Privacy Constrained Energy Management in Microgrid Systems

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***Abstract*—We propose privacy-preserved joint supply- and demand-side energy management strategies for a microgrid system that consists of several cells and a control center, with each cell composed of a smart meter, a distributed energy generator and some energy consuming customers. It is assumed that the cells can cooperate by exchanging their locally generated energy and they can obtain external energy, both through the control center. The problem is formulated as privacy-constrained linear optimization problem to optimality regulate the energy utilization within the microgrid system for reducing the energy shortage which needs to be fulfilled by externally imported energy. To solve this problem, we develop a privacy-preserving scheme that performs the proposed dual decomposition-based algorithm in a distributed fashion with the limited information exchanges. Finally, the optimal energy schedule is obtained without the privacy violation and simulation results are provided to demonstrate the superior performance of the proposed techniques over the traditional methods.**

I. INTRODUCTION

Nowadays, the microgrid systems that employ the environment-friendly distributed energy generators are developed to reduce the carbon footprint while providing quality and economic energy supplies [1]. The reliability and efficiency of the microgrid system's operation are critical and rely highly on the effective and efficient energy management, which aims to balance the electricity supply and demand by cooperating between different generators and consumers [2][3]. In the meantime, the issue of privacy is another important consideration for smart-grid systems, especially when intra-system cooperations exist. It is argued in [4] and [5] that the customers' energy demand patterns may expose their habits and behaviors and should be kept confidential from the electricity suppliers except for what is absolutely needed for billing.

Based on the typical domestic smart-grid model, in this paper, we consider customer cooperation mechanisms in a system that consists of a group of self-fed customers (also called cells) and a control center, under the general smart-grid system structure described in [6][7], taking into consideration the privacy constraints. We consider the microgrid system that is composed of renewable energy sources, e.g., solar panels and wind turbines, smart meters and consumers. To cope with the limitation of unstable availability and capacity of the renewable energy resources, a complementary traditional energy source, e.g., power plant, is also employed together with the buffer batteries.

The goal of the energy management, consisting of the *transmission schedule* and *demand schedule*, is to balance the energy supply and demand at every time instant to reduce the traditional (non-green) energy importing while the grid operational reliability is enforced. The optimal energy management requires striking a balance between the inter-cell cooperation and unnecessary energy loss, e.g., path-loss and potential battery overflow. Maintaining such a balance becomes more complicated due to the fact that privacy, which is related to the consumers' behaviors, is involved in the demand-side energy management. Therefore, designing an optimal energy scheduling policy that takes into account the privacy constraints poses significant challenges for the power grid operator.

The remainder of the paper is organized as follows. In Section II, we describe the structure of the microgrid system and formulate the privacy constratined energy scheduling problem. In Section III, we provide an iterative distributed algorithm to optimally solve the formulated energy scheduling problem. Simulation results are provided in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. The Microgrid System

Consider a microgrid system consisting of a control center, smart meters, distributed generators, and consumers, with complementary energy supply, as shown in Fig. 1. The control center is unique in the microgrid and directly connected to smart meters and the macrogird system (conventional energy source). The operations of the microgrid control center is directed by the energy transmission schedule, i.e., the supply-side energy management.

Each smart meter is connected to a group of consumers and a distributed generator. Each distributed generator is also equipped with a battery for buffering the generated energy. A smart meter together with the linked (and controlled) consumers and the distributed generator constitutes a *cell*. One typical example is a house or a factory with a smart meter and a distributed generator.

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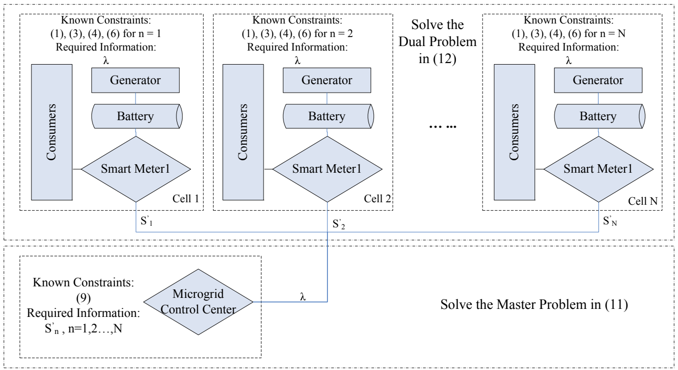

Fig. 1. Block diagram of the microgrid system and the operations.

