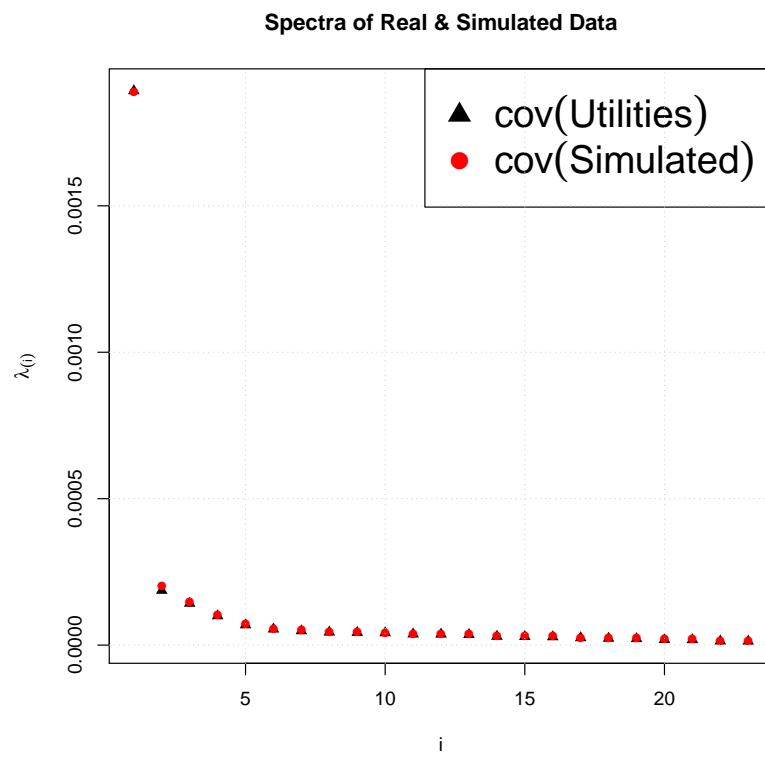


Figure 1: Eigenvalues of Utilities: real & simulated

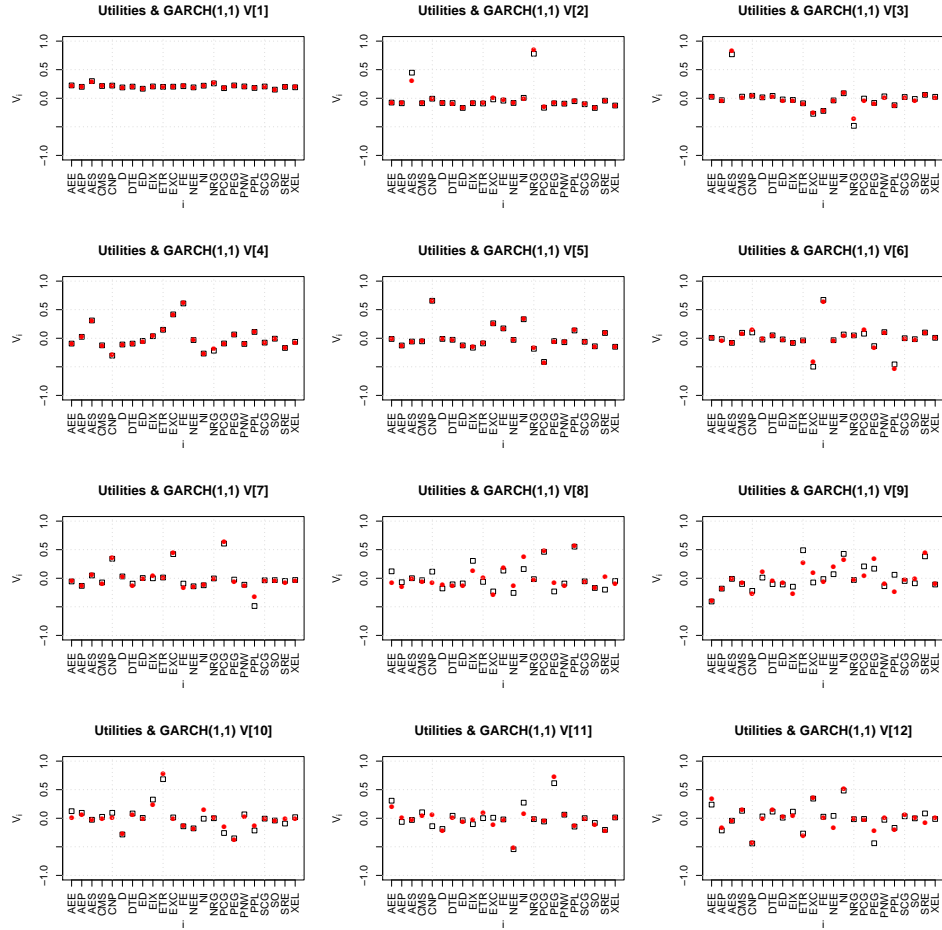


Figure 2: Eigenvectors of Utilities: real & simulated

