

CLASSIFICATION

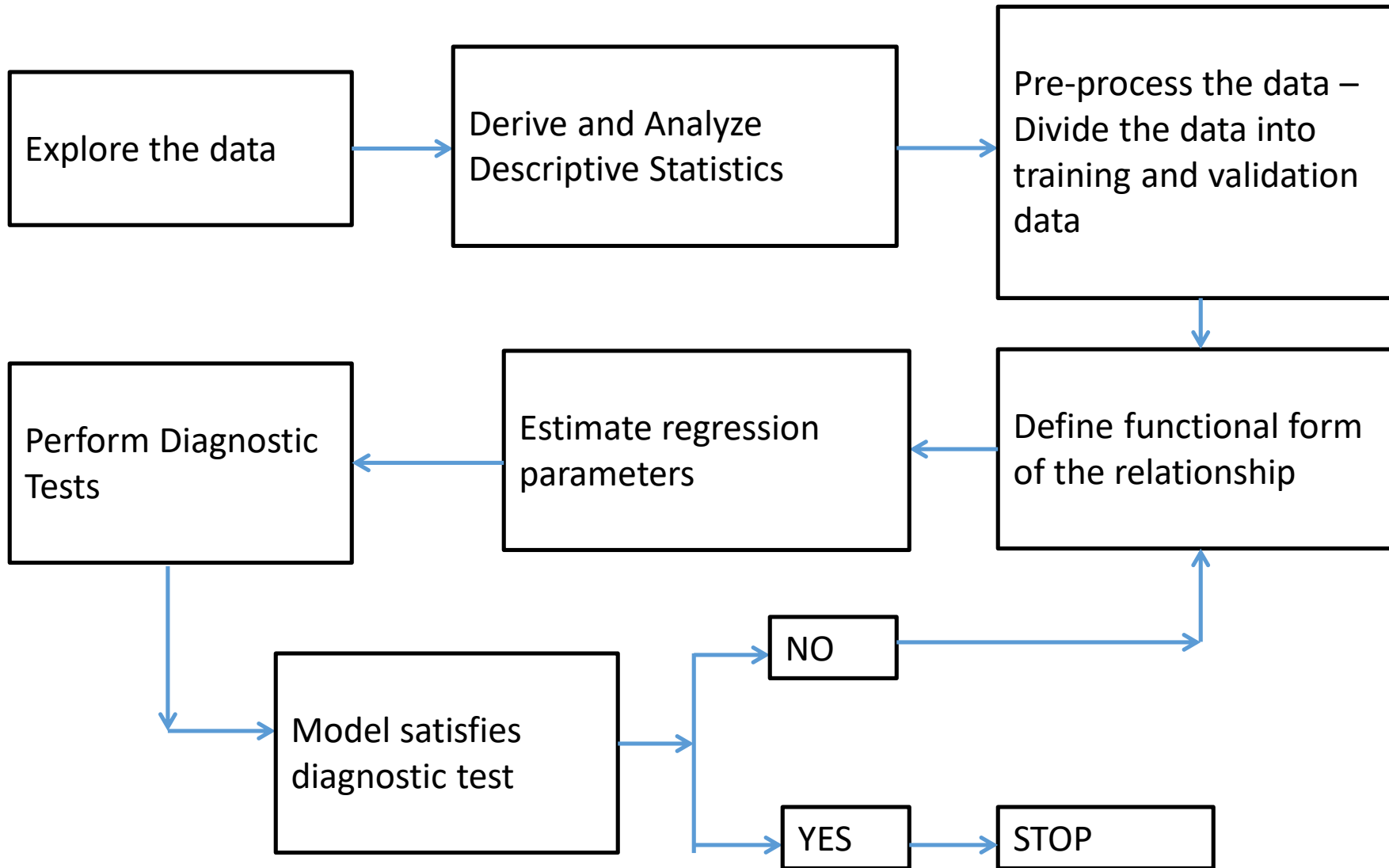
LOGISTIC REGRESSION

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Logistic Regression - Introduction

- Logistic regression models estimate how probability of an event may be affected by one or more explanatory variables.
- Logistic regression is a technique used for predicting “**class probability**”, that is the probability that the case belongs to a particular class.
- Binomial (or binary) logistic regression is a model in which the dependent variable produces 2 outputs.
- In multinomial logistic regression model, the dependent variable can take more than two values.
- Linear Regression v/s Logistic Regression