We assume that the power transmission link exists among the cells in the microgrid and the energy generated in each cell can not only feed the in-cell consumers but also export to the consumers outside the cell. We further assume that portions of the consumers' demands are elastic, that is, these energy demands are allowed to be met within a certain time window. The smart meter acts as a local controller to manage intra-cell energy demand, e.g., to assign the time window and power quota for consumers' elastic demands, based on the energy demand schedule, i.e., the demand-side management.

B. Energy Management

Assume that the energy management occurs in a time-slotted fashion. The beginning of a time slot is reserved for information collection, exchange and schedule calculation. Then the schedule is applied for energy management. Throughout the paper, the set of cells is denoted as $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$, and the set of time slots is $\mathcal{K} \triangleq \{1, 2, \ldots, K\}$.

The supply-side energy management regulates the energy flows between different cell i and j in the microgrid, denoted by $t_{i,j}^k$, the conventional energy imported to cell n, denoted by p_n^k, and the reserved energy for in-cell consumers, denoted by r_n^k, in the k-th time slot. Then, the consumers' demand in cell n in the k-th time slot, denoted by d_n^k, is given by

$$d_n^k = r_n^k + l_n p_n^k + l_n \sum_{i=1, i \neq n}^{N} l_i t_{i,n}^k, \ n \in \mathcal{N}, \ k \in \mathcal{K} \ , \quad (1)$$

where l_n is the path loss incurred by the transmission line between the control center and cell n.

We assume that the generated energy is only available at the end of the time slot and predictable for the subsequent K time slots. We denote the predicted value as E_n^k and introduce the auxiliary variables $\hat{p}_n^k, \ n \in \mathcal{N}, \ k \in \mathcal{K}$, which denote the virtual energy consumptions to prevent the battery from overflowing. Then, we then have the following relationship between the battery levels at the beginning of two consecutive time slots:

$$\begin{cases} B_n^{k+1} = B_n^k - \sum_{j=1, j \neq n}^{N} t_{n,j}^k - r_n^k - \hat{p}_n^k + E_n^k \\ 0 \leq B_n^{k+1} \leq B_n^{\max} \end{cases} \quad (2)$$

and its equivalent form:

$$\sum_{k=1}^{\kappa} (r_n^k + \sum_{j=1, j \neq n}^{N} t_{n,j}^k + \hat{p}_n^k) \geq B_n^0 + \sum_{k=1}^{\kappa-1} E_n^k - B_n^{\max}, \ \kappa \in \mathcal{K} \ , \quad (3)$$

$$\sum_{k=1}^{\kappa} (r_n^k + \sum_{j=1, j \neq n}^{N} t_{n,j}^k + \hat{p}_n^k) \leq B_n^0 + \sum_{k=1}^{\kappa-1} E_n^k, \ \kappa \in \mathcal{K} \ . \quad (4)$$

The demand-side energy management partially controls the energy consumptions of the consumers in the same cell, e.g., switches on or off the consumers. In view of the controllability of the demands, we classify the consumers' demand into the following two categories: *schedulable demand*, which is mentioned as elastic demand in the previous section, denoted by $\hat{c}_n^k$; and *un-schedulable demand*, which is not allowed to be scheduled by the smart meter, denoted by c_n^k. Therefore, the total demands can be written as

$$d_n^k \triangleq c_n^k + \hat{c}_n^k, \ n \in \mathcal{N}, \ k \in \mathcal{K} \ . \quad (5)$$

The schedulable demands are constrained by certain conditions, namely *schedulable demand constraints*. For example, one typical mapping from the consumer's preference to the constraint is that a specific task is required to be completed within a designated time window, e.g., charging the electric vehicle between 12pm and 5am. Such constraint can be formulated as

$$\sum_{k=K_1}^{K_2} \hat{c}_n^k = C_n \ , \quad (6)$$

where $[K_1, K_2]$ defines the time window that is available for energy scheduling and C_n is the total required energy.

