Forest Fire Simulation

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1. Forest fire model

Before explaining the forest fire model, let define some notations first.

- $S = \{(x, y) \in \mathbb{N}^2 | 1 \leq x \leq n \text{ and } 1 \leq y \leq n\}$ is set of points, representing the coordinate of trees.
- $B_1\big((a, b)\big) = \{(a-1, b), (a, b-1), (a, b+1), (a+1, b)\}$ is set of coordinate of **direct neighboring trees** of the tree with coordinate (a, b).
- $B_2\big((a, b)\big) = \{(x, y) \in \mathbb{N}^2 | 0 \leq |x-a| \leq 1 \text{ and } 0 \leq |y-b| \leq 1\ \}$ is set of coordinate of **diagonal neighboring trees** of the tree with coordinate (a, b).

The forest fire model can be constructed in 2 ways; depending on the definition of neighboring trees as described above. The following list describes the mechanism of the model. Note that (b) is used in direct neighboring trees definition and (c) is used in diagonal neighboring trees definition.

a. The fire begins in the center of the forest i.e. the tree with coordinate $\left(\left\lfloor\frac{n}{2}\right\rfloor, \left\lfloor\frac{n}{2}\right\rfloor\right)$
b. (direct neighboring trees) For every $t \in \mathbb{N}$, for every burning tree (x, y), neighboring trees $B_1\big((x, y)\big)$ has probability p to catching fire. Note that it takes each tree b hours to burn completely.
c. (diagonal neighboring trees) For every $t \in \mathbb{N}$, for every burning tree (x, y), neighboring trees $B_2\big((x, y)\big)$ has probability p to catching fire. Note that it takes each tree b hours to burn completely.
d. The model terminates when there is no burning tree or every tree is burnt or $t = T$ where T is the maximum time allowed in the model.

2. Experiment

The experiment is conducted by using following parameters.

Parameter	Meaning	Type	Values
Size	Size of forest	constant	50
B	Time needed to completely burn a tree	variable	1, 2,…,7
P	Probability that neighboring trees catching fire	variable	0.00,0.01,…,1.00
max_time	maximum time allowed in the model	constant	10000
nb_simus	Repeated time	constant	1000
diag_neigh	Type of neighboring trees definition	constant	False

In order to investigate the relationship between parameter b and p with the number of non-burnt trees, the provided function **variable_proba_propagation** is called 7 times, for b = 1,2,…,7. For each time other parameters i.e. size, p, max_time, nb_simus, diag_neigh are fixed as mentioned in the above table. Note that the original codes are modified in order to generate output files for more convenience data analysis.

