Cost of Differential Privacy in Demand Reporting for Smart Grid Economic Dispatch

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***Abstract*—Increasing dynamics of electrical loads presents uncertainty and hence new challenges for power grid controls and optimization. In economic dispatch control (EDC) for minimizing generation cost, demand reporting by customers is a promising approach for managing the uncertainty, but it raises important privacy concerns. Adding random noise to aggregate queries of demand reports can provide differential privacy (DP) for the individual customers. But the noisy query results can adversely impact the EDC's optimality. In this paper, we analyze the privacy cost in demand reporting in terms of how DP-induced noise will increase the total generation cost. Our analysis shows that the noise amounts for different customers are intricately coupled with one another in determining the total cost. In view of the coupling, we apply the principle of Shapley value to attribute fair shares of the total cost to the power grid buses. For efficient sharing of the privacy cost, in a manner scalable to large power systems with many buses, we additionally propose heuristic algorithms to approximate the Shapley value. Trace-driven simulations based on a 5-bus power system model validate our analysis and illustrate the performance of the proposed cost sharing algorithms.**

I. Introduction

In the era of smart grids, electrical loads are becoming more dynamic and harder to predict due to, for example, the proliferation of demand-side renewables (e.g., solar panels), batteries for energy storage, and smart appliances that can react to their environments, as well as real-time energy pricing increasingly required by law for customers [1]. The unpredictability of load introduces new challenges for grid operators to optimize the efficiency of their power networks. Economic dispatch control (EDC) [2] is an important example problem. EDC plans the active power outputs of generators to meet future demand at the lowest total generation cost. In today's power grids, EDC uses a forecast-demand vector of the buses to send scheduled power outputs to the generators. Higher load dynamics will necessarily reduce the forecast accuracy and hence undermine the EDC's optimality [3]. To combat the uncertainty, demand reporting from end customers is a promising solution, in which smart meters installed at the consumption points report their future demand for the purpose of the EDC. Demand reporting has also been used in other demand response applications such as bid-based market clearing [4].

Demand reporting, however, leads to natural privacy concerns, because knowledge of power consumption may generally reveal sensitive information such as users' daily activities. For example, non-intrusive load monitoring (NILM) may infer a household's detailed tasks from a trace of its power consumption [5]. To mitigate the privacy concern, differential privacy (DP) [6] is a rigorous information-theoretic approach to prevent leakage of individual records by statistical (e.g., aggregate) queries on a database of these records. DP is relevant to the privacy of demand reporting for EDC, because EDC requires only *aggregate* demand forecast per-bus for its decisions. Hence, following the principle of DP, we may add random noise to the aggregate (future) demand reported by the customers and reveal only this noisy version of the aggregate [5], [7], [8], [9], [10]. The amount of noise added is commensurate with the required level of privacy protection. Using the noisy per-bus aggregates for the EDC will lead to suboptimal control, however. The resulting increased generation cost represents the cost of privacy. It is important to understand this privacy cost.

Based on the above key observations, we study two fundamental problems in this paper. First, how to quantify the system-wide total DP cost of demand reporting for EDC, in which different groups of customers may require different levels of privacy? Second, how to attribute fair shares of the total DP cost to the heterogeneous groups of customers so that, for example, customers having more stringent privacy requirements will have to bear a higher privacy cost because they impose higher inaccuracy of the input to the EDC? Our analysis and cost sharing algorithms will provide an important basis for practical implementation of demand reporting for EDC and other smart grid control applications, by allowing customers to acquire sufficient privacy protection on a fair cost basis. The attribution of privacy cost in this study is meant to apply in the context of an incentive program that motivates customers to take part in the demand reporting by passing on generation cost savings to the customers, although detailed design of this incentive program is beyond the scope of this paper. Hence, we do not expect that customers will make net payments for buying privacy for their demand reports. Rather, it is expected that they will receive compensation as reward for their participation in the demand reporting. The privacy cost may then serve as a negative adjustment to this compensation, so that the net incentive compensation for a more privacy-stringent customer will be lower but still positive.

