Solutions to Practice Final

$$f(x) = \begin{cases} \dfrac{-28}{x+3} & \text{if } x \le 1 \\ \\ \dfrac{35}{x-6} & \text{if } x > 1 \end{cases}$$

$\lim_{x \to 1^-} f(x) = -7$ because you can evaluate the top since it's to the left of 1.

$$\frac{-28}{1+3} = \frac{-28}{4} = -7$$

$\lim_{x \to 1^+} f(x) = -7$ also by plugging into bottom expression

$f(1) = \dfrac{-28}{1+3} = -7$ also.

Since the above are all defined and equal, we know that f is continuous at $x=1$.

Not continuous at $x = -3, 6$ due to the denominators being zero

$$\lim_{h \to 0} \frac{10\sin(9+h) - 10\sin(9)}{h}$$

$f(x) = 10 \sin x$

$a = 9$

p. 273 #153

$s(t) = -16t^2 + 560t$

$s'(t) = v(t) = -32t + 560$

$v(3) = -32(3) + 560 = -96 + 560 = 464 \ m/s$

$v'(t) = a(t) = -32$ ← the acceleration is constantly $-32 \ m/s^2$.

$$\begin{array}{r} -32 \\ \times \ 3 \\ \hline -96 \end{array} \qquad \begin{array}{r} 560 \\ -96 \\ \hline 464 \end{array}$$

p. 332 #354

$P(t) = 500{,}000(1 + 5\%)^t$
$= 500{,}000(1.05)^t$

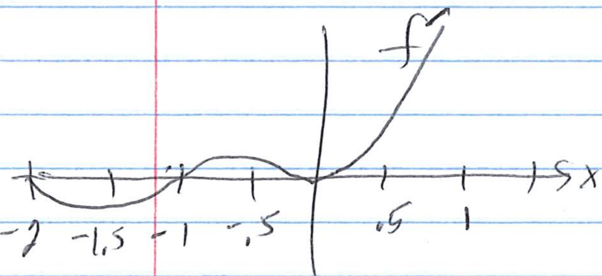
$P'(t) = 500{,}000(1.05)^t \ln 1.05$
$= 24395.08 \, (1.05)^t$

is the rate of people/year

When $t = 10$

$P'(10) = 500{,}000(1.05)^{10} \ln 1.05$
$= 39{,}737$ people/year growth

p. 404 #206



a) f is increasing where f' is above the x-axis:

$-1 < x < 0 \ \cup \ x > 0$

decreasing:

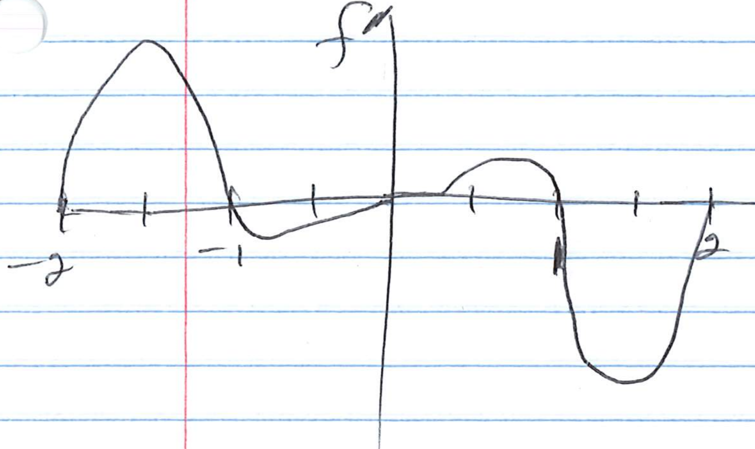
$-2 < x < -1$

b) extrema, first find critical points, where $f' = 0$

$x = -2, -1, 0$

	dec		inc		inc	
f'	−		+		+	
		−2		−1 MIN		0 ← neither min nor max

p. 404 #207



a) f is increasing

$-2 < x < -1, \; 0 < x < 1$

f is decreasing

$-1 < x < 0, \; 1 < x < 2$

b) critical points at

$x = -2, -1, 0, 1, 2$

INC DEC INC DEC

$+ \quad - \quad + \quad -$

$-2 \quad -1 \quad 0 \quad 1 \quad 2$

MAX MIN MAX

p. 470 #379

Use L'Hôpital's Rule

$$\lim_{x \to 1} \frac{x-1}{\ln x} = \lim_{x \to 1} \frac{1}{1/x} = \lim_{x \to 1} x = 1$$

p. 437 #265

$$\lim_{x \to -\infty} \frac{(x^4 - 4x^3 + 1)/x^4}{(2 - 2x^2 - 7x^4)/x^4}$$ divide top and bottom by x^4

$$\lim_{x \to -\infty} \frac{1 - 4/x + 1/x^4}{2/x^4 - 2/x^2 - 7} = \frac{1 - 0 + 0}{0 - 0 - 7} = -\frac{1}{7}$$

