

Information-Theoretic Privacy in Smart Metering Systems Using Cascaded Rechargeable Batteries

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Abstract—A rechargeable battery may alleviate the issue of privacy loss in a smart metering system by distorting a household's load profile. However, existing studies involve a single rechargeable battery, whereas in a network scenario, there could be multiple batteries connected together. In this letter, we study the extension where a user's electricity load is input into a network of two rechargeable batteries, connected in series, and operating individually. This battery network attempts to mask the user load from the utility provider. We focus on the case of independent identically distributed load profile and a system of ideal batteries with no conversion loss, and use normalized mutual information (leakage rate) as the privacy metric. We derive upper and lower bounds on the leakage rate in terms of (single-letter) mutual information expressions. On the achievability side, our information-theoretic upper bound captures the novel tension between minimizing the leakage across each individual battery and the effect of their joint interaction. For the lower bound, we show that a system with a single battery, whose storage capacity is the sum of the two individual batteries, can achieve a leakage rate at least as small as our proposed setup. Furthermore, we use simulations to compare achievable leakage of our proposed scheme with several baseline schemes. The achievable leakage rates obtained in this study could help us to elucidate the privacy performance of a network of batteries.

Index Terms—Privacy, rechargeable battery, security, smart metering systems.

I. INTRODUCTION

SMART meters are essential in the modern electrical grids [1]. They deliver power usage data to the utility companies for energy management. However, this leads to the possibility of privacy loss from the customers' point of view [2]. One way to alleviate the issue of privacy loss is the implementation of a rechargeable battery [3]. Using a rechargeable battery, the load profile of the household users can be distorted by charging and discharging the battery. Varodayan and Khisti [4] assume a battery model with a discrete storage state and an independent identically distributed (i.i.d.) load profile, and introduce mutual

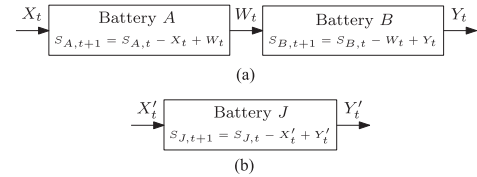


Fig. 1. Two battery systems. (a) C-C System. (b) 2C System.

information (MI) as the privacy metric. A characterization of optimal battery policies for such system has been obtained in [5] and [6]. It is shown that the optimal charging policy satisfies an invariance property that makes the analysis of the leakage rate tractable. Gündüz and Gómez-Vilardebó [7] consider a variant involving an energy harvesting device coupled with the rechargeable battery. A rate distortion framework to capture the tension between utility and privacy has been introduced in [8]; however, that framework does not incorporate instantaneous battery constraints. Other methods for load obfuscation have been proposed in the literature, e.g., use of water heater [9].

In this letter, we focus on the privacy protection by implementing rechargeable batteries. While previous studies have focused on the case of a single battery, we consider two batteries connected in series. The user load is input into the first battery, whose output in turn is fed into the second battery, which is connected to the external grid. Both the batteries operate independently and satisfy the ideal conservation rule. The charging policy for each battery can be selected by the user to minimize information leakage to the utility company. The joint interaction between the two batteries makes this problem a nontrivial extension of the single-battery case. We show that a policy that minimizes the leakage across each individual battery is suboptimal and derive information theoretic upper and lower bounds on the leakage rate.

II. PROBLEM FORMULATION

A. System and Variable Definitions

1) *Notation*: The probability that the random variable X takes on the value of x is denoted by $P_X(x)$. We use subscript t to denote the time index. Thus, X_t denotes the random variable X sampled at time t and likewise $S_{A,t}$ denotes the random variable S_A at time t . We denote X_m^n as a shorthand for $(X_m, X_{m+1}, \dots, X_n)$ and $X^T = X_1^T$. Here, T denotes the time-horizon of interest.