In answering our research questions, we address the following three key challenges. **First**, existing studies on DP for smart meter data aggregation have considered only a single homogeneous level of required DP [5], [7], [8], [9], [10]. We believe that, whereas heterogeneous customers in the real world will desire different levels of privacy protection, a demand reporting scheme that admits a range of the provided DP will be more responsive to their needs. Moreover, existing studies do not address key features of the physical system, e.g., the grid's topology, required by the EDC. In this paper, we analyze heterogeneous DP protection and its impact based on realistic power grid topology. **Second**, inaccuracy of demand reports will mislead the EDC into a generation dispatch that does not balance the supply and demand, thereby causing under-/over-frequency. Load-frequency control (LFC), a closed-loop control system that regulates the grid frequency, will kick in to fix the mismatch in practice. Thus, we define the DP cost as discrepancy between the post-LFC generation cost and the generation cost without DP protection. However, because of complex dynamics of the LFC, it is non-trivial to derive the relationship between the post-LFC generation cost and the noise amounts in demand reports. **Third**, as our analysis shows, the noise amounts for different privacy groups are deeply coupled with one another in determining the total DP cost. This property precludes independent attribution of the DP cost for each group in isolation. We must account for any network effects of the DP between interdependent groups.

In addressing the above challenges, we make the following contributions:

- We propose a demand reporting scheme in which each customer chooses an ϵ value from a predefined offered set according to the ϵ-DP definition, and all the customers choosing the same ϵ constitute each privacy group. We extend the approach in [6] to achieve ϵ-DP for each group.
- Based on a power engineering model of the LFC, we derive an analytic expression for the total privacy cost. We prove that it is always non-negative. This expression is a prerequisite for computing the fair DP cost shares for the different buses.
- We apply the principle of Shapley value to attribute fair shares of the total privacy cost to the buses. However, although the Shapley value is an effective and well accepted conceptual device, its implementation does not scale well to a large power system with many buses. Thus, we propose heuristic algorithms of low complexity for the DP cost attribution problem among the buses and the privacy groups, and compare their performance with the Shapley value-based approach.
- We conduct extensive simulations based on a 5-bus power system model and real load traces to validate our analysis and illustrate the performance of cost sharing algorithms.

The balance of the paper is organized as follows. Section II reviews related work. Section III presents preliminaries and states our problem. Section IV presents the proposed demand-reporting scheme to support a range of DP requirements. Section V analyzes the total privacy cost. Section VI presents algorithms for attribution of fair shares of this privacy cost among buses or groups of customers. Section VII presents our simulation results. Section VIII concludes.

II. RELATED WORK

DP has been applied in smart metering to protect customers' privacy [5], [7], [8], [9], [10]. Its implementation is mainly based on distributing additive random noises [7], [8] among the smart meter readings to achieve DP of aggregate queries, such that demand-response aggregators cannot identify the data of individual customers. Since smart meters may fail, fault tolerance in modular addition-based encryption of aggregate meter readings is an important problem [7]. Won *et al.* [9] propose a proactive fault-tolerant aggregation protocol based on future ciphertexts, to ensure DP at higher communication efficiency and lower errors than previous fault tolerance approaches. Gulisano *et al.* [10] argue that DP-induced noises to metering data may adversely affect certain data analytics applications, and propose approaches to limiting the noise amounts. In battery-based load hiding, Zhao *et al.* [5] show that by enforcing household battery charging/discharging amounts to follow a binomial distribution, it is possible for the aggregation of power consumption over time to satisfy (ϵ, δ)-DP.

Although the research discussed above analyzes privacy in a power grid context, none of the results address how DP for customers' demand reports may impact the optimality of EDC as an important power grid control problem. We provide a novel analysis of the system's privacy cost that considers key characteristics of the physical control (e.g., power system topology and interaction between EDC and LFC), and present original insights of attributing fair shares of the overall cost among heterogeneous privacy groups of the customers.

EDC and LFC in power grids face new challenges due to increased supply/demand uncertainty arising from renewable generation and dynamic load. To mitigate the loss of efficiency stemming from this uncertainty, recent work [3], [11] has proposed to synchronize the EDC and LFC, whereas traditionally the two control mechanisms operate at different time scales. In [3], for example, the EDC is incorporated into generation-side LFC such that they both run at the same pace. In [11], load-side LFC is additionally integrated. Although synchronization of the EDC with LFC can improve the control in the face of uncertainty, such integrated EDC-LFC faces significant deployment barriers in that it will require major redesign of existing power grid control systems and electricity markets. In contrast, in this paper we leverage the increasing availability of advanced metering infrastructure (AMI) to allow accurate control without major changes to the EDC and LFC.

