

Chapter 4. Question 7 (Group3)

Bayes' Theorem and Normal Density: Dividend Probability Calculation

Step 1: Define Events

- Let $D = \text{Yes} \rightarrow$ company issues a dividend
- Let $D = \text{No} \rightarrow$ company does not issue a dividend
- Let $X =$ last year's percent profit

We are asked to find:

$$P(D = \text{Yes} \mid X = 4)$$

Step 2: Given Probabilities

Prior probabilities:

$$P(D = \text{Yes}) = 0.8, \quad P(D = \text{No}) = 0.2$$

Conditional distributions: Both are normal with the same variance $\sigma^2 = 36$ (so $\sigma = 6$)

- If dividend issued: $X \mid D = \text{Yes} \sim N(10, 36)$
 - If no dividend: $X \mid D = \text{No} \sim N(0, 36)$
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Step 3: Bayes' Theorem

$$P(D = \text{Yes} \mid X = 4) = \frac{f_{\text{Yes}}(4) \cdot P(D = \text{Yes})}{f_{\text{Yes}}(4) \cdot P(D = \text{Yes}) + f_{\text{No}}(4) \cdot P(D = \text{No})}$$

Step 4: Compute Normal Densities

Normal density formula:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f_{\text{Yes}}(4) = \frac{1}{\sqrt{2\pi \cdot 36}} e^{-\frac{(4-10)^2}{2 \cdot 36}} = \frac{1}{6\sqrt{2\pi}} e^{-0.5} \approx 0.066 \cdot 0.6065$$

$$f_{\text{No}}(4) = \frac{1}{6\sqrt{2\pi}} e^{-\frac{(4-0)^2}{72}} = \frac{1}{6\sqrt{2\pi}} e^{-0.2222} \approx 0.066 \cdot 0.8007$$

Step 5: Plug into Bayes' Formula

The common factor $1/(6\sqrt{2\pi})$ cancels out:

$$P(D = \text{Yes} \mid X = 4) = \frac{0.8 \cdot e^{-0.5}}{0.8 \cdot e^{-0.5} + 0.2 \cdot e^{-0.2222}}$$

Approximations:

$$e^{-0.5} \approx 0.6065, \quad e^{-0.2222} \approx 0.8007$$

$$P(D = \text{Yes} \mid X = 4) = \frac{0.8 \cdot 0.6065}{0.8 \cdot 0.6065 + 0.2 \cdot 0.8007} = \frac{0.4852}{0.6453} \approx 0.75$$

Final Answer

$P(D = \text{Yes} \mid X = 4) \approx 0.75$

Interpretation: Given a company had a 4% profit last year, there is about a 75% probability that it will issue a dividend this year.
