Computer Graphics Assignment #1

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1 Question

Answer the inverse Fourier transform of the functions:

$$F_1(\omega_x, \omega_y) = \delta(\omega_x - \sqrt{3})\delta(\omega_y - 1) + \delta(\omega_x + \sqrt{3})\delta(\omega_y + 1)$$
$$F_2(\omega_x, \omega_y) = i\delta(\omega_x - \sqrt{3})\delta(\omega_y - 1) - i\delta(\omega_x + \sqrt{3})\delta(\omega_y + 1)$$

2 Answer

2.1 Equation

2.1.1 Inverse Fourier transform of F_1

$$\mathscr{F}^{-1}\{F_1(\omega_x, \omega_y)\} = \mathscr{F}^{-1}\Big\{\delta(\omega_x - \sqrt{3})\delta(\omega_y - 1) + \delta(\omega_x + \sqrt{3})\delta(\omega_y + 1)\Big\} \tag{2.1}$$
$$= \mathscr{F}^{-1}\Big\{\delta(\omega_x - \sqrt{3})\delta(\omega_y - 1)\Big\} + \mathscr{F}^{-1}\Big\{\delta(\omega_x + \sqrt{3})\delta(\omega_y + 1)\Big\} \tag{2.2}$$

By applying the property of the dirac delta function $\int_{-\infty}^{\infty} f(x)\delta(x - x_0)\, dx = f(x_0)$, we have

$$\mathscr{F}^{-1}\Big\{\delta(\omega_x - \sqrt{3})\delta(\omega_y - 1)\Big\} = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} \delta(\omega_y - 1)e^{i\omega_y y}\int_{-\infty}^{\infty} \delta(\omega_x - \sqrt{3})e^{i\omega_x x}\, d\omega_x\, d\omega_y \tag{2.3}$$
$$= \frac{1}{4\pi^2}\int_{-\infty}^{\infty} \delta(\omega_y - 1)e^{i\omega_y y}\Big(e^{\sqrt{3}ix}\Big)\, d\omega_y \tag{2.4}$$
$$= \frac{1}{4\pi^2}\Big(e^{\sqrt{3}ix}\Big)\big(e^{iy}\big) \tag{2.5}$$

and similarly,

$$\mathscr{F}^{-1}\Big\{\delta(\omega_x + \sqrt{3})\delta(\omega_y + 1)\Big\} = \frac{1}{4\pi^2}\Big(e^{-\sqrt{3}ix}\Big)\big(e^{-iy}\big) \tag{2.6}$$

Therefore, by combining result from (2.5) and (2.6) and applying the Euler's formula, we have

$$\mathscr{F}^{-1}\{F_1(\omega_x, \omega_y)\} = \frac{1}{4\pi^2}\Big(e^{\sqrt{3}ix+iy} + e^{-\sqrt{3}ix-iy}\Big) \tag{2.7}$$
$$= \frac{1}{4\pi^2}\Big(\cos\Big(\sqrt{3}x + y\Big) + i\sin\Big(\sqrt{3}x + y\Big) + \cos\Big(-\sqrt{3}x - y\Big) + i\sin\Big(-\sqrt{3}x - y\Big)\Big) \tag{2.8}$$
$$= \frac{1}{4\pi^2}\Big(\cos\Big(\sqrt{3}x + y\Big) + i\sin\Big(\sqrt{3}x + y\Big) + \cos\Big(\sqrt{3}x + y\Big) - i\sin\Big(\sqrt{3}x + y\Big)\Big) \tag{2.9}$$
$$= \frac{1}{2\pi^2}\cos\Big(\sqrt{3}x + y\Big) \tag{2.10}$$

2.1.2 Inverse Fourier transform of F_2

Since F_1 and F_2 are the same, except the coefficient i and the minus sign in the latter, we can modify the expression in (2.7):

$$\mathscr{F}^{-1}\{F_2(\omega_x,\omega_y)\} = \frac{i}{4\pi^2}\left(e^{\sqrt{3}ix+iy} - e^{-\sqrt{3}ix-iy}\right) \tag{2.11}$$

$$= \frac{i}{4\pi^2}\left(\cos\left(\sqrt{3}x+y\right) + i\sin\left(\sqrt{3}x+y\right) - \cos\left(-\sqrt{3}x-y\right) - i\sin\left(-\sqrt{3}x-y\right)\right) \tag{2.12}$$

$$= \frac{i}{4\pi^2}\left(\cos\left(\sqrt{3}x+y\right) + i\sin\left(\sqrt{3}x+y\right) - \cos\left(\sqrt{3}x+y\right) + i\sin\left(\sqrt{3}x+y\right)\right) \tag{2.13}$$

$$= -\frac{1}{2\pi^2}\sin\left(\sqrt{3}x+y\right) \tag{2.14}$$

2.2 Graph

We can visualize the inverse Fourier transform of F_1 and F_2, described in (2.10) and (2.14), by using Python.