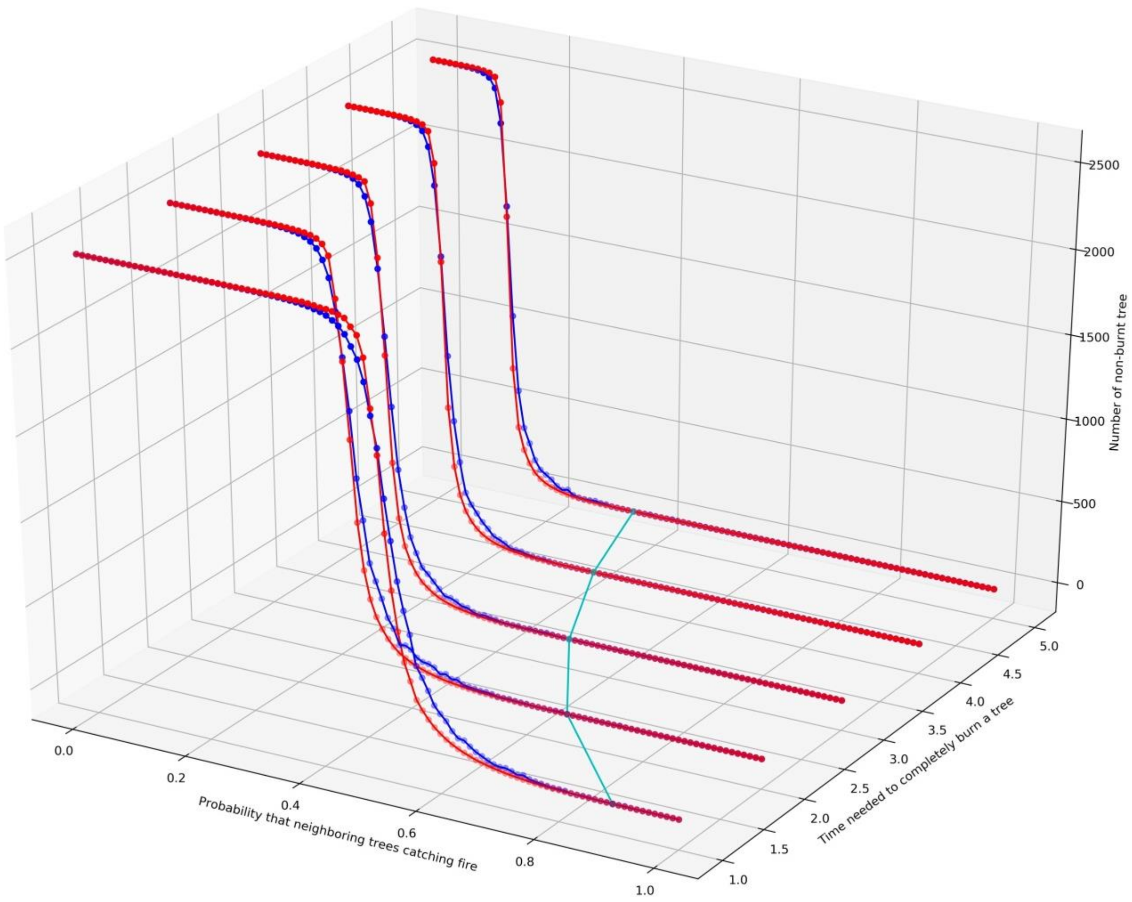
However, it is obvious that if the value of parameter **diag_neigh** is **True**, the number of non-burnt trees must be decreased for the same set of other parameters. In order to investigate this particular parameter, the above experiment is conducted once again with diag_neigh = True.

In the provided source codes, the function **multiple_simulations** returns the values of average number of non-burnt trees and its Q1, Q2, Q3. Note that Q1, Q2, and Q3 are the first, second, and third quartile in the list of number of non-burnt trees; conducted with particular set of parameters totally nb_simus rounds. In this experiment, the completely burnt forest occurs when Q2 in the list of number of non-burnt trees is zero i.e. in nb_simus rounds of simulations, more than half of them results in zero non-burnt trees.

