

## UNIT - III

### Logic and Knowledge Representation

→ First-Order Logic :-

- Representation -
- Syntax & Semantics of FOL -
- Using FOL Logic -
- Knowledge Engineering in FOL -

→ Inference in FOL :-

- Propositional vs First-Order Inference -
- Unification & Lefting -
- Forward chaining -
- Backward chaining -
- Resolution

## FOL (First Order Logic)

→ It is powerful framework  
→ Also called as First Order Predicate logic (FOL)  
→ Knowledge representation  
→ We use 2 techniques to represent knowledge:  
    - Propositional logic  
    - Predicate logic

→ Result of propositional logic is True or False.

- but not both simultaneously

→ In order to represent complex sentences or Natural language sentences propositional logic is not sufficient.  
ex:  $x$  is an even number.

→ We require predicate logic to represent the complex sentences

→ Predicate logic is an extension of proposition logic.

### \* Representation Components of Predicate logic

1. Objects (or) Constants : 5, John

2. Variables : x, y

3. Predicates : represents relationship b/w the objects.

ex:  $\frac{\text{John}}{\text{object}}$      $\frac{\text{loves}}{\text{Predicate}}$      $\frac{\text{Sophy}}{\text{object}}$   $\Rightarrow$  loves (John, Sophy)

4. Quantifiers:

① Universal Quantifier ( $\forall$ ) - for all, for every, for each, everything, anything, for any,

## ② Existential Quantifier ( $\exists$ ):

There exists, for some, for atleast one, there exists some.

## 5. Connectives:

$\wedge$  (AND),  $\vee$  (OR),  $\neg$  (negation), conditional ( $\rightarrow$ ), biconditional ( $\leftrightarrow$ ).

## \* Representation of FOL

→ FOL creates a powerful set of ontological commitments i.e  
- Properties, objects, functions, relations

Properties: It is used to differentiate one object with another (eg: big, small)

Objects: Items with separate identities (names)

Functions: One type of relation

Relations: Relation  $\bowtie$  exists b/w objects.

## \* Representation of FOL

- It means writing real-world knowledge in the form of logical statements.
- It is used to represent objects, their Properties, and relationships b/w them.
- It helps to AI agents to understand & make decisions.
- It is more powerful than propositional logic.
- It is a way to represent knowledge about the world using:
  - objects
  - properties
  - Relations
  - Rules.

Elements	Representation	Meaning
Constants	Ram, India	Specific things
Variables	x, y	General things
Predicates	Student(x), Loves(Ram, Priya)	Properties or relations
Functions	FatherOf(x)	Maps input to object
Quantifiers	forall, exists	Show quantity
Connectives	AND, OR, NOT, IMPLIES	Logical symbols

## Quantifiers

- $\forall x \rightarrow$  universal Quantified (for all)

Ex:  $\forall x \text{ Human}(x) \rightarrow \text{Mortal}(x)$   
 (All humans are mortal)

- $\exists x \rightarrow$  Existential quantifier (there exists)

Ex:  $\exists x \text{ Animal}(x) \wedge \text{wild}(x)$   
 (Some animals are wild)

## Connectives

Symbol	Meaning	Example.
$\wedge$	AND	$A \wedge B$
$\vee$	OR	$A \vee B$
$\rightarrow$	Implies	$A \rightarrow B$
$\sim$	NOT	$\sim A$
$\Leftrightarrow$	Iff	$A \Leftrightarrow B$

## Examples

Statement	FOL Representation
①. Ravi is a student	$\text{Student}(\text{Ravi})$
②. All humans are mortal	$\forall x \text{ Human}(x) \rightarrow \text{Mortal}(x)$
③. Some birds can fly	$\exists x \text{ Bird}(x) \wedge \text{Fly}(x)$
④. Ram loves ice cream	$\text{Loves}(\text{Ram}, \text{IceCream})$
⑤. Rani is mother of Ravi	$\text{MotherOf}(\text{Rani}, \text{Ravi})$

## \* Syntax and Semantics of FOL

### Syntax

\* Same as Representation

### Semantics

→ Semantics = Meaning of FOL Statements

Term	Meaning	Example
Domain	Set of all objects	{Ram, Ravi, Cat}
Interpretation	Assigning meaning to constants, functions, Predicates	student(Ram) = True
Truth Value	Stmt is True or False depending on interpretation	$\forall x \text{ Student}(x) \rightarrow$ True or False

### Example:

let domain = {Ram, Ravi}

let Student(Ram) = True

Student(Ravi) = False

Then:

- $\forall x \text{ Student}(x) = \text{False}$
- $\exists x \text{ Student}(x) = \text{False}$

## \* Using FOL Logic

→ It means converting English sentences (facts, rules) into FOL statements.

→ It helps AI to store knowledge

### Steps to use FOL logic

1. Identify constants

2. Identify variables

3. Define predicates (Properties or relations)

4. Use quantifiers ( $\forall, \exists$ )

5. Use connectives ( $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$ )

### Examples

English Sentence

FOL logic

①. Ram is a student

$\text{student}(\text{Ram})$

②. All humans are mortal

$\forall x \text{Human}(x) \rightarrow \text{Mortal}(x)$

③. Some animals are wild

$\exists x \text{Animal}(x) \wedge \text{Wild}(x)$

④. If it rains, the ground  
is wet

$\text{Rains}(x) \rightarrow \text{Wet}(\text{Ground})$

⑤. Every person has a mother

$\forall x \exists y \text{Mother}(y, x)$

⑥. Ravi loves pizza

$\text{Loves}(\text{Ravi}, \text{Pizza})$

## Benefits of using FOL

- Helps to represent complex knowledge.
  - useful in :-NLP
    - Expert Systems
    - Planning.
  - More expressive than propositional logic.
- 
- ⇒ Converts real-world facts to logic
  - ⇒ uses tools constants, variables, predicates, quantifiers.

