Computer Graphics Assignment #3

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1 Question

1. Explain the aliasing problem caused by discretization with an image prepared by yourself.
    - Prepare one image whose size is 512×512.
    - Using "SignalFreq" program, produce an image that has aliased texture with resampling.
    - Explain the Nyquist frequency in the case of the resampling.
2. Explain one anti-aliasing technique to remove the alias on the image produced in the above.
    - Design an appropriate filter and describe it by equations in signal and frequency domains.
    - Explain the procedure of the anti-aliasing, which uses the filter.
    - Explain the reason why the aliasing is removed by the procedure.

In both answers, include efficient images or graphs to help the understanding of the explanations.

2 Answer

2.1 Aliasing and Nyquist Frequency

Given the delta function series $\delta_T(x)$, the fourier transform of the discrete sampling function $f_s(x)$ can be derived by Convolution theorem:

$$\begin{aligned}
\mathscr{F}\{f_s(x)\} &= \int_{-\infty}^{\infty} f(x)\delta_T(x)e^{-i\omega x}\,dx && (2.1)\\
&= F(\omega) * \mathscr{F}\left\{\sum_{n=-\infty}^{\infty} \delta(t-nT)\right\} && (2.2)\\
&= f^* \sum_{n=-\infty}^{\infty} F(\omega - nf^*) && (2.3)
\end{aligned}$$

where $f^* = 1/T$ denotes the Nyquist frequency.

From equation (2.3), we can observe that if $F(\omega) > 0$ when $\omega > f^*/2$, each copy of F will overlap with each other. This situation leads to a phenomenon called *aliasing*, where we cannot distinguish low-frequency components from high-frequency ones due to overlapping in the frequency domain. For example, consider an image with its frequency domain representation as shown in Figure 1a. When we increase sampling interval (or equivalently reduce sampling rate f^*), aliasing happens as illustrated in Figure 1b, especially on the roof tiles below the nameplate of the station.

(a) An original 512×512 image with its frequency domain representation

(b) Aliasing happens with a lower Nyquist frequency f^*

Figure 1: Relationship between aliasing effect and Nyquist frequency

2.2 Anti-aliasing

Suppose that the sampling rate is fixed, we can design a low-pass filter to remove the aliasing on the image. For example, we can use a Gaussian filter (2.4a) and (2.4b) to smooth the image, assuming that $\sigma_x = \sigma_y = \sigma$.

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{2.4a}$$

$$\mathscr{F}\{G(x, y)\} = e^{-\frac{\sigma^2}{2}\left(\omega_x^2+\omega_y^2\right)} \tag{2.4b}$$

We apply the filter to an image by performing convolution between both of them. Or equivalently, sliding a finite representation of kernel across the image and adding results together in a discretized space. The principle behind this procedure is smoothing and removing high-frequency components from the image. Given that the standard deviation σ is chosen appropriately, it is possible to reduce high-frequency components such that $F(\omega) = 0$ when $\omega > f^*/2$; thus, mitigating the aliasing effect. Figure 2 demonstrate the application of the Gaussian filter.

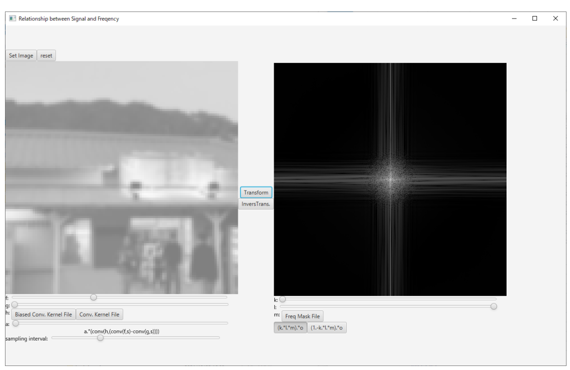

Figure 2: The smoothed image with less aliasing and its frequency domain representation