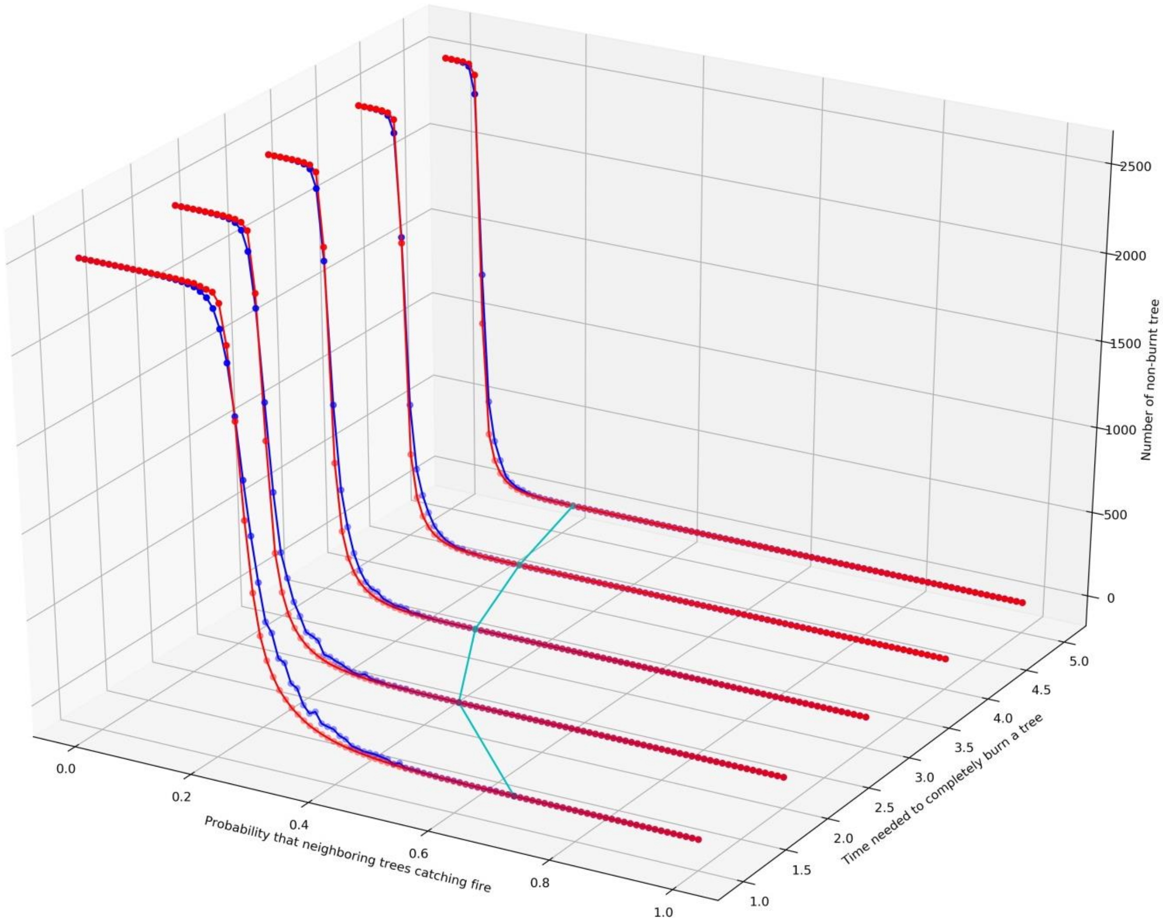
For example, $(\bar{x}, Q1, Q2, Q3) = (0.926, 0, 1, 1)$ does not represent a completely burnt forest despite the average number of non-burnt trees is lower than 1. However, $(\bar{x}, Q1, Q2, Q3) = (0.619, 0, 0, 1)$ represents a completely burnt forest.

3. Result

Let denote the number of non-burnt trees with respect to pair (p, b) as $N(p, b)$. The relationship of parameter b, p, and $N(p, b)$ with respect to **direct neighboring trees** definition is shown in graph below. The red line indicates the average value of $N(p, b)$ while the blue line indicates the corresponding Q2 in nb_simus times of simulation. The cyan line indicates the **ending probability** $p_{\text{end}}(b)$ such that if $p \geq p_{\text{end}}(b)$, then the forest is always completely burnt with respect to variable parameter b.



The relationship of parameter b, p, and $N(p, b)$ with respect to **diagonal neighboring trees** definition is shown in graph below. The red line indicates the average value of $N(p, b)$ while the blue line indicates the corresponding Q2 in nb_simus times of simulation. The cyan line indicates the **ending probability** $p_{\text{end}}(b)$ such that if $p \geq p_{\text{end}}(b)$, then the forest is always completely burnt with respect to variable parameter b.



4. Discussion

According to the S-curve graphs in the previous section, at the beginning, $N(p, b)$ for a fixed value of parameter b is roughly constant as the value of parameter p increases. This behavior continues for a certain interval of p denoted by **threshold probability** $p_t(b)$. After that, $N(p, b)$ dramatically decreases and become constant again. In the graph, there are 3 interesting observations listed below:

- Fixing p as a constant, $b \propto N(p, b)$. And, fixing b as a constant, $p \propto N(p, b)$.
- The value of threshold probability $p_t(b) \propto \frac{1}{b}$.
- The definition of neighboring trees greatly affects the value of $N(p, b)$. However, this outcome is intuitively obvious since the number of neighboring trees in diagonal definition is more than the direct one. Therefore, the value of $N(p, b)$ in later case is more than the first one.