2) *C-C System*: Consider a smart-metering system of two batteries, as shown in Fig. 1(a), which will be referred to as the C-C system, since the system could be interpreted as two batteries connected in series and with capacity C each. The C-C system is placed between the user and the utility company in order to distort the load profile. The arrows indicate

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the direction of power demand. The input to the C-C system, $x \in \{0, 1, \dots, m_x\}$, is the i.i.d. power demand sent from the user. The first battery sends the intermediate power demand, $w \in \{0, 1, \dots, m_w\}$, to the second battery in cascade. The output demand of the C-C system, $y \in \{0, 1, \dots, m_y\}$, is sent to the utility company. We assume that the user demand can always be satisfied, and hence $m_x \leq m_w \leq m_y$. For simplicity, we let $m_x = m_w = m_y$ and let the two batteries have the same capacity, C . We then let $s_A \in \{0, 1, \dots, C\}$ be the energy stored in the first battery in cascade, which will be referred to as the state of battery A ; likewise, $s_B \in \{0, 1, \dots, C\}$ for the second battery in cascade. We assume that there are no cooperation between the two batteries such that the output of each battery depends only on the history of the input, output, and state of the same battery. Following this assumption, we let $q_{W_t|X^t, S_A^t, W^{t-1}}^A$ and $q_{Y_t|W^t, S_B^t, Y^{t-1}}^B$ be the conditional probability distributions that dictate the charging policies of battery A and battery B at time t , respectively, and let $q^A := \{q_{W_t|X^t, S_A^t, W^{t-1}}^A : t = 1, 2, \dots\}$ and $q^B := \{q_{Y_t|W^t, S_B^t, Y^{t-1}}^B : t = 1, 2, \dots\}$.

3) *2C System*: Fig. 1(b) shows a system of one rechargeable battery with capacity $2C$ equal to the sum of the individual battery capacities in the C-C system, which we will be referring to as the 2C system. Since we will be comparing between the privacy performance of the C-C system and the 2C system, we apply the same input distribution to both systems to keep the comparison fair. Therefore, $x' \in \{0, 1, \dots, m_x\}$ is the i.i.d. input demand to the 2C system, and $P_X = P_{X'}$. The output demand of the 2C system, $y' \in \{0, 1, \dots, m_y\}$, is sent to the utility company. We let $s_J \in \{0, 1, \dots, 2C\}$ be the energy stored in the battery. We let $q_{Y'_t|X'^t, S_J^t, Y'^{t-1}}^J$ be the conditional probability distribution that dictates the charging policy of the single battery at time t and let $q^J := \{q_{Y'_t|X'^t, S_J^t, Y'^{t-1}}^J : t = 1, 2, \dots\}$.

4) *Battery State Update Equations*: Given the definition of the input, output, and state for the two systems above, let us discuss the underlying constraints that govern those variables. We assume that we have control over the distribution of initial battery state, since setting the initial battery state is a part of the startup operation for the metering system. Also, the distribution P_X is given by nature. For simplicity, we assume that the batteries are ideal such that there are no conversion losses or other inefficiencies. Hence, the state transition of the battery A and battery B for the C-C system, as shown in Fig. 1(a), are governed by the following battery update equations:

$$S_{A,t+1} = S_{A,t} - X_t + W_t \quad (1)$$

$$S_{B,t+1} = S_{B,t} - W_t + Y_t. \quad (2)$$

Definition 1 (Feasible Charging Policies): A charging policy is $q^A := \{q_{W_t|X^t, S_A^t, W^{t-1}}^A : t = 1, 2, \dots\}$ is feasible for battery A if for each time t the battery state $S_{A,t}$ satisfying (1) is contained in $\{0, 1, \dots, C\}$. Let Q_A denote the set of all feasible charging policies for battery A . The set Q_B for battery B is defined in a similar fashion.

For the 2C system, the state update equations take the following form:

$$S_{J,t+1} = S_{J,t} - X'_t + Y'_t, \quad (3)$$

and the set of feasible charging policies can be defined in an analogous fashion.