III. PRELIMINARIES

In this section, we first present preliminaries of DP and EDC. Then, we introduce the problem of EDC based on demand reporting. The notation convention of this paper is as follows. Take a symbol x as an example. $\mathbf{X}$ denotes a matrix, $\mathbf{x}$ a column vector, $x[t]$ the tth sample of a time series x (we omit the time index $[t]$ when it is clear), $\widetilde{x}$ the Laplace transform of x, $\dot{x}$ the derivative of x with respect to time. $\mathbb{R}^p$ and $\mathbb{R}^{p\times q}$ denote the sets of p-dimensional real column vectors and real $p \times q$ matrices, respectively. $\mathbb{D}$ denotes the domain of data sets. $\|\cdot\|$ denotes cardinality.

A. Differential Privacy (DP)

In this paper, we use ϵ-DP [6] as our privacy definition. It is formally defined as

Definition 1 ([6]). *A randomized algorithm $\mathcal{A} : \mathbb{D} \to \mathbb{R}^t$ gives ϵ-DP if for all data sets $D_1 \in \mathbb{D}$ and $D_2 \in \mathbb{D}$ differing on at most one element, and all $S \subseteq Range(\mathcal{A})$, $Pr(\mathcal{A}(D_1) \in S) \leq e^{\epsilon} \cdot Pr(\mathcal{A}(D_2) \in S)$.*

In other words, a differentially private algorithm $\mathcal{A}$ produces indistinguishable output for any two datasets which differ on a single element. Thus, an adversary cannot infer the value of a single user's data in the dataset. The parameter ϵ characterizes the level of privacy. A smaller ϵ implies better privacy. Prior research [12] shows that, by adding independent and identically distributed (i.i.d.) Laplacian noise to the output of a function $\mathcal{F}$, we may achieve ϵ-DP. Let $\mathrm{Lap}(\lambda)$ denote a zero-mean Laplace distribution of probability density function (PDF) $f(x|\lambda) = \frac{1}{2\lambda} e^{\frac{|x|}{\lambda}}$. We have the following lemma.

Lemma 1 ([12]). *For all function $\mathcal{F} : \mathbb{D} \to \mathbb{R}^t$, the following algorithm $\mathcal{A}$ gives ϵ-DP: $\mathcal{A}(D) = \mathcal{F}(D) + [x_1, x_2, \ldots, x_r]^\intercal$, where the x_i are drawn i.i.d. from $\mathrm{Lap}(S(\mathcal{F})/\epsilon)$ and $S(\mathcal{F})$ denotes the global sensitivity of $\mathcal{F}$.*

We also introduce an *infinite divisibility property* [13] of the Laplace distribution. Denote by $\mathrm{Gamma}(n, \lambda)$ a Gamma distribution whose PDF is $f(x|n, \lambda) = \frac{(1/\lambda)^{1/n}}{\Gamma(1/n)} x^{\frac{1}{n}-1} \mathrm{e}^{-x/\lambda}$, where $\Gamma(1/n)$ is the Gamma function evaluated at $1/n$. Let $\gamma_{1,i}$ and $\gamma_{2,i}$ denote two random variables that are drawn i.i.d. from $\mathrm{Gamma}(n, \lambda)$. Then, for any natural number n, $x = \sum_{i=1}^{n} \gamma_{1,i} - \gamma_{2,i}$ follows $\mathrm{Lap}(\lambda)$.

B. Economic Dispatch Control (EDC)

EDC is the determination of active power outputs of generators to meet the system load at the lowest generation cost, subject to various transmission and operational constraints. It is a form of tertiary generation control that updates the setpoints of LFC periodically (e.g., every five minutes or longer [14], [15]). A formal formulation of EDC is as follows.