C. Privacy Control

Privacy is an important issue in smart-grid systems. In this paper, following [4] [5], we treat the consumers' behaviors and preferences as the privacy information, which are reflected in terms of the consumers' energy demands d_n^k and $\hat{c}_n^k$, as well as the related constraints. Such privacy information can be further categorized by their sensitivities, i.e, information with different sensitivities may be concealed to different persons [8]. In particular, we assume that consumers' total energy demands d_n^k, which are needed for billing and energy delivery, can be revealed to the microgrid control center. On the other hand, the consumers' schedulable demands $\hat{c}_n^k$ and the related constraints, which contain more details of their behaviors and are only related to energy scheduling, should be kept confidential from both the control center and other cells. We summarize the different types of the privacy information in the microgrid system in Table I.

TABLE I
PROPERTIES OF PRIVATE INFORMATION

Type	Info. related to	Cell n	Ctrl. center	Other cells
I	$\hat{c}_n^k$	Non-private	Private	Private
II	$d_n^k, t_{n,\cdot}^k, t_{\cdot,n}^k$	Non-private	Non-private	Private

As seen in Table I, the Type-I privacy information is related to the consumer's behavior and is considered strictly private that cannot be shared outside the cell, including the control center. The Type-II privacy information includes the information related to the energy schedule and can only be released to the control center for energy scheduling, delivery and billing. In addition, we assume that the information related to the working status of the distributed generator is non-private and accessible to every cell as well as the control center.

D. Problem Formulation

It is well known that man-made CO_2 emission causes negative environmental effects, e.g., global warming [3]. From the microgrid's perspective, importing less energy from the macrogrid leads to lower CO_2 emission and economic cost, which is the main goal of the energy scheduling policy.

By defining $\mathcal{T} \triangleq \left\{t_{i,j}^k, r_i^k, p_j^k \mid i,j \in \mathcal{N},\ i \neq j\right\}$ as the energy schedule that directs the microgrid control center to regulate the energy exchange, our objective is to minimize the total imported energy to the microgrid within K time slots, i.e.,

$$\mathcal{T}^* = \arg \min_{\mathcal{T}\geq 0, \mathcal{A}\geq 0} I(\mathcal{T}) \ , \tag{7}$$

where

$$I(\mathcal{T}) = \sum_{k=1}^{K} \alpha(k) \sum_{n=1}^{N} p_n^k \ , \tag{8}$$

and $\alpha(k) \leq 1$ is a discount factor that is non-increasing in k and close to 1, $\mathcal{A} \triangleq \{\hat{\boldsymbol{p}}_n \mid n \in \mathcal{N}\}$ and $\hat{\boldsymbol{p}}_n$ is a vector formed by stacking up $\hat{p}_n^k,\ k \in \mathcal{K}$.

A feasible schedule $\mathcal{S}$ and auxiliary variables $\mathcal{A}$ must meet various constraints, i.e., the *battery capacity constraints* in (3) and (4) to ensure the physical realizability of the battery, the *demand fulfilment constraints* in (1) to ensure the service quality, the *schedule window constraints* in (6) to ensure the consumers' satisfaction, and the *privacy constraints* in Table I to protect the consumers' privacy. Specifically, unlike the (in)equations constraints, the privacy constraints in Table I takes effect in the process of calculating and implementing the energy schedule.

III. Computing the Optimal Energy Schedule

A. Dual Decomposition

The problem in (7) is a privacy-constrained LP problem and is not feasible to be solved directly due to the *hidden constraints* issue, i.e., the privacy constraints may prevent the solver from knowing some of ther necessary constraints. Specifically, the Type-I privacy constraints hide all constraints that are not related to cell n when the problem is solved in cell $n \in \mathcal{N}$; and the Type-II privacy constraints hide the schedule window constraints when the problem is solved in the microgrid control center.