B. Objective Function

We wish to hide X^T given that the utility company has access to Y^T by adjusting the charging policies for each of the two aforementioned systems, while adhering to the constraint imposed by the battery update equation. The privacy metric that we are using is the MI, $I(X^T; Y^T)$, which captures the total amount of information in X^T leaked through Y^T . Leakage rate is the information leaked per unit time, and we define the leakage rate for the C-C system as follows:

$$L_{C-C} := \limsup_{T \rightarrow \infty} \frac{1}{T} I_{C-C}(S_{A,1}, S_{B,1}, X^T; Y^T). \quad (4)$$

The unit for MI is bits if the logarithm is base 2 so the unit of leakage rate is bits/s. The reason why the initial battery states are in the expression is because they do not affect the asymptotic leakage rate while simplifying the analysis, as explained in [5]. Also, we focus on the case of an infinite time horizon, and hence, we set $T \rightarrow \infty$. We define the leakage rate for the 2C system as follows:

$$L_{2C} := \limsup_{T \rightarrow \infty} \frac{1}{T} I_{2C}(S_{J,1}, X'^T; Y'^T). \quad (5)$$

Lower leakage rate indicates better privacy performance of the system. From the definition of MI in [10], the leakage rates in (4) and (5) can be calculated by marginalizing the following two joint distributions induced for the C-C system and 2C system, respectively:

$$\begin{aligned} & P_{S_{A,1}^T, X^T, W^T, S_{B,1}^T, Y^T} \\ &= P_{S_{A,1}} P_{X_1} q_{W_1|X_1, S_{A,1}}^A P_{S_{B,1}} q_{Y_1|W_1, S_{B,1}}^B \\ &\quad \cdot \prod_{t=2}^T \mathbb{1}_{(S_{A,t} = S_{A,t-1} - X_{t-1} + W_{t-1})} P_{X_t} q_{W_t|X^t, S_A^t, W^{t-1}}^A \\ &\quad \cdot \prod_{t=2}^T \mathbb{1}_{(S_{B,t} = S_{B,t-1} - W_{t-1} + Y_{t-1})} q_{Y_t|W^t, S_B^t, Y^{t-1}}^B, \end{aligned} \quad (6)$$

$$\begin{aligned} & P_{S_J^T, X'^T, Y'^T} = P_{S_{J,1}} P_{X'_1} q_{Y'_1|X'_1, S_{J,1}}^J \\ &\quad \cdot \prod_{t=2}^T \mathbb{1}_{(S_{J,t} = S_{J,t-1} - X'_{t-1} + Y'_{t-1})} P_{X'_t} q_{Y'_t|X'^t, S_J^t, Y'^{t-1}}^J \end{aligned} \quad (7)$$

where $P_{X_t} = P_X$ since we are assuming i.i.d. input distributions, and $\mathbb{1}_{U=V}$ for the two random variables U and V is evaluated to 1 when $U = V$, 0 otherwise.

In the following, we formally give the problem statement.

Problem 1: We wish to characterize the minimum leakage rate $L^* := \inf_{q^A, q^B, P_{S_{A,1}}, P_{S_{B,1}}} L_{C-C}$ of the C-C system. Here, the infimum over q^A and q^B is over the set of feasible charging policies stated in Definition 1. The distributions $P_{S_{A,1}}$ and $P_{S_{B,1}}$ can be arbitrary over the support of $\{0, 1, \dots, C\}$. As our main result, we will establish an upper bound and a lower bound for L^* .

III. BOUNDS ON THE MINIMUM INFORMATION LEAKAGE

A. Class of Invariant Policies

In this section, we define a class \mathcal{P}_{inv} of policies and initial battery distributions which is used in the derivation of an upper bound. In this class, the policies are stationary and

memoryless, where the battery output depends only on the current battery state and battery input (i.e., for all time t , $q_{W_t|X_t, S_{A,t}}^A = q_{W_t|X_t, S_A}^A$ for battery A and $q_{Y_t|W_t, S_{B,t}}^B = q_{Y_t|W_t, S_B}^B$ for battery B). In addition, in this class, the following conditions are satisfied:

$$P_{S_{A,2}|W_1}(s_{A,2}|w_1) = P_{S_{A,1}}(s_{A,2}) \quad \forall s_{A,2}, w_1 \quad (8)$$

$$P_{S_{B,2}|Y_1}(s_{B,2}|y_1) = P_{S_{B,1}}(s_{B,2}) \quad \forall s_{B,2}, y_1. \quad (9)$$