Consider a power grid with N buses and L transmission lines. Denote by $\mathcal{G}$ and $\mathcal{L}$ the sets of generator buses and load buses, respectively. For a generator or load bus, say i, denote by p_i^g and p_i^l the active power generation and consumption, respectively, where $p_i^g \geq 0$ and $p_i^l \leq 0$ following the convention that power injection/draw is positive/negative. To simplify the discussion, we assume that a load bus is not connected with any generator (i.e., $p_i^g = 0$ if $i \in \mathcal{L}$) and a generator bus is not connected with any load (i.e., $p_i^l = 0$ if $i \in \mathcal{G}$). Denote by $\mathbf{p}^g$ and $\mathbf{p}^l$ the vectors composed of p_i^g and p_i^l for all buses, respectively. EDC is subject to the following three constraints. First, the following linearized nodal power flow constraint is widely adopted in power system analysis [16]:

$$\mathbf{p}^g + \mathbf{p}^l = \mathbf{A}^\intercal \mathbf{B} \mathbf{A} \boldsymbol{\theta}, \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{L\times N}$ is the incidence matrix characterizing the power grid topology, $\mathbf{B} \in \mathbb{R}^{L\times L}$ is a diagonal matrix whose diagonal elements represent the susceptance of the corresponding line, and $\boldsymbol{\theta} \in \mathbb{R}^N$ is a vector of the voltage phase angles at the buses. Second, the difference between the phase angles of any two connected buses should be within $[-\pi/2, \pi/2]$ to ensure the grid's stability [17]. This constraint can be represented compactly as

$$-\boldsymbol{\pi}/2 \leq \mathbf{A}\boldsymbol{\theta} \leq \boldsymbol{\pi}/2, \tag{2}$$

where $\boldsymbol{\pi} = [\pi, \ldots, \pi]^\intercal$. Lastly, each generator's output is within its capacity, i.e.,

$$0 \leq p_i^g \leq \overline{p}_i^g, \quad \forall i \in \mathcal{G}, \tag{3}$$

where $\overline{p}_i^g$ represents the capacity of the generator at bus i. Let $C_i(p_i^g)$ denote the generation cost of this generator. EDC solves the following constrained optimization problem:

$$\underset{\mathbf{p}^g, \boldsymbol{\theta}}{\text{minimize}} \sum_{i\in\mathcal{G}} C_i(p_i^g), \text{ subject to} \,(1), (2), (3). \tag{4}$$

Note that the input to Eq. (4) is $\mathbf{p}^l$. In this paper, we use $\mathrm{EDC}(\mathbf{p}^l)$ to denote the solution to Eq. (4).

C. Demand Reporting based EDC (DR-EDC)

In today's deregulated electricity markets, EDC is deeply integrated with real-time pricing systems [2]. Specifically, a profit-neutral independent system operator (ISO) takes as input offers of supply from generators and demand bids from utilities to compute real-time locational prices to clear the market. Meanwhile, it uses demand forecast from the bids as $\mathbf{p}^l$ to solve the EDC problem in Eq. (4). Thus, improving the quality and accuracy of the demand forecast will improve the cost optimality and reliability of the EDC [18]. This is especially important in the era of smart grids, where there are increased demand dynamics and uncertainty.

In this paper, we consider a new scheme of EDC that we call demand-reporting based EDC (or DR-EDC). At the beginning of each DR-EDC cycle, the smart meters of customers report their demand in one or more future cycles to their respective utilities. The aggregated future demand is then used as input to the EDC. Note that an EDC cycle is typically five minutes or longer [14], [15], and existing AMIs can already sustain five-minute reporting intervals [19]. Compared with conventional EDC driven by utilities' per-bus demand forecasts, the DR-EDC driven by direct demand reports from the customers can better manage load uncertainty. It is because the customers' own smart meters have much better knowledge of the future consumption than the utilities, e.g., they have direct access to control and scheduling decisions of home automation systems, smart appliances, battery systems, etc. The more accurate control will reduce generation costs (as illustrated in Section VII-A), which will translate generally into monetary rewards for the customers as incentives or rebates. However, there is the concomitant need to understand how the privacy requirements of customers will impact the performance limits of the DR-EDC and hence their net contributions, which is a main concern of this paper.

IV. Differentially Private Demand Reporting

This section proposes a demand-reporting scheme that provides ϵ-DP under different ϵ values for different groups of users. In the following, Section IV-A describes our system and threat models. Section IV-B describes the proposed scheme.