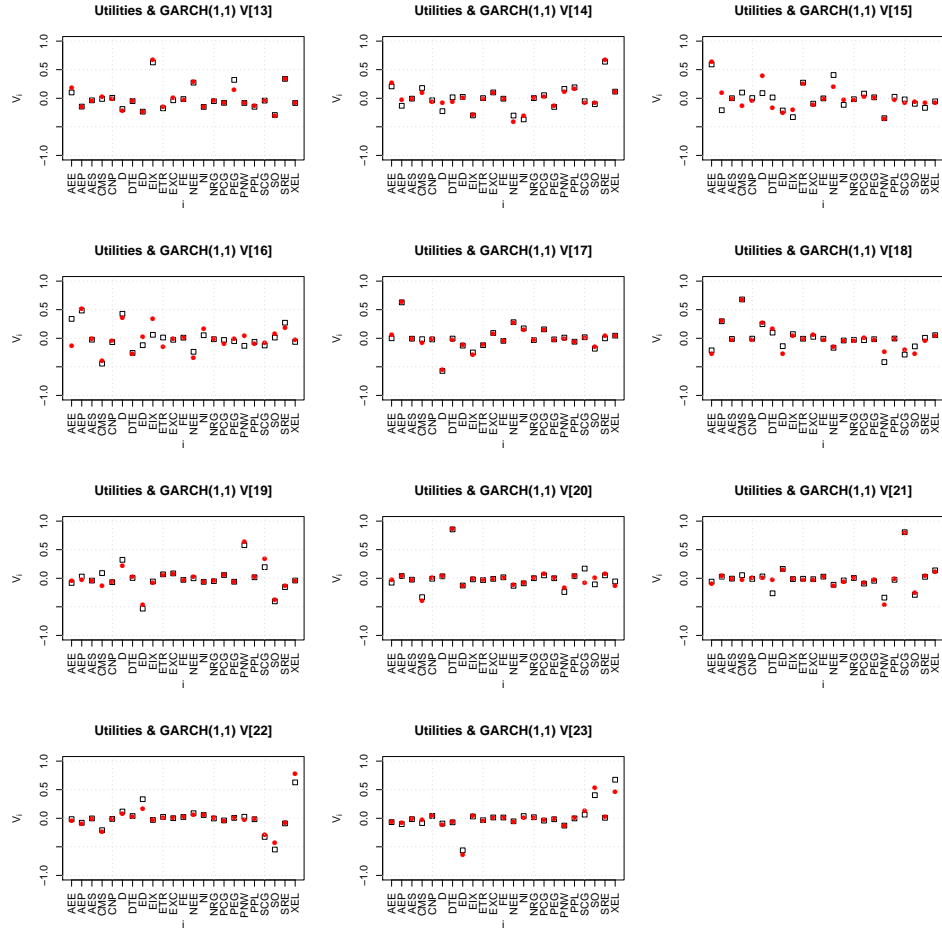


Figure 3: Eigenvectors of Utilities: real & simulated continued

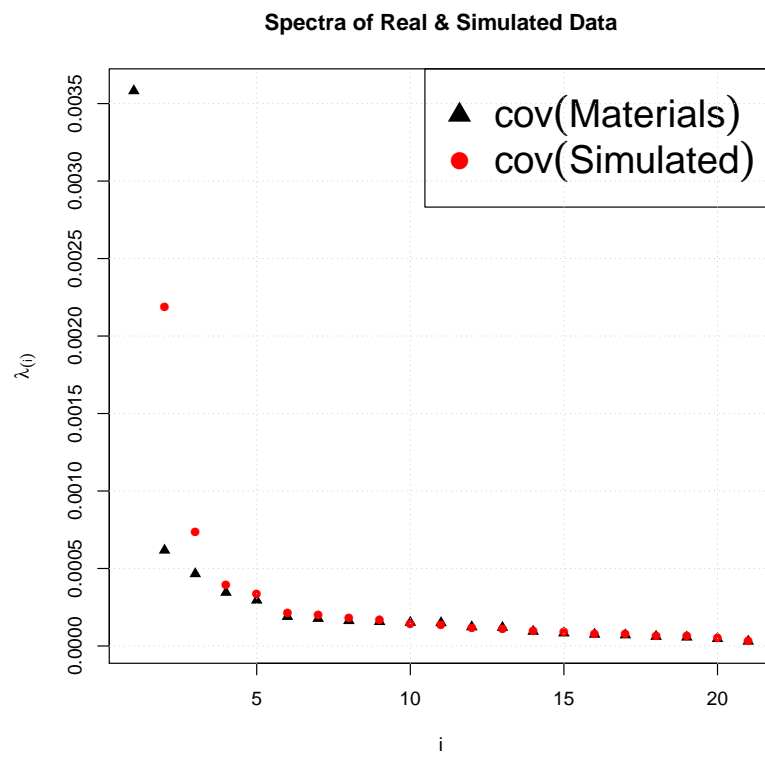


Figure 4: Eigenvalues of Materials: real & simulated