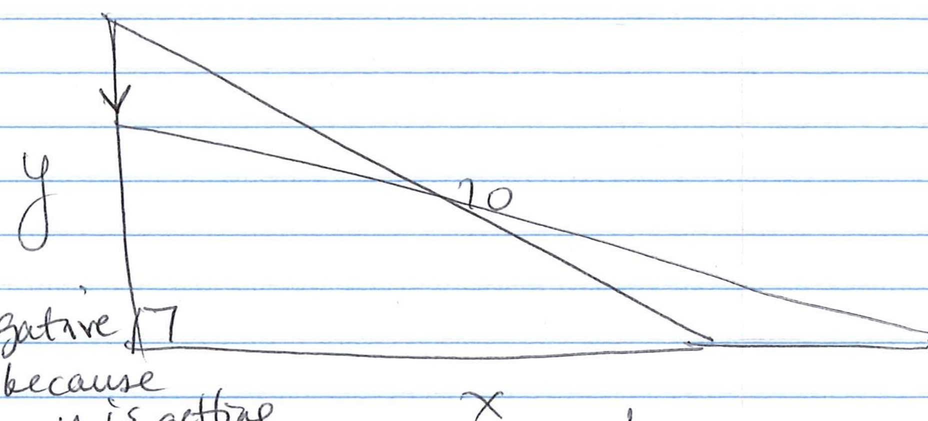
p. 437 #266

$$\lim_{x \to \infty} \frac{3x \;/ x}{\sqrt{x^2+1} \;/ \sqrt{x^2}}$$

$\sqrt{x^2} = x$, so you are doing the same to top and bottom

$$\lim_{x \to \infty} = \frac{3}{\sqrt{1 + \frac{1}{x^2}}} = \frac{3}{\sqrt{1}} = \frac{3}{1} = 3$$

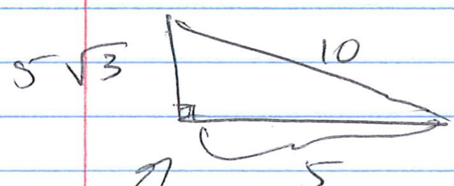
p. 350 #5



$\frac{dy}{dt} = -2 \text{ ft/sec}$

negative because y is getting smaller in length

$\left.\frac{dx}{dt}\right|_{x=5} = ?$



not drawn to scale

$$x^2 + y^2 = 10^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

when $x = 5$

$$2(5)\frac{dx}{dt} + 2(5\sqrt{3})\left(-2\frac{ft}{sec}\right) = 0$$

$$10\frac{dx}{dt} - 20\sqrt{3} = 0$$

$$\frac{dx}{dt} = \frac{+20\sqrt{3}}{10} = +2\sqrt{3} \text{ ft/sec}$$

Problem under ladder problem:

a) $f' = \frac{1}{4}(4-x^2)\underbrace{e^{-x^2/8}}_{\text{This is always positive since } e^{-x^2/8} = \frac{1}{e^{x^2/8}}} = 0$ to find critical points

So, $4-x^2$ must equal 0 for f' to be zero

$4-x^2=0$

$x^2=4$

$x=\pm\sqrt{4}=\pm 2$

f': $-$ | $+$ | $-$

-2 MIN, 2 MAX

Use f'' to see if they are max or min

$f''(-2) = \frac{1}{16}((-2)^3-12(-2))e^{-4/8}$

$= \frac{1}{16}(-8+24)e^{-1/2}$

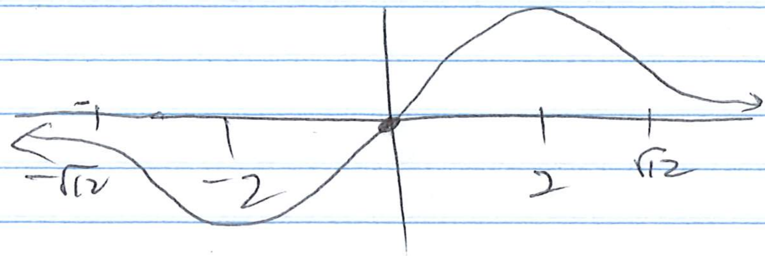
$= e^{-1/2} = \frac{1}{\sqrt{e}}$ which is ~~always~~ positive

So at $x=-2$ we have a min because concavity is positive.

$f''(2) = \frac{1}{16}(2^3-12(2))e^{-4/8}$

$= -e^{-1/2}$ ~~also~~ is negative, so MAX at $x=2$

~~possible~~ Graph:



$y=0$ is the horizontal asymptote

2nd inflection points @

$$f''(x)=0=\frac{1}{16}\left(x^3-12x\right)e^{-x^2/8}$$

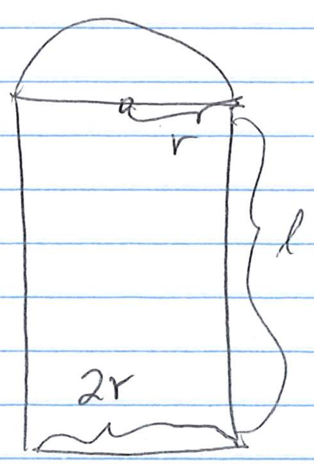
$$0=\underbrace{x\left(x^2-12\right)}_{\text{So what makes this part }=0?}\underbrace{e^{-x^2/8}}_{\text{always positive}}$$

$X=0,\ +\sqrt{12},\ -\sqrt{12}$

$\sqrt{12}\approx 3.5$

p. 453 # 35

Things to assume: 1.) "The window" is the semicircle and the rectangle



2.) The square panes below need not go down 3. It could be ~~more~~ >3, or <3, so let's call the length l.