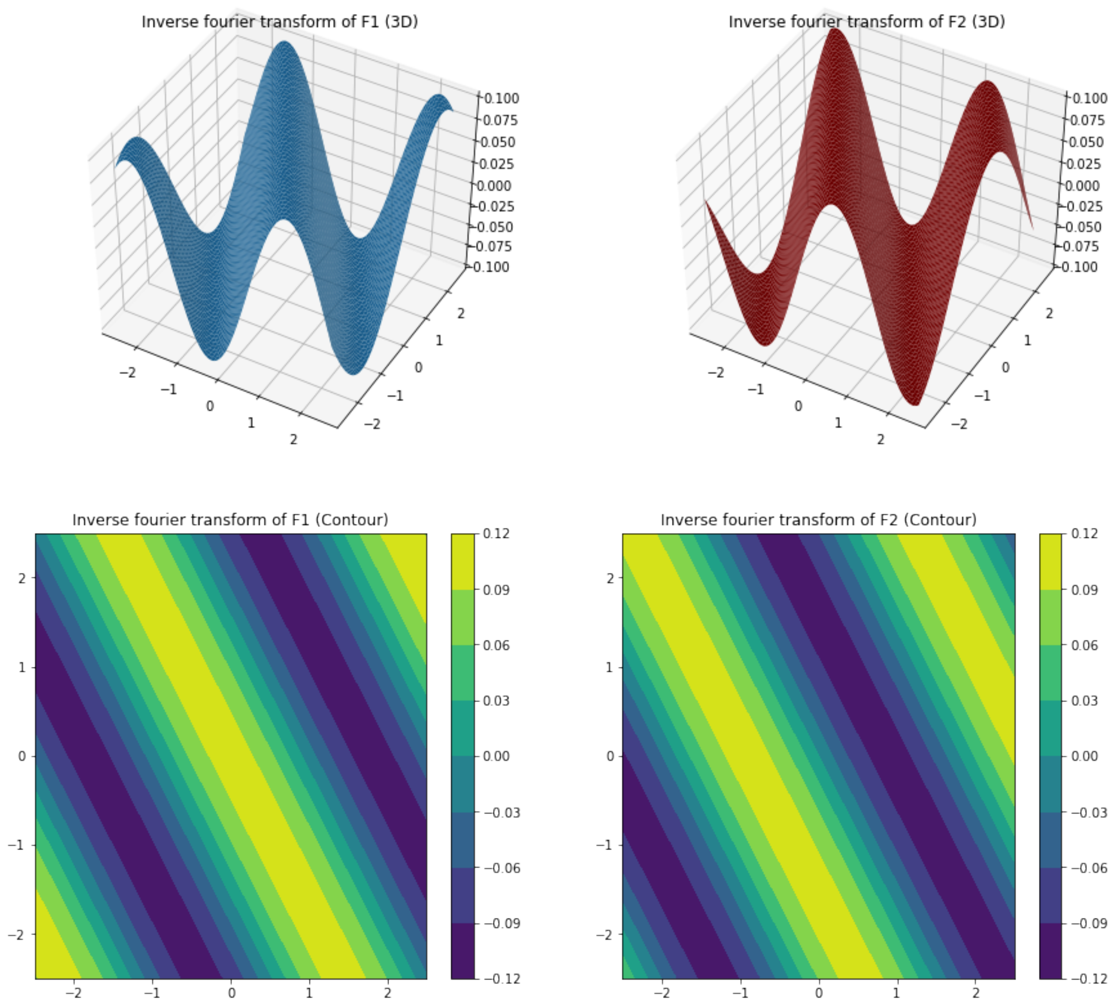

Figure 1: Visualization of $\mathscr{F}^{-1}\{F_1(\omega_x,\omega_y)\}$ and $\mathscr{F}^{-1}\{F_2(\omega_x,\omega_y)\}$

2.3 Text (Full sentences)

- $\mathscr{F}^{-1}\{F_1(\omega_x, \omega_y)\}$ is a cosine function with an amplitude of $\frac{1}{2\pi}$ in the direction of 30 degrees from x-axis. Its period is $\left(\frac{2\pi}{\sqrt{3}}, 2\pi\right)$.
- Similarly, $\mathscr{F}^{-1}\{F_2(\omega_x, \omega_y)\}$ is a sine function with an amplitude of $\frac{1}{2\pi}$ in the direction of 30 degrees from x-axis. Its period is $\left(\frac{2\pi}{\sqrt{3}}, 2\pi\right)$. It is also reflected over the unit vector $\left(\sqrt{3}, 1\right)$.