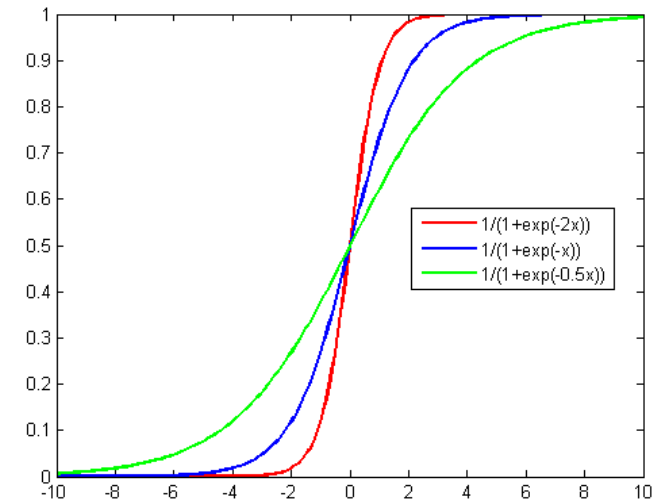
Logistic Regression Model Development



Logistic Regression - Introduction

- The name logistic regression emerges from logistic distribution function.

$$\frac{e^Z}{1 + e^Z}$$



- Mathematically, logistic regression attempts to estimate conditional probability of an event (or class probability).

Logistic Function (Sigmoidal function)

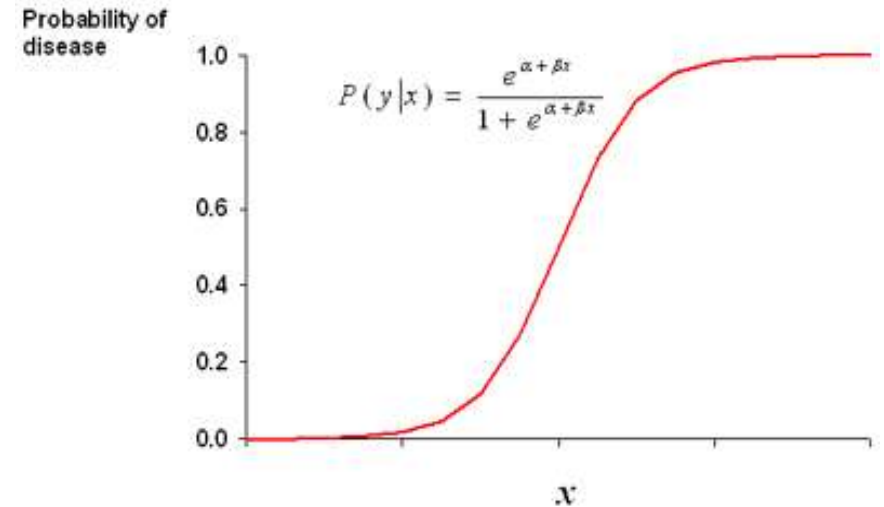
$$P(Y = 1) = \pi(z) = \frac{1}{1 + e^{-z}} = \frac{e^z}{1 + e^z}$$

$$z = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n$$

Logistic Regression with one Explanatory Variable

$$P(Y = 1 | X = x) = \pi(x) = \frac{\exp(\alpha + \beta x)}{1 + \exp(\alpha + \beta x)}$$

- $\beta = 0$ implies that $P(Y|x)$ is same for each value of x
- $\beta > 0$ implies that $P(Y|x)$ increases as the value of x increases
- $\beta < 0$ implies that $P(Y|x)$ decreases as the value of x increases



Logistic Transformation

- The logistic regression model is given by:

$$\pi_i = \frac{e^{(\beta_0 + \beta_1 X_i)}}{1 + e^{(\beta_0 + \beta_1 X_i)}}$$

$$\frac{\pi_i}{1 - \pi_i} = e^{(\beta_0 + \beta_1 X_i)}$$

$$P(\text{Heart Disease} = 1) = \frac{e^{\beta_0 + \beta_1 \text{Age}}}{1 + e^{\beta_0 + \beta_1 \text{Age}}}$$

$$\ln\left(\frac{\pi_i}{1 - \pi_i}\right) = \beta_0 + \beta_1 X_i$$

Function with linear properties (Link Function)

Logistic Regression of challenger data

Let:

- $Y_i = 0$ denote no damage
- $Y_i = 1$ denote damage to the O-ring
- $P(Y_i = 1) = \Pi_i$ and $P(Y_i = 0) = 1 - \Pi_i$.
- We have to estimate $P(Y_i = 1 | X_i)$.