The above invariance conditions imply that $P_{S_{A,t}, W_t, X_t|W^{t-1}} = P_{S_{A,1}, W_1, X_1}$ and, in particular, the sequence W^T is an i.i.d. sequence, which can be proved by following similar steps in [5]. This allows [5, Lemma III.1] to hold, which reduces $\frac{1}{T}I(S_{A,1}, X^T, W^T)$ to $I(S_{A,1}, X_1, W_1)$ and $\frac{1}{T}I(S_{B,1}, W^T, Y^T)$ to $I(S_{B,1}, W_1, Y_1)$. These reductions are used in the derivation of an upper bound in the following section.

B. Upper Bound

Now, we propose an upper bound for the optimal leakage rate of the C-C system in the following theorem.

Theorem 1: The minimum leakage rate for C-C system can be upper-bounded by the following single-letter expression:

$$\begin{aligned} L^* \leq & \left\{ I(S_{A,1}, X_1; W_1) + I(S_{B,1}, W_1; Y_1) \right. \\ & \left. - I(S_{A,1}, S_{B,1}, X_1, Y_1; W_1) \right\} \\ & \forall \{q^A, q^B, P_{S_{A,1}}, P_{S_{B,1}}\} \in \mathcal{P}_{\text{inv}}. \end{aligned} \quad (10)$$

Proof: Assuming $\{q^A, q^B, P_{S_{A,1}}, P_{S_{B,1}}\} \in \mathcal{P}_{\text{inv}}$:

$$\begin{aligned} & I(S_{A,1}, S_{B,1}, X^T; Y^T) \\ & \stackrel{\text{a)}}{=} I(S_{A,1}, X^T; W^T) + I(S_{B,1}, W^T; Y^T) \\ & \quad - I(S_{A,1}, S_{B,1}, X^T, Y^T; W^T) \\ & \stackrel{\text{b)}}{=} T \cdot I(S_{A,1}, X_1; W_1) + T \cdot I(S_{B,1}, W_1; Y_1) \\ & \quad - I(S_{A,1}, S_{B,1}, X^T, Y^T; W^T) \\ & = T \cdot I(S_{A,1}, X_1; W_1) + T \cdot I(S_{B,1}, W_1; Y_1) \\ & \quad - \sum_{t=1}^T I(S_{A,t}, S_{B,t}, X^T, Y^T; W_t|W^{t-1}) \\ & \stackrel{\text{c)}}{=} T \cdot I(S_{A,1}, X_1; W_1) + T \cdot I(S_{B,1}, W_1; Y_1) \\ & \quad - \sum_{t=1}^T I(S_{A,t}^t, S_{B,t}^t, X^T, Y^T; W_t|W^{t-1}) \\ & \stackrel{\text{d)}}{\leq} T \cdot I(S_{A,1}, X_1; W_1) + T \cdot I(S_{B,1}, W_1; Y_1) \\ & \quad - \sum_{t=1}^T I(S_{A,t}, S_{B,t}, X_t, Y_t; W_t|W^{t-1}) \\ & \stackrel{\text{e)}}{\leq} T \cdot I(S_{A,1}, X_1; W_1) + T \cdot I(S_{B,1}, W_1; Y_1) \\ & \quad - \sum_{t=1}^T I(S_{A,t}, S_{B,t}, X_t, Y_t; W_t) \end{aligned}$$

$$\begin{aligned} & \stackrel{\text{f)}}{=} T \cdot I(S_{A,1}, X_1; W_1) + T \cdot I(S_{B,1}, W_1; Y_1) \\ & \quad - T \cdot I(S_{A,1}, S_{B,1}, X_1, Y_1; W_1), \end{aligned} \quad (11)$$

