Obligatory Excercise 2 2014

TMA4275 - Lifetime analysis
NTNU

10057

April 4, 2014

a)

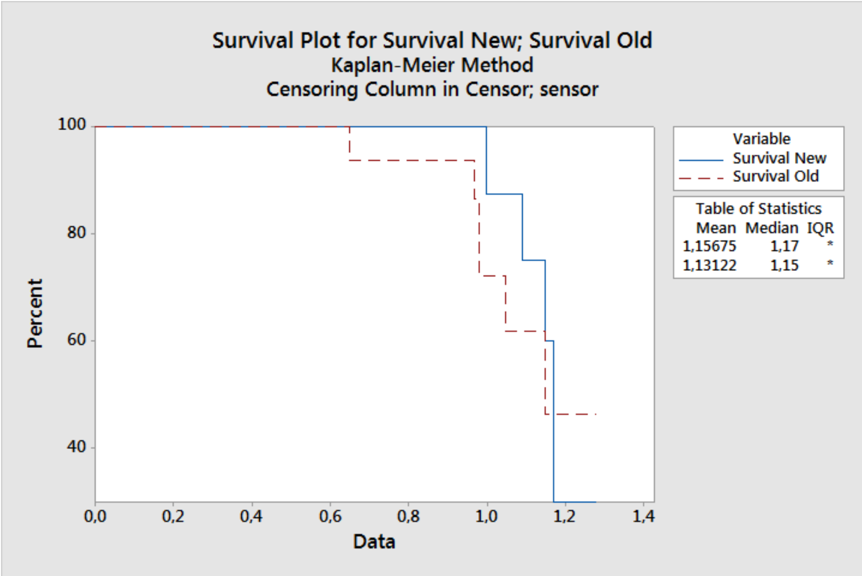


Figure 1: A figure of the survival plot for new and old tires

As we can see on the figure the median and the mean of the new tires are slightly higher than for the old tires. And by the log-Rank test done by minitab, we can conclude that the old tires are not as good as the new tires, if we use 95 % confidence.

b)

Regression Table

		Standard			95,0%	Normal CI
Predictor	Coef	Error	Z	P	Lower	Upper
Intercept	-5,18665	2,84774	-1,82	0,069	-10,7681	0,394816
Tire age	-0,0560298	0,0717152	-0,78	0,435	-0,196589	0,0845294
Wedge gauge	0,618075	0,233062	2,65	0,008	0,161282	1,07487
Interbelt gauge	0,677893	0,305756	2,22	0,027	0,0786229	1,27716
EB2B	0,735087	0,529309	1,39	0,165	-0,302340	1,77251
Peel force	1,97579	0,780630	2,53	0,011	0,445789	3,50580
Carbon black	2,79784	2,07499	1,35	0,178	-1,26906	6,86475
W x P	-1,23881	0,475896	-2,60	0,009	-2,17155	-0,306075
Shape	16,7698	4,32613			10,1144	27,8044

Table 1: Regression with Life Data, from minitab.

With Log-Likelihood $= 10,056$. We see that wedge gauge, Interbelt gauge, Peel force and W x p are significant covariants on a 5 % level. The shape-parameter is bigger than one, suggesting that the failure rate increases over time, which seems reasonable when comparing to figure 1.

The values in our table are quite different from the ones in the article, probably because of the difference in models. But some of the p-values and the z and t values in the different models are quite similar. Also the significant covariants are the same.

c)

Predictor	Coef	Standard Error	Z	P	95,0% Lower	Normal CI Upper
Intercept	-1,35968	0,316173	-4,30	0,000	-1,97937	-0,739994
Wedge gauge	0,570659	0,184221	3,10	0,002	0,209593	0,931725
Interbelt gauge	0,467936	0,144704	3,23	0,001	0,184322	0,751551
Peel force	1,62726	0,452442	3,60	0,000	0,740486	2,51403
W x P	-1,09370	0,292382	-3,74	0,000	-1,66676	-0,520641
Shape	16,7486	4,12438			10,3364	27,1387

Table 2: Regression with Life Data with non-significant data taken out, from minitab.