## \* Knowledge Engineering

- It is the process of building a knowledge base for an AI system.
- In FOL, this means Collecting facts, rules & relationships and expressing them FOL Logic.
- ⇒ Used in expert systems, chatbots, planners etc.
- It means constructing (building) knowledge for an AI System.
- In FOL, we represent facts and rules using logic.  
ex: All humans are mortal  
 $\forall x \text{Human}(x) \rightarrow \text{Mortal}(x)$

### Steps

1. Define the Problem: what should the system solve?  
→ ~~the~~ what is actual ~~problem~~ issue?
2. Choose the Vocabulary:
  - Decide constants, predicates, functions
  - ex: - Constant: Ravi
  - Predicate: HasFever(x)

### 3. Write FOL sentences :

- convert knowledge into FOL Logic

ex:  $\forall x \text{ HasFever}(x) \rightarrow \text{Sick}(x)$

### 4. Add Facts : Substitute x value.

ex:  $\text{HasFever}(\text{Ravi})$

### 5. Ask questions / queries :

ex: Is Ravi sick?

### 6. Use Inference to get answers: (use logic)

- system applies logic & gives output.

ex: If a person has fever and cough, they have flu!

②. constants : Ravi, Doctor

Predicate :  $\text{HasFever}(x)$ ,  $\text{HasCough}(x)$ ,  $\text{HasFlu}(x)$

#

③.  $\forall x (\text{HasFever}(x) \wedge \text{HasCough}(x)) \rightarrow \text{HasFlu}(x)$

④. •  $\text{HasFever}(\text{Ravi})$   
•  $\text{HasCough}(\text{Ravi})$

⑤. ~~Ques~~ Does Ravi have flu?

⑥. Yes

### Uses

- Robots
- Expert Systems
- Chatbots
- Game logic

## \* Inference in FOL

- Inference means deriving new facts from known facts and rules.
- It means finding new facts from known facts.
- It helps AI to think & answer questions

### Used

- To solve problems
- To answer questions
- To make decisions
- To think

### Types of Inference

#### ①. Forward Chaining

- Starts from known facts
- Applies rules to generate new facts
- Repeats until the goal is reached.
- ⇒ Uses rules to find new facts.
- ⇒ Goes forward step by step.

Ex: Human (Ram)

Rule: Human(x)  $\rightarrow$  Mortal(x)

Result: Mortal (Ram)

- ⑥ Backward Chaining
- Starts from the goal.
  - works backwards
  - moves backwards step by step.
  - used in expert systems, AI reasoning

ex: Goal - Mortal (Ram)?

• Rule:  $\forall x \text{ Human}(x) \rightarrow \text{Mortal}(x)$

• Rule: Is Ram a Human? - Yes

→ check Is Ram a Human? - Yes

⇒ So, Mortal (Ram) → True.

### Techniques

- Unification

- Resolution

- Modus Ponens [If  $A \rightarrow B$  & A is True then  
B is true]

\* Unification → matching 2 FOL stmts.

- It is the process of matching two logical expressions.  
- by finding common values for variables.  
→ It is used in inference rules  
- resolution.

Example:

- Expression 1 : loves (Ram, X)  
- Expression 2 : loves (Ram, Plaza)
- ⇒ Unifier :  $X = \text{Plaza}$
- These two can be unified using substitution.

Expression 1	Expression 2	Result
$P(x)$	$P(\text{John})$	$x = \text{John}$
$Q(a, y)$	$Q(a, \text{Ravi})$	$y = \text{Ravi}$
<del><math>P(\text{Ravi}, P(x, x))</math></del> <del><math>P(\text{Ravi}, S(t))</math></del>	$P(\text{Ram}, S(t))$	Not unifiable (x used differently)

Uses

- in resolution  
- in backward chaining.

\* Lifting → Using resolution in FOL

→ Lifting is the process of applying inference rules to FOL

- instead of propositional logic.

Ex: Parent(x, y) ∨ Loves(x, y)

~ Parent(Ram, Sita)

① x = Ram, y = Sita

② Parent(Ram, Sita) ∨ Loves(Ram, Sita)

~ Parent(Ram, Sita)

Result: Loves(Ram, Sita)

## Propositional Logic

- 1. Uses full sentence.
- 2. less expressive
- 3. Easier
- 4. less powerful
- 5. No internal structure.
- 6. No quantifiers used
- 7. Uses resolution.
- 8. ex:  $A \rightarrow B$
- 9. Used in simple problems
- 10. Used in basic reasoning

## First-order Logic

- 1. Uses Variables & Predicates
- 2. More expressive
- 3. Not easier
- 4. More powerful
- 5. It has internal structure.
- 6. Use quantifiers
- 7. Uses resolution.
- 8. ex:  $\forall x (\text{Human}(x) \rightarrow \text{Mortal}(x))$
- 9. Used in NLP, Expert System, AI agent
- 10. Used in complex knowledge representation, deep reasoning.