20ft border does not change.
However the area inside it needs to be maximized.

Border $= 20 = \pi r + 2l + 2r$

πr ↑ $\frac{1}{2}$ circle; $2l$ ↑ two lengths; $2r$ ↑ bottom edge

Area $= A = \frac{1}{2}\pi r^2 + l(2r)$

Let's rewrite A without l; let's ~~stop~~ replace l with something in terms of r.

$$20 = \pi r + 2l + 2r$$

$$20 - \pi r - 2r = 2l$$
$$20 - r(\pi + 2) = 2l$$
$$10 - \frac{r}{2}(\pi + 2) = l = 10 - \frac{r\pi}{2} - r$$

$$A = \frac{1}{2}\pi r^2 + \left(10 - \frac{r\pi}{2} - r\right)(2r)$$
$$= \frac{\pi}{2} r^2 + 20r - \pi r^2 - 2r^2$$
$$= -\frac{\pi}{2} r^2 + 20r - 2r^2$$

What are the endpoints for $A(r)$?

When $r = 0$ the radius is min.

When $r = ?$ the radius is max. → If $l = 0$,

$r = \frac{20}{\pi + 2}$ because of the very top equation

So, we have our max and min, now we need to find any critical point(s).

$$A' = 2\left(-\frac{\pi}{2}\right)r + 20 - 4r = -\pi r + 20 - 4r$$

let $A' = 0$ $\qquad 0 = -\pi r + 20 - 4r$

$$-20 = -r(\pi + 4)$$

$$\frac{20}{\pi+4} = r$$ is our only critical point.

Testing $r = 0, \frac{20}{\pi+2}, \frac{20}{\pi+4}$, $\qquad A = -\frac{\pi}{2}r^2 + 20r - 2r^2$

$$A(0) = 0$$

$$A\left(\frac{20}{\pi+2}\right) = -\frac{\pi}{2}\left(\frac{20}{\pi+2}\right)^2 + 20\left(\frac{20}{\pi+2}\right) - 2\left(\frac{20}{\pi+2}\right)^2$$

$$= -\left(\frac{20}{\pi+2}\right)^2\left(\pi/2 + 2\right) + \frac{20^2}{\pi+2} = 23.768$$

$$A\left(\frac{20}{\pi+4}\right) = -\frac{\pi}{2}\left(\frac{20}{\pi+4}\right)^2 + 20\left(\frac{20}{\pi+4}\right) - 2\left(\frac{20}{\pi+4}\right)^2$$

$$= -\left(\frac{20}{\pi+4}\right)^2\left(\frac{\pi}{2} + 2\right) + \frac{20^2}{\pi+4} = 28.005$$ ✓

GIVES MAX AREA

$$\int (x-1)(x+1)\,dx$$

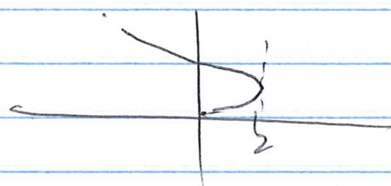
$$= \int x^2 - 1\,dx$$

$$\frac{x^3}{3} - x + C$$

p. 331 #35b (I regret giving you this one since it requires a graphing calculator.)

I gave you this only because it requires implicit differentiation.

Graph:



The tangent line at $x=2$ looks as though it's a vertical line. I cannot verify that algebraically.

Let's show how to find $\frac{dy}{dx}$:

$$x^3 - x\ln y + y^3 = 2x+5$$

$$3x^2 \cancel{\frac{dx}{dx}} - \underbrace{\left(\ln y + x\,\frac{1}{y}\boxed{\frac{dy}{dx}}\right)}_{\text{product rule}} + 3y^2\boxed{\frac{dy}{dx}} = 2\cancel{\frac{dx}{dx}}$$

$$3x^2 - \ln y - \frac{x}{y}\boxed{\frac{dy}{dx}} + 3y^2\boxed{\frac{dy}{dx}} = 2$$

$$-\frac{x}{y}\boxed{\frac{dy}{dx}} + 3y^2\boxed{\frac{dy}{dx}} = 2 - 3x^2 + \ln y$$

$$\boxed{\frac{dy}{dx}}\left(3y^2 - \frac{x}{y}\right) = 2 - 3x^2 + \ln y$$

$$\frac{dy}{dx} = \frac{2 - 3x^2 + \ln y}{3y^2 - \frac{x}{y}}$$