Log-Likelihood $= 6,621$. Compared to table 3 in the article all values are quite different. Probably because of the difference in model.

d)

	Lower quartile	median	Upper quartile	1.0	1.2	1.5
Bad tire	0.84	1.12	1.50	0.83	0.24	0,00
Good tire	1.12	2.63	6,20	0.99	0.99	0.99

Table 3: Table with interesting values for good and bad tires.

So clearly a "good tire" lasts longer than a "bad tire".

e)

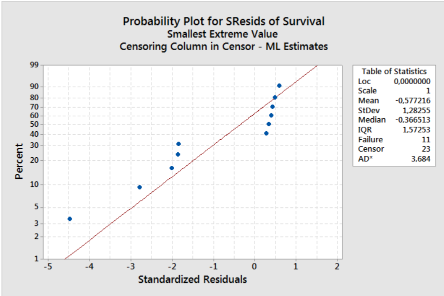
We now want to compare the model in section b) with the model in section c) and the Weibull model using no covariants. For this we want compare the log-likelihood values.

H_0 : models are the same versus H_1 : the models are different.

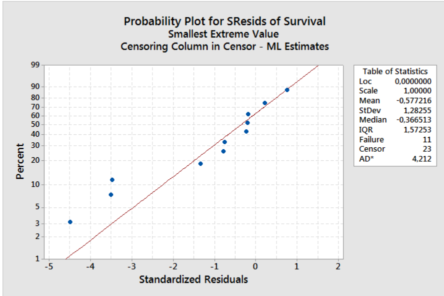
Comparing the models in b) and c) yields

$\chi^2_4 = 9.49$ on a significant level of $\alpha = 0.05$. Twice difference in log-likelihood is $2(10.056 - 6.621) = 6.87$. $P(\chi^2_5 > 6.87) = 0.15$ And we therefore do not reject H_0. And conclude that the reduced model from c) is sufficient when estimating the lifetime of the tires.

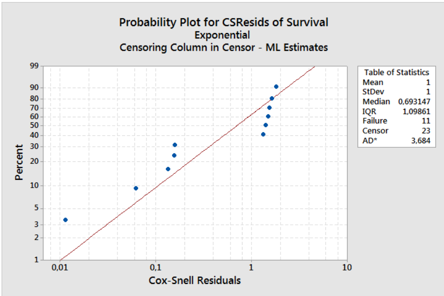
In the model with no covariants minitab gives log-likelihood $= -4.958$. $2(10.056-(-4.958)) = 30.028$, $P(\chi^2 > 30.028) < 0.001$. We therefore reject the hypothesis that the model without covariants, and the model with all covariants are equal.



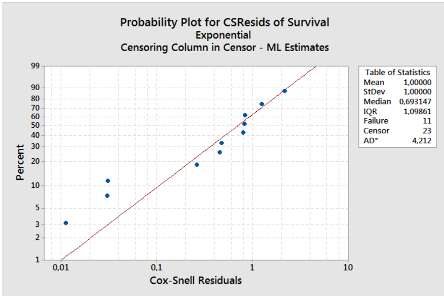
(a) Residual standard plot of the full model



(b) Residual standard plot of the reduced model



(c) Residual Cox-Snell plot of the full model



(d) Residual Cox-Snell plot of the reduced model

Figure 2: Plots of reciduals

f)

The Cox-model gives the following expression

$$z_0(t;x) = z_0(t)e^{\beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4}$$

The main difference between the cox model from the reduced model and the full model is the number of β-s. But the models should give the same probability for the same time. The hazard-rate for the Weibull-regression is

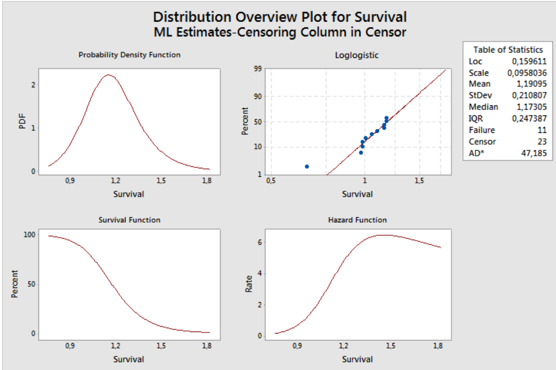
$$z_0(t) = z_0(t)e^{-\alpha(\beta_1 x_1 + \beta_2 * x_2 + \beta_3 x_3 + \beta_4 x_4)}$$

So we see that the difference is the "$-\alpha$" term. Which only means that the β in one is the same as $-\alpha\beta$ in the other. The estimates for β-s in the Weibull regression model is given in table 1 and 2, for the full, and reduced model.