To resolve the problem of "hidden constraints", we first denote $u_n^k \triangleq \sum_{i=1,i\neq n}^{N} l_i t_{i,n}^k$ and $v_n^k \triangleq \sum_{j=1,j\neq n}^{N} t_{n,j}^k$ as the incoming and outgoing green energy to/from cell n in the k-th time slot, respectively, subjecting to the energy flow equilibrium condition:

$$\sum_{n=1}^{N} p_n^k + \sum_{n=1}^{N} l_n v_n^k = \sum_{n=1}^{N} u_n^k,\ k \in \mathcal{K} \ , \tag{9}$$

and define $\mathcal{S} \triangleq \left\{S_n \mid S_n = \left(T_n, \hat{c}_n\right),\ n \in \mathcal{N}\right\}$ where $T_n^k \triangleq \left(u_n^k, v_n^k, r_n^k, p_n^k\right)$, and $\boldsymbol{T}_n$ and $\hat{\boldsymbol{c}}_n$ are vectors formed by stacking up T_n^k and $\hat{c}_n^k$, $k \in \mathcal{K}$, respectively. Then we convert the problem in (7) into a partial *Lagrange dual problem* by moving the constraints in (9) into the objective function using a set of *Lagrangian multipliers* [10]:

$$L(\mathcal{S}, \boldsymbol{\lambda}) = I(\mathcal{S}) + \sum_{k=1}^{K} \lambda_k \Big(\sum_{n=1}^{N} p_n^k + \sum_{n=1}^{N} l_n v_n^k - \sum_{n=1}^{N} u_n^k \Big) \ , \tag{10}$$

where $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_K] \in \mathbb{R}^K$ are the *dual variables*.

Instead of solving the original problem, we can successively solve the following two problems to obtain the energy schedule $\mathcal{T}^*$:

$$\max_{\boldsymbol{\lambda}} g(\boldsymbol{\lambda}) \ , \tag{11}$$

where

$$g(\boldsymbol{\lambda}) = \inf_{\mathcal{T}\geq 0, \mathcal{A}\geq 0} L(\mathcal{S}, \boldsymbol{\lambda}) \ , \tag{12}$$

with the constraints in (3), (4), (1), (6), and privacy constraints in terms of u_n^k and v_n^k; and a group of (in)equations

$$\begin{cases} \sum_{i=1,i\neq n}^{N} l_i t_{i,n}^k = u_n^k & n \in \mathcal{N},\ k \in \mathcal{K} \\ \sum_{j=1,j\neq n}^{N} t_{n,j}^k = v_n^k & n \in \mathcal{N},\ k \in \mathcal{K} \\ t_{i,j}^k \geq 0 & i,j \in \mathcal{N},\ k \in \mathcal{K} \end{cases} . \tag{13}$$

Based on the decomposition, we can further develop an iterative algorithm to solve the optimization problem in (7) in a distributed fashion [9][10]: as shown in Fig. 1, at each iteration, the smart meters solve the dual problem in (12) to obtain the optimal S_n; and the microgrid control center solves the master problem in (11) to obtain the dual variables $\boldsymbol{\lambda}$.

B. Solving the Master Problem at Control Center

The master problem in (11) is only constrained by privacy constraints and we can solve it by employing the sub-gradient method [9]. Specifically, the dual variables can be recursively updated as follows [10]:

$$\lambda_k^{(q)} = \lambda_k^{(q-1)} - \beta(q-1) \cdot h_k(\mathcal{S}^{(q-1)}) \ , \tag{14}$$

where q is the iteration number, $\beta(q)$ is the step size, and $h_k(\mathcal{S})$ corresponds to the constraint that is moved to the objective function, i.e.,

$$h_k(\mathcal{S}) = \sum_{n=1}^{N} p_n^k + \sum_{n=1}^{N} l_n v_n^k - \sum_{n=1}^{N} u_n^k \ . \tag{15}$$

With a diminishing step size, e.g., $\beta(q) = 1/q$, we have $L(\mathcal{S}, \boldsymbol{\lambda}^{(q+1)}) \geq L(\mathcal{S}, \boldsymbol{\lambda}^{(q)})$, with equality if all constraints are met, and finally $g(\boldsymbol{\lambda}^{(q)})$ achieves its maximum [10][9].