where a) holds due to a chain of manipulations of MI terms and the fact that $X^T \rightarrow W^T \rightarrow Y^T$ form a Markov chain; b) and f) hold due to [5, Lemma III.1] by assuming $\{q^A, q^B, P_{S_{A,1}}, P_{S_{B,1}}\} \in \mathcal{P}_{\text{inv}}$; c) holds due to the fact that $(S_{A,t}^t, S_{B,t}^t)$ is a deterministic function of $(S_{A,1}, S_{B,1}, X^{t-1}, W^{t-1}, Y^{t-1})$ given by equations (1) and (2); d) holds because the addition of letter increases MI; e) is due to $(X_t, S_{A,t}, W_t) \perp W^{t-1}$ from the invariance property, and hence, $W^{t-1} \perp W_t$ holds, which implies $I(S_{A,t}, S_{B,t}, X_t, Y_t; W_t) \leq I(S_{A,t}, S_{B,t}, X_t, Y_t; W_t|W^{t-1})$. ■

C. Lower Bound

In the following theorem, we derive a lower bound on the optimal leakage rate of the C-C system. binary input case

Theorem 2: The minimum information leakage of the C-C system is lower-bounded by that of the 2C system, i.e.,

$$L^* \stackrel{\text{a)}}{\geq} \inf_{q^J, P_{S_{J,1}}} L_{2C} \stackrel{\text{b)}}{=} \min_{P_{S_{J,1}}} I_{2C}(S_{J,1} - X_1; X_1). \quad (12)$$

Proof: Step b) is due to [5, Lemma III.3]. We will prove a) by proving the following claim.

Claim 1: Given initial state distributions $P_{S_{A,1}}$ and $P_{S_{B,1}}$ and a set of policies $\{q_{W_t|X^t, S_{A,t}^t, W^{t-1}}^A, q_{Y_t|W^t, S_{B,t}^t, Y^{t-1}}^B : t = 1, 2, \dots\}$ for the C-C system, there exist initial state distribution $P_{S_{J,1}}$ and a set of policies $\{q_{Y_t|X^t, S_{J,t}^t, Y^{t-1}}^J : t = 1, 2, \dots\}$ for the 2C system such that

$$P_{S_{A,1}^N + S_{B,1}^N, X^N, Y^N} = P_{S_{J,1}^N, X'^N, Y'^N} \quad \forall N \in \mathbb{N}. \quad (13)$$

Note that this claim completes the proof since

$$L^* = \frac{1}{T} I_{C-C}(S_{A,1}, S_{B,1}, X^T; Y^T) \quad (14)$$

$$= \frac{1}{T} I_{C-C}(S_{A,1}, S_{B,1}, S_{A,1} + S_{B,1}, X^T; Y^T) \quad (15)$$

$$\geq \frac{1}{T} I_{C-C}(S_{A,1} + S_{B,1}, X^T; Y^T) \quad (16)$$

$$= \frac{1}{T} I_{2C}(S_{J,1}, X'^T; Y'^T) \quad (17)$$

$$\geq \inf_{q^J, P_{S_{J,1}}} \frac{1}{T} I_{2C}(S_{J,1}, X'^T; Y'^T) \quad (18)$$

$$= \inf_{q^J, P_{S_{J,1}}} L_{2C} \quad (19)$$

where the RHS of (14)–(16) are evaluated under optimal initial distributions and set of policies for the C-C system that minimize the information leakage and the RHS of (17) is evaluated under the initial distribution and the set of policies for the 2C system that satisfy (13).

