Chapter 3, Question 3 – Solution

We are given a dataset with five predictors:

- $X_1 = \text{GPA}$
- $X_2 = \text{IQ}$
- $X_3 = \text{Level}$ (1 for College, 0 for High School)
- $X_4 = \text{Interaction between GPA and IQ}$ (i.e., $X_1 \cdot X_2$)
- $X_5 = \text{Interaction between GPA and Level}$ (i.e., $X_1 \cdot X_3$)

The fitted least squares model is:

$$\hat{Y} = 50 + 20X_1 + 0.07X_2 + 35X_3 + 0.01X_4 - 10X_5$$

where $\hat{Y}$ represents the predicted starting salary (in thousands of dollars).

(a) Which statement is correct, and why?

Let's compare average salaries for college graduates ($X_3 = 1$) and high school graduates ($X_3 = 0$), assuming fixed values of GPA ($X_1 = x_1$) and IQ ($X_2 = x_2$).

For high school graduates ($X_3 = 0$, $X_5 = 0$):

$$\hat{Y}_{HS} = 50 + 20x_1 + 0.07x_2 + 0.01(x_1 \cdot x_2)$$

For college graduates ($X_3 = 1$, $X_5 = x_1$):

$$\hat{Y}_{College} = 50 + 20x_1 + 0.07x_2 + 35 + 0.01(x_1 \cdot x_2) - 10x_1$$

Simplify:

$\hat{Y}_{College} = \hat{Y}_{HS} + 35 - 10x_1$

Difference in predicted salary:

$\hat{Y}_{College} - \hat{Y}_{HS} = 35 - 10x_1$

From this:

- If $35 - 10x_1 > 0$, then college graduates earn more on average. This occurs when $x_1 < 3.5$.
- If $35 - 10x_1 < 0$, then high school graduates earn more on average. This occurs when $x_1 > 3.5$.

Thus, **for a fixed IQ and GPA, high school graduates earn more than college graduates only if GPA exceeds 3.5**.

Therefore, the correct answer is:

iii. *For a fixed value of IQ and GPA, high school graduates earn more, on average, than college graduates provided that the GPA is high enough.*

(b) Predict the salary of a college graduate with IQ = 110 and GPA = 4.0

Given:

- $X_1 = 4.0$
- $X_2 = 110$
- $X_3 = 1$
- $X_4 = X_1 \cdot X_2 = 4.0 \times 110 = 440$
- $X_5 = X_1 \cdot X_3 = 4.0 \times 1 = 4.0$

Substitute into the model:

$Y^\wedge=50+20(4.0)+0.07(110)+35(1)+0.01(440)-10(4.0)=50+80+7.7+35+4.4-40=137.1 Y^\wedge$
$=50+20(4.0)+0.07(110)+35(1)+0.01(440)-10(4.0)=50+80+7.7+35+4.4-40=137.1$

Predicted salary = **$137,100**.

(c) True or false: Since the coefficient for the GPA/IQ interaction term is very small, there is very little evidence of an interaction effect.

False.

The size of the coefficient alone does **not** indicate statistical significance. The coefficient’s magnitude is influenced by the scale of the variables (e.g., GPA is often between 0–4, IQ around 100). Whether an effect is “small” or “large” must be assessed in the context of the data and its practical impact, and statistical significance is determined by hypothesis testing (e.g., pp-value), not merely by the coefficient’s numerical value. A small coefficient can still be statistically significant if it is precisely estimated.

Summary of Answers:

(a) **iii**
(b) **$137,100**
(c) **False** – coefficient size does not directly measure evidence of an effect.