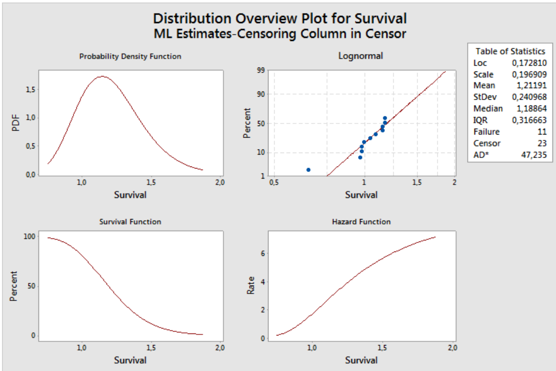
g)

When deriving the Cox-Snell residuals, we start by taking the logarithm of the survival function to the model we use, which is weibull in this case.

$$\ln T_i = \beta_0 + \sum_{k=1}^{n} \beta_k x_k + \sigma U_i$$



(a) loglogistics plot over interesting data



(b) LogNormal plot over interesting data

Figure 3: Plots of different fitted modells.

$$R_{T_i} = 1 - \phi(\frac{\ln t - \beta_0 - \sum_{k=1}^{n} \beta_k x_k}{\sigma})$$

$$V_i = -\ln R_{T_i}(T_i) = -\ln[1 - \phi(\frac{\ln t - \beta_0 - \sum_{k=1}^{n} \beta_k x_k}{\sigma})]$$

For the Weibull -distribution we have

$$\ln T = \beta_0 + \sum_{i=1}^{k} + \frac{1}{\alpha} W$$

which is the same as for the cox-model, and the of course gives the same residuals. This explains why the residual plots are the same for the two models. I also think the reduced model gives a better fit for the model.

When comparing all the plots from plot 2 and plot 3, I would say the reduced cox-snell/weibull gives the best estimate. Loglogistics and lognormal gives a distinct "s"-shape, which means that the model is probably wrong.

h)

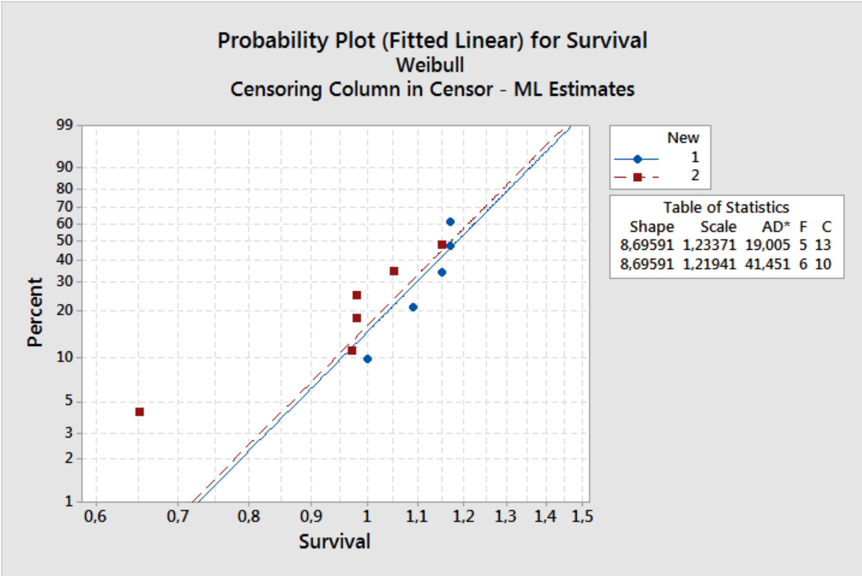


Figure 4: Probability plot for accelerated lifetime testing

From plot 4 we can see that the reduced model gives a better approximation for the regression than the full model, since it is closer to the line. We also have one point way of in the full model.