We prove Claim 1 by using induction. Let us assume arbitrary initial state distributions $P_{S_{A,1}}$ and $P_{S_{B,1}}$ and a set of policies $\{q_{W_t|X^t, S_{A,t}^t, W^{t-1}}^A, q_{Y_t|W^t, S_{B,t}^t, Y^{t-1}}^B : t = 1, 2, \dots\}$ for the C-C system. Note that from (6), we have

$$\begin{aligned} & P_{S_{A,1}, X_1, W_1, S_{B,1}, Y_1} \\ & = P_{S_{A,1}} P_{X_1} q_{W_1|X_1, S_{A,1}}^A P_{S_{B,1}} q_{Y_1|W_1, S_{B,1}}^B. \end{aligned} \quad (20)$$

From (20), we can derive

$$P_{S_{A,1}+S_{B,1},X_1,Y_1} = P_{S_{A,1}+S_{B,1}} P_{X_1} P_{Y_1|X_1,S_{A,1}+S_{B,1}} \quad (21)$$

because $S_{A,1}$, $S_{B,1}$, and X_1 are mutually independent. Now, we choose $P_{S_{J,1}}$ and $q_{Y'_1|X'_1,S_{J,1}}^J$ for the 2C system as

$$P_{S_{J,1}} = P_{S_{A,1}+S_{B,1}}, \quad (22)$$

$$q_{Y'_1|X'_1,S_{J,1}}^J = P_{Y_1|X_1,S_{A,1}+S_{B,1}}. \quad (23)$$

Then, we have

$$P_{S_{J,1},X'_1,Y'_1} = P_{S_{J,1}} P_{X'_1} q_{Y'_1|X'_1,S_{J,1}}^J \quad (24)$$

$$= P_{S_{A,1}+S_{B,1}} P_{X_1} P_{Y_1|X_1,S_{A,1}+S_{B,1}} \quad (25)$$

$$= P_{S_{A,1}+S_{B,1},X_1,Y_1} \quad (26)$$

since $P_{X_1} = P_{X'_1}$ according to our model. Thus, (13) holds for $N = 1$.

Now, to use induction, for $k \in \mathbb{N}$, let us assume that there exist initial state distribution $P_{S_{J,1}}$ and a set of policies $\{q_{Y'_t|X'_t,S_{J,1}}^J : t = 1, 2, \dots, k\}$ for the 2C system such that (13) holds for $N = k$. For the C-C case, the joint distribution $P_{S_{A,1}+S_{B,1},X^{k+1},Y^{k+1}}$ can be written as follows:

$$\begin{aligned} P_{S_{A,1}+S_{B,1},X^{k+1},Y^{k+1}} &\stackrel{a)}{=} P_{S_{A,1}+S_{B,1},X^k,Y^k} \cdot P_{X_{k+1}} \\ &\cdot \mathbb{1}_{S_{A,k+1}+S_{B,k+1}=S_{A,k}+S_{B,k}+Y_k-X_k} \\ &\cdot P_{Y_{k+1}|X^{k+1},S_{A,k+1}+S_{B,k+1},Y^k}. \end{aligned} \quad (27)$$

Note a) comes from (1) and (2). Now, we choose the policy for time $(k+1)$ for the 2C system as follows:

$$q_{Y'_{k+1}|X'^{k+1},S_{J,k+1},Y^k}^J = P_{Y_{k+1}|X^{k+1},S_{A,k+1}+S_{B,k+1},Y^k}. \quad (28)$$

Then,

$$\begin{aligned} P_{S_{J,k+1},X'^{k+1},Y'^{k+1}} &= P_{S_{J,k},X'^k,Y'^k} \cdot P_{X'_{k+1}} \\ &\cdot \mathbb{1}_{S_{J,k+1}=S_{J,k}+Y'_k-X'_k} \cdot q_{Y'_{k+1}|X'^{k+1},S_{J,k+1},Y'^k}^J. \end{aligned} \quad (29)$$

Now, by comparing (27) and (29) and noting that $P_{S_{A,1}+S_{B,1},X^k,Y^k} = P_{S_{J,k},X'^k,Y'^k}$ by the induction assumption, $P_{X_{k+1}} = P_{X'_{k+1}}$ from our model, and (28) due to our choice, we conclude that (13) holds for $N = k+1$.