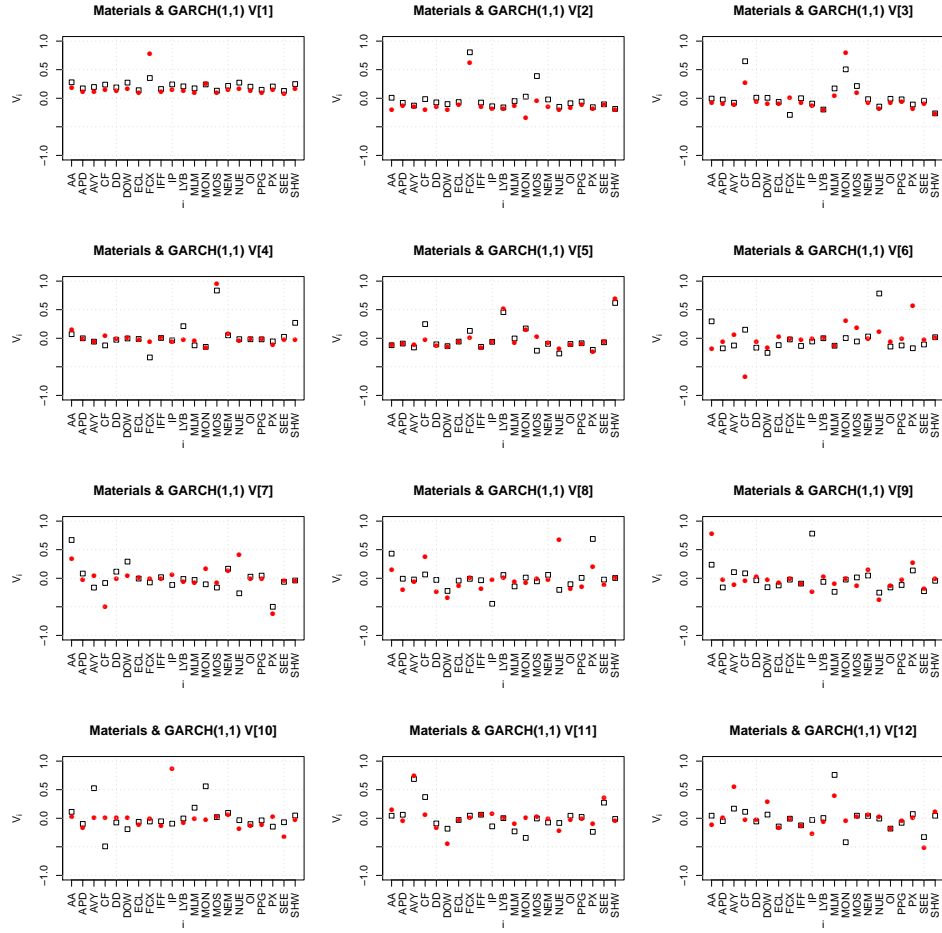


Figure 5: Eigenvectors of Materials: real & simulated

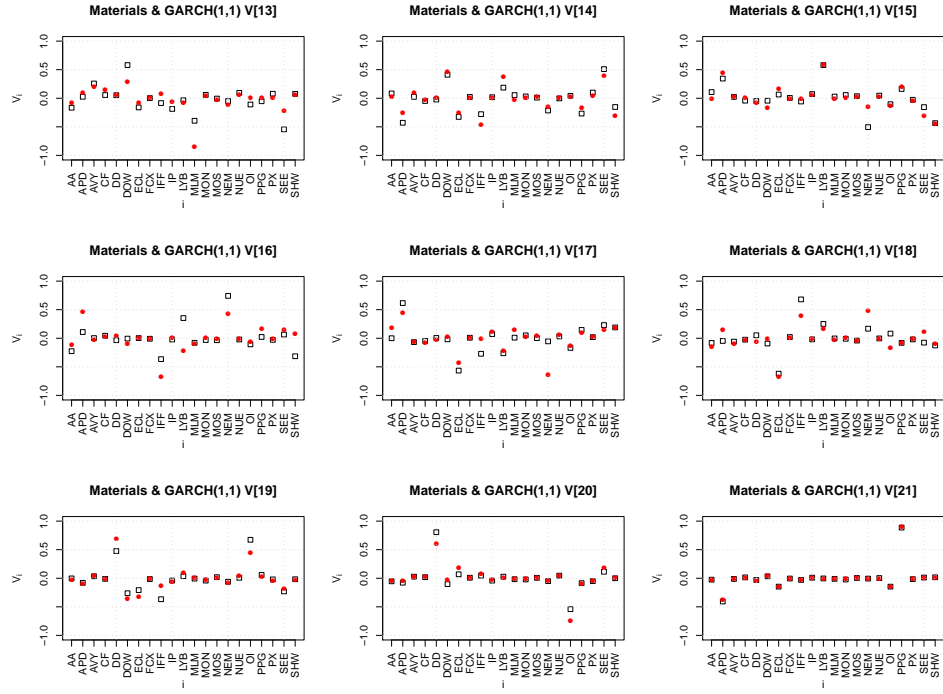


Figure 6: Eigenvectors of Materials: real & simulated continued