Variables in the Equation

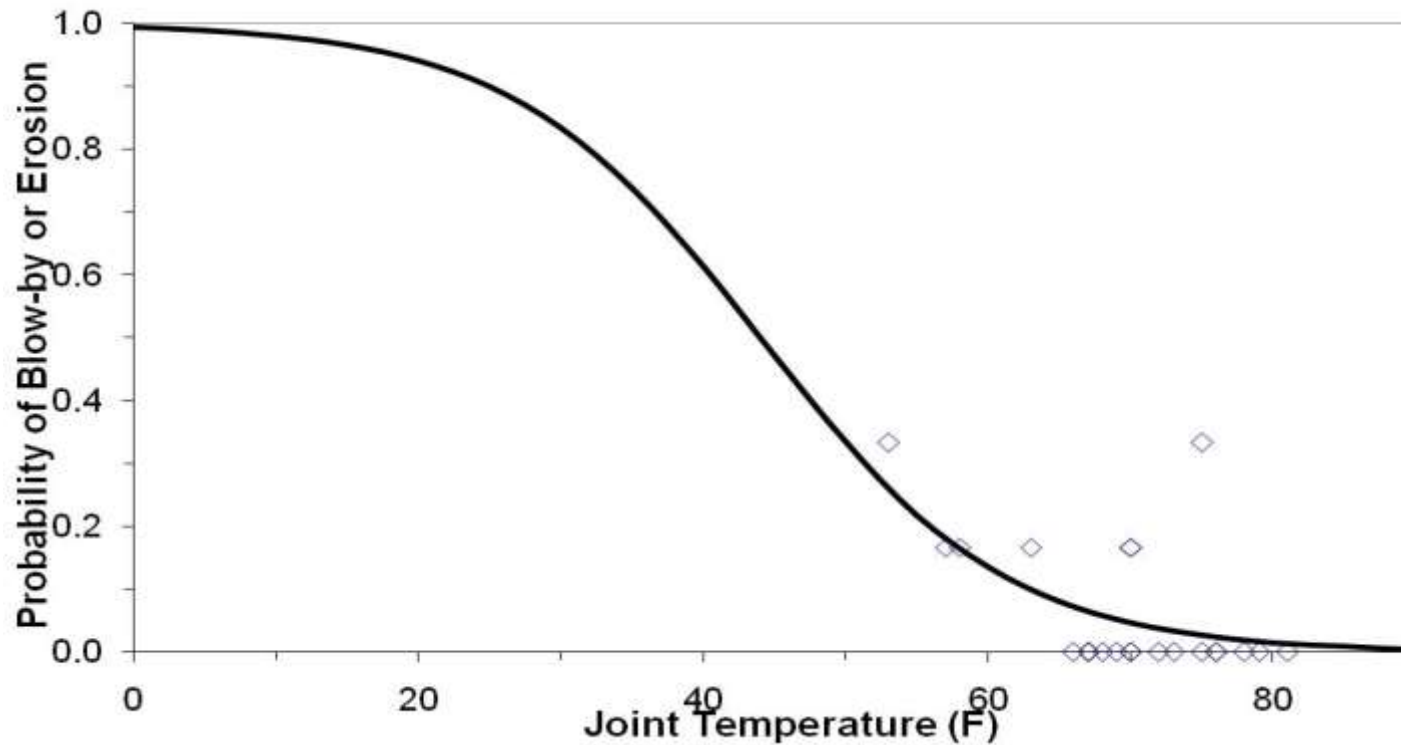
		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1 ^a	LaunchTemperature	-.236	.107	4.832	1	.028	.790
	Constant	15.297	7.329	4.357	1	.037	4398676

a. Variable(s) entered on step 1: LaunchTemperature.

$$\ln\left(\frac{\pi_i}{1 - \pi_i}\right) = 15.297 - 0.236X_i$$

Probability of failure estimate

$$\pi_i = \frac{e^{15.297 - 0.236 X_i}}{1 + e^{15.297 - 0.236 X_i}}$$



Using Logistic Regression Predict the class label for the following dataset as Positive or Negative at Cut-off probability 0.5, & parameters $B_0 = -42.54$, $B_1 = 2.95$ and $B_2 = 10.4$,

Observation#	1	2	3	4	5
X1	1	2	3	4	5
X2	3	3	4	4	3
Y (class label)	?	?	?	?	?

1. $0.00001237 = -ve$ class label
2. $3.53 \times 10^{-24} = -ve$ class label
3. $0.99 = +ve$ class label
4. $0.91 = +ve$ class label
5. $0.95 = +ve$ class label