Then we use the total schedule demand $\boldsymbol{d}_n$ to replace $\hat{\boldsymbol{c}}_n$ and define the equivalent form of the energy schedule $S_n' =$

$(\boldsymbol{T}_n, \boldsymbol{d}_n)$, where $\boldsymbol{T}_n$ is used to update the dual variable and $\boldsymbol{d}_n$ is used to calculate the objective value. The master problem, therefore, is solvable in the microgrid control center without privacy violations.

With the energy schedule $\mathcal{S} \triangleq \{S'_n \mid n \in \mathcal{N}\}$ which is collected from each cell, the microgrid control center uses (14) to update the dual variables $\boldsymbol{\lambda}$ and calculate the objective value $L(\mathcal{S}, \boldsymbol{\lambda})$, until the dual solution converges. Specifically, at the m-th iteration, the control center collects $S_n^{'(m)}$ from every cell first. Then, it updates the dual variables $\boldsymbol{\lambda}$ according to (14) and calculates the objective value $L^{(m)} \triangleq L(\mathcal{S}^{(m)}, \boldsymbol{\lambda})$. Given a pre-specified threshold ϵ and an integer M, if

$$\max_{i=m-M,m-M+1,\ldots,m-1} |L^{(i)} - L^{(m)}| < \epsilon \,, \tag{16}$$

then solve (13) and stop the algorithm.

C. Solving the Dual Problem at Smart Meters

The Lagrangian dual problem in (12), which is constrainted by (3), (4), (1), (6) and the privacy constraints related to all $n \in \mathcal{N}$, can be further decomposed by different cell index n:

$$\mathcal{P}_n = \begin{cases} \min_{S'_n \geq 0, \hat{\boldsymbol{p}}_n \geq 0} L_n(S'_n, \boldsymbol{\lambda}_n) \\ \text{ s.t. } \quad (3), (4), (1), \text{and } (6) \text{ for } n \\ \qquad\quad \text{Privacy constraints in Table I for } n \end{cases}, \tag{17}$$

where

$$L_n(S'_n, \boldsymbol{\lambda}) = \sum_{k=1}^{K} \left(\alpha(k) p_n^k + \lambda_k (p_n^k + l_n v_n^k - u_n^k) \right) . \tag{18}$$

For any cell n, the problem in (17) is only related to the privacy information of the consumers in cell n. No constraints are hidden if this sub-problem is solved in the corresponding cell n. Moreover, the standard LP tools can be employed to solve (17) [9]. Specifically, at the m-th iteration, cell n retrieves $\boldsymbol{\lambda}^{(m)}$ from the control center first; then it solves the LP problem (17) and obtains the dual solution $S_n^{'(m)}$ which is then sent to the microgrid control center.

D. Recovery of Feasible Primary Solution

The primal feasibility of the solution, which is obtained by employing the sub-gradient method on the dual problem, does not always hold [11]. An infeasible schedule usually leads to energy surplus or shortage at the microgrid control center, resulting in energy waste or unplanned importing, respectively.

To ensure the feasibility of the obtained schedule, a step that maps the infeasible solution to a feasible one can be implemented at the microgrid control center at the end of each outer primal-dual iteration. Specifically, we denote $\tilde{\mathcal{S}}^{(m)}$ as the mapped primal feasible solution obtained at the end of the m-th outer primal-dual iteration. Then the mapping is given by [11]

$$\tilde{\mathcal{S}}^{(m+1)} = \frac{m}{m+1} \cdot \tilde{\mathcal{S}}^{(m)} + \frac{1}{m+1} \cdot \mathcal{S}^{(m)} \,. \tag{19}$$

We define $I^{(m)} \triangleq I(\tilde{\mathcal{S}}^{(m)})$ and set the optimal solution $\mathcal{S} = \tilde{\mathcal{S}}^{(m^*)}$ where m^* is the iteration index corresponding to the smallest $I^{(m)}$ among the last M iterations.