By induction, Claim 1 is proved. \blacksquare

IV. SIMULATION RESULTS

In Fig. 2, the following five expressions are plotted and compared across various total battery size for the binary input case $x \in \{0, 1\}$ and the ternary input case $x \in \{0, 1, 2\}$:

- 1) Lower bound in Theorem 2.
- 2) Minimum of the upper bound in Theorem 1.
- 3) Upper bound in Theorem 1 computed for policies and initial state distributions that minimize the leakage rate for each battery individually.
- 4) L_{2C} evaluated using the well-known best effort algorithm (BEA) in [3] as the charging policy.
- 5) L_{2C} evaluated by applying the algorithm proposed by Yang *et al.* in [11], which minimizes the output variance by holding the output to a constant as much as possible.

According to Theorems 1 and 2, the optimal leakage rate of the C-C system should lie somewhere between 1) and the 2).

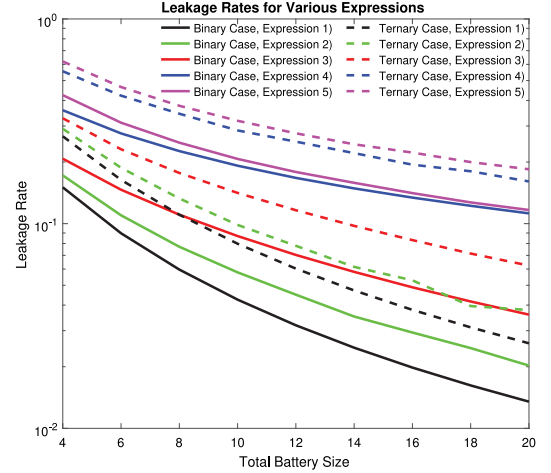


Fig. 2. Leakage rate versus total battery size for binary input.

Expressions 1), 2), and 3) are evaluated using `fmincon()` in MATLAB, where $0.5e-6$ is selected as the step size and interior-point is selected as the algorithm. Expression 1) is optimized over $P_{S_{J,1}}$. Expressions 2) and 3) are optimized over joint $\{q^A, q^B, P_{S_{A,1}}, P_{S_{B,1}}\} \in \mathcal{P}_{\text{inv}}$. In addition, the invariance condition is a linear constraint when the initial state distribution is fixed, as mentioned in [12] and [5]. However, since both the initial state distribution and policy are treated as variables, this constraint becomes nonlinear. Hence, we use the `nlcon` setting in `fmincon()` to constrain the optimization to $\{q^A, q^B, P_{S_{A,1}}, P_{S_{B,1}}\} \in \mathcal{P}_{\text{inv}}$, i.e., (8) and (9) are set as constraints. Also for simplicity, we assume that both batteries in the C-C system use the same charging policy for 2) and 3). Expression 4) is evaluated by the simulation technique described in [13] with the simulation length set to $T = 10^6$. Expression 5) is obtained by setting the cost to 0, since this letter focuses on the minimization of the privacy loss.

In Fig. 2, we show the achievable leakage rate for binary and ternary valued inputs as a function of the battery size. The binary input is taken to be equiprobable and the ternary input is taken to be binomial with parameter 0.5. Interestingly, the C-C system achieves a lower leakage rate than the two existing algorithms being compared: the algorithm in [11] as well as BEA. This is shown as 4) and 5) both lie above 2). In addition, Fig. 2 shows that the policy yielding the optimal leakage rate for a single battery does not necessarily lead to the minimum upper bound of the leakage rate for the C-C system, since 3) is above 2) for both input cases.

V. CONCLUSION

In this letter, we studied the privacy performance of a smart metering system with cascaded rechargeable batteries using MI as the privacy metric. We showed that the proposed system achieves better privacy performance than BEA and the algorithm proposed by Yang *et al.* However, we found that the uncooperative cascaded system may potentially achieve higher information leakage rate compare to the optimal leakage rate achieved by the single-battery system. Future work could involve the extension to a smart metering system with three or more rechargeable batteries in series.

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