This is the Title of my Thesis

Your Name

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PROJECT / MASTER THESIS

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0.1 Introduction

We tried to find the time estiamte for the signal to be transmitted in several different ways. A good way to start was by finding a time scale.

0.2 Modeling a time scale

We wanted to find a scaling for time on order to get a feel about the time it takes before a signal is transmitted. In order to do that, we use the diffusion equation

$$\frac{\partial c^*}{\partial t^*} = \kappa \Delta c^*.$$

Since we were given a radius, and a height, we chose to use cylindrical coordinates, thus Δ becomes

$$\Delta f = \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial f}{\partial r^*}\right) + \frac{1}{(r^*)^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial (z^*)^2}.$$

We scale the equation so that

$$\begin{aligned} c^* &= Cc, \\ r^* &= Rr, \\ z^* &= hz, \\ t^* &= Tt. \end{aligned}$$

With the scaling the equation becomes

$$\frac{\partial Cc}{\partial Tt} = \kappa\left(\frac{1}{Rr}\frac{\partial}{\partial Rr}\left(Rr\frac{\partial Cc}{\partial Rr}\right) + \frac{1}{(Rr)^2}\frac{\partial^2 Cc}{\partial \phi^2} + \frac{\partial^2 Cc}{\partial (hz)^2}\right).$$

We assume that the neurotransmitters are released in the centre of the axon, because of rotational symmetry c becomes independent of ϕ. Since $R >> h$ it follows that $1/R^2 << 1/h^2$, and we assume $1/R^2 \approx 0$. After these simplifications the equation becomes

$$\frac{1}{T}\frac{\partial c}{\partial t} = \kappa\frac{1}{h^2}\frac{\partial^2 c}{\partial z^2}.$$

To simplify we decide that all derivatives are ~ 1, thus we obtain

$$\frac{1}{T} = \kappa\frac{1}{h^2}.$$

Solving for T gives us a timescale

$$T = \frac{h^2}{\kappa} = \frac{(15 \cdot 10^{-9}\text{m})^2}{0.3 \cdot 10^{-12}\text{m}^2/\text{s}} = 0.75\text{ms}.$$

This is by no means a correct time for the signal to be transmitted, but we could guess that the real time will be of order 1ms.

0.3 2D diffusion equation

Domain, initial and boundary values

Initially we chose a random point where we released all the neurotransmitters. The initial function for the concentration of neurotransmitters is zero everywhere except from that point. On the boundary we assumed that the particles could freely diffuse outside of the domain.
To simplify the diffusion equation we neglect the height of the synaptic cleft. Furthermore, we chose to view the resulting disc as a square with size $4r^2$, due to the difficut nature of the laplacian in cylindrical coordinates. We assume that the receptors are equally distributed over the square.
If the number of bounded receptor doesn't change over a time interval Δt, we assume equilibrium and that a signal is being transmitted.

Numerical scheme

We used the finite difference method to discretize the modelling equations. OBS!(ref.)

$$c_{j,k}^{i+1} = c_{j,k}^{i} + \frac{\Delta t}{2}\left[\kappa\left(\frac{c_{j+1,k}^{i} - 2c_{j,k}^{i} + c_{j-1,k}^{i}}{(\Delta y)^2} + \frac{c_{j,k+1}^{i} - 2c_{j,k}^{i} + c_{j,k-1}^{i}}{(\Delta x)^2}\right) - k_1 c_{j,k}^{i} P_{j,k}^{i} + k_2(1 - P_{j,k}^{i})\right]$$

$$P_{j,k}^{i+1} = P_{j,k}^{i} + \Delta t\left[- k_1 c_{j,k}^{i} P_{j,k}^{i} + k_2(1 - P_{j,k}^{i})\right]$$

Results

Using the values given in **?** with $k_1 = 10^4$ and $k_2 = 10$, and

- number of steps in x- and y-direction, $nx = ny = 10$
- number of time steps $nt = 4 \cdot 10^5$, simulated over 1 second

This results in a signaling time of $t_s = 33.8$ms. The following figures shows the distribution of neurotransmittors and free receptors at the signaling time.

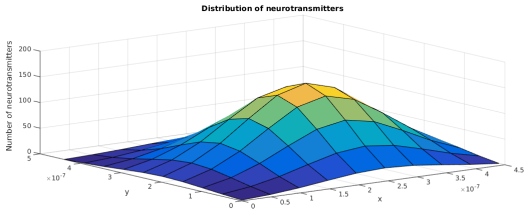

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Figure 1: Neurotransmitters

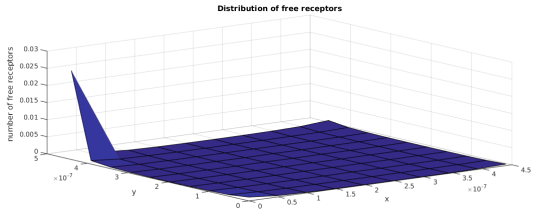

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Figure 2: Receptors

Figure 3: Distribution at signal time.

Discussion

As we can see from figure **??** practically all receptors are bounded at this time, so this may be considered an upper bound for the signaling time. This is a two dimensional model, and thus lacs a certain accuracy. Given more time the model could have been extended to three dimensions, but as a simple representation of what happens during neurotransmission in the synaptic cleft, we consider this model is applicable.

Given more time, this model could also have been used to model the clearance time.