Finally we summarize the distributed algorithm for solving the energy scheduling problem (7) as follows.

Algorithm 1 - Partial Dual Decomposition Algorithm

1: Initialization (implemented by the control center)
$\boldsymbol{\lambda}^{(0)} = \mathbf{0}, I^{(0)} = 0, m = 1$; specify ϵ and M

2: Local optimization (implemented at each cell)
FOR $n \in \mathcal{N}$
  Retrieve $\boldsymbol{\lambda}^{(m-1)}$
  Solve (17) using an LP solver to obtain $S_n^{(m)}$
  Submit $S_n^{(m)}$
ENDFOR

3 Global optimization (implemented at the control center)
Use (14), (10) to update $\boldsymbol{\lambda}^{(m)}$, $L^{(m)}$
Map $S_n^{(m)}$ to $\tilde{S}^{(m)}$ by (19)
IF $\max_{i=m-M,\ldots,m-1} |L^{(i)} - L^{(m)}| < \epsilon$, **GOTO STEP 4**
ELSE
  Activate cells; $m \leftarrow m + 1$; **GOTO STEP 2**
ENDIF

4: Schedule recovery
$\mathcal{S}^* = \tilde{\mathcal{S}}^{(m^*)}$; recover $t_{i,j}^k$ by solving (13)

Without the privacy constraints, the problem (7) in term of $\mathcal{S}$ is a linear programming problem. Then, the strong duality holds and there exists a saddle point for the Lagrangian dual problem in (12). If the step size $\beta(q)$ in (14) is properly chosen and the optimal solution to $\mathcal{P}_n$ is unique in $\mathcal{S}$, then the above Algorithm 1 can obtain the primal optimal and feasible solution to (7) [9]; if the solution to $\mathcal{P}_n$ is not unique in $\mathcal{S}$, one primal feasible and optimal solution can be obtained after the mapping step (19) with a properly chosen step size $\beta(q)$ [11]. Therefore, Algorithm 1 can provide the globally optimal primal solution to (7). Moreover, as shown in Fig. 1, there is no privacy violation during the process of information exchange and schedule calculation.

IV. Simulation Results

We assume that there are $N = 4$ cells in the microgrid system. We set the scheduling period as 6 hours and divide it into $K = 6$ time slots of equal durations. For each cell n we set the battery capacity $B_n^{\max} = 10$ and the initial energy level $B_n^0 = 5$. Assume that both the generated energy E_n^k and un-schedulable energy c_n^k follow a uniform distribution. Also, we assume that in (6) the total schedulable demand is $C_n = 12$ and $K_1 = 1$, $K_2 = 6$. In addition, we set the discount factor as $\alpha(k) = 0.99^k$, step size as $\beta(q) = 1/q$, and path losses as $l_1 = 0.984$, $l_2 = 0.983$, $l_3 = 0.910$ and $l_4 = 0.905$.

For comparison, we consider two simple scheduling strategies, namely, the *energy transmission scheduling only* (ETS-only) strategy and the *demand scheduling only* (DS-only) strategy. The ETS-only strategy assumes that all demands are un-schedulable and only the energy transmission schedule is performed. To obtain the ETS-only schedule, we distribute the schedulable demands to each available time slot evenly and solve (7) in the control center only. On the other hand, the DS-only strategy assumes that each cell cannot receive energy from outside and only optimizes its own schedulable demands.

To obtain the DS-only schedule, we apply the fixed non-cooperative transmission schedule ($v_n^k = 0,\ k \in \mathcal{K},\ n \in \mathcal{N}$) and solve $\mathcal{P}_n$ in each smart meter. Moreover, we also consider a "no schedule" strategy where we distribute the schedulable demands to each available time slot evenly and no intercell-cooperation is allowed. The convergence threshold is set as $\epsilon = 10^{-3}$.

We consider a typical scenario where the generated energy E_n^k and the energy demand c_n^k follow a uniform distribution over the interval $[6-a, 6+a]$, with the value of a varies from 1 to 3. In this scenario, the energy generation and consumption are balanced in the long term but they fluctuate at different time slots. The required external energy is caused by the fluctuations of the energy supply and demand in this example. The total amount of imported energy given by various scheduling strategies, as well as by optimal schedule without the privacy constraints, is shown in Fig. 2.

As expected, it is seen from Fig. 2 that the proposed dual decomposition method and the optimal schedule give the best performance. The ETS-only and the DS-only strategies perform worse than the optimal schedule because they do not fully exploit inter-cell cooperation in the microgrid system and significant amount of energy is overflowed due to the battery capacity limit. As dynamic range a increases, the performances of all methods degrade due to the fluctuations of the energy supplies and demands.

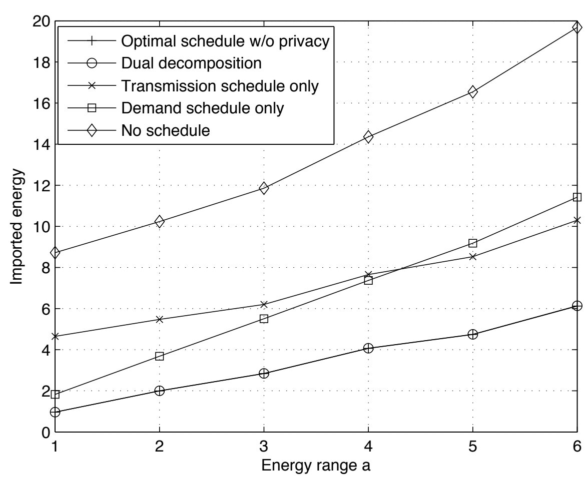

Fig. 2. Performance comparisons among different energy schedules.

We next evaluate the convergence rate of the dual decomposition method. In this simulation, we assume that the generated energy E_n^k and energy demand c_n^k follow a uniform distribution over the interval $[4, 8]$. We record the objective value of the primal problem in (7) and dual problem (11) at each iteration, and plot them in Fig. 3. It is seen that the dual decomposition method converges at around the 50-th iteration. Moreover, upon convergence, the dual decomposition method exhibits zero dual gap, i.e., the optimal performance is achieved.

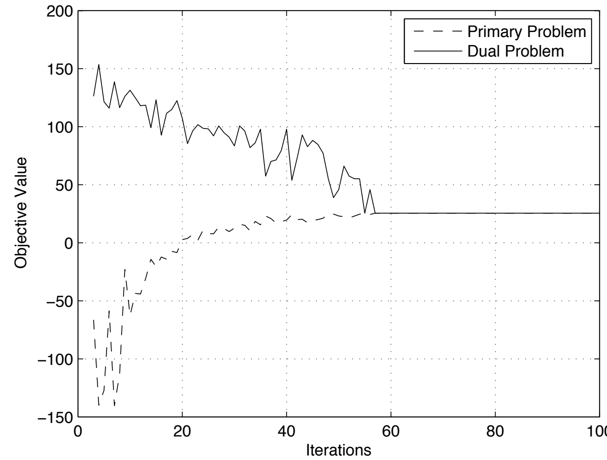

Fig. 3. Convergence performance of the dual decomposition method.

V. Conclusions

We have considered the privacy-preserved scheduling problem for a microgrid system equipped with the renewable energy sources and smart meters. This energy scheduling problem is formulated as a privacy-constrained LP problem. We have developed the dual decomposition method to optimally solve this problem. The proposed algorithm takes into account the privacy constraint and can be implemented in a distributed fashion. Simulation results have demonstrated that the proposed method can provide substantial reduction in the non-renewable energy consumption, while meeting the energy demands of the consumers as well as their privacy constraints.

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