A. System and Threat Models

1) System model: For simplicity, we assume that the demand (or load) of each customer does not change during an DR-EDC cycle. Denote by $\mathcal{N}_i$ the set of customers attached to load bus i, $p^l_{i,j}[t]$ the demand report sent by the jth customer in $\mathcal{N}_i$ at the beginning of the tth DR-EDC cycle. We assume that each bus i is served by an *aggregator*, denoted by A_i. A_i receives the demand reports from the customers in $\mathcal{N}_i$ and reports the aggregated demand to the ISO for the EDC. The demand aggregation among $\mathcal{N}_i$ can be implemented based on homomorphic encryption [20] to prevent the aggregator and any intermediate nodes in the aggregation tree from knowing the individual $p^l_{i,j}$. In this paper, we assume that all the customers participate in the demand reporting. But our analysis can be readily extended to address non-participating customers.

2) Threat model: The adversary can be any "curious" entity having access to the per-bus aggregated demand reports, but not the actual per-bus power consumption that is available to the ISO only. For example, the adversary can be an DR-EDC aggregator or a subscriber to the per-bus aggregated demand reports (e.g., the DR-EDC operator). The adversary aims to infer the demand reports of individual customers from the aggregates. Our scheme provides a guaranteed level of DP to the customers against such an adversary. Our threat model is similar to those used in representative DP research for smart metering (e.g., [7], [8]).

B. DP-Assured Demand Reporting Scheme

This section presents a demand reporting scheme that guarantees DP. It is based on the basic principle discussed in [8], although our scheme is more general in that it supports multiple levels of privacy requirement. Moreover, our scheme differentiates customers according to the load buses they are on, which is needed for analyzing the impact of DP on the EDC in Section V. The proposed scheme works as follows.

1) Formation and maintenance of privacy groups: Our scheme offers K different pre-defined ϵ values denoted by $\epsilon_1, \epsilon_2, \ldots, \epsilon_K$, where $0 < \epsilon_1 < \epsilon_2 < \ldots < \epsilon_{K-1}$ and $\epsilon_K = \infty$. As discussed in Section III-A, a smaller ϵ implies better privacy. In particular, from Lemma 1, with an infinitely large ϵ (i.e., ϵ_K), the variance of the zero-mean Laplace distribution $\mathrm{Lap}(S(\mathcal{F})/\epsilon)$ is zero, which suggests there is zero noise and hence no DP protection. Each customer chooses an ϵ from the K values offered, according to her needs. In practice, smart meters owned by the customer can be configured and programmed to suitable settings automatically. At bus i, the set of customers choosing ϵ_k is denoted by $\Phi_{i,k}$, where $1 \le k \le K$. Thus, $\sum_{k=1}^{K} \|\Phi_{i,k}\| = \|\mathcal{N}_i\|$. In this paper, $\Phi_{i,k}$ is also called the kth *privacy group* of the bus i and k represents the *privacy level*. The customers in $\mathcal{N}_i$ send their selected privacy levels to the aggregator A_i. Then, for each $k \in [1, K]$, the aggregator sends the size of the kth privacy group, i.e., $\|\Phi_{i,k}\|$, to the customers in $\Phi_{i,k}$. When a new customer joins the bus i or an existing customer leaves it or changes her privacy level, A_i will inform relevant customers in $\mathcal{N}_i$ of the change, to ensure that each customer knows the current size of the privacy group that she belongs to.

2) Noising demand reports: At the tth DR-EDC cycle, for the kth privacy group at the bus i, we define the function $\mathcal{F}$ in Lemma 1 as $\mathcal{F}_{i,k}(p^l_{i,j}[t'] | j \in \Phi_{i,k}, t' \in [1,t]) = [\sum_{j\in\Phi_{i,k}} p^l_{i,j}[1], \ldots, \sum_{j\in\Phi_{i,k}} p^l_{i,j}[t]]^\intercal \in \mathbb{R}^t$. At the beginning of the tth DR-EDC cycle, each customer j in $\Phi_{i,k}$ draws two i.i.d. samples, $\gamma_{j,1}[t]$ and $\gamma_{j,2}[t]$, from the Gamma distribution $\mathrm{Gamma}(\|\Phi_{i,k}\|, S(\mathcal{F}_{i,k})/\epsilon_k)$. Recall that the customer obtains the parameter $\|\Phi_{i,k}\|$ of the Gamma distribution during the formation or maintenance of the privacy groups. By extending the approach in [8] to address privacy groups, $S(\mathcal{F}_{i,k})$ can be set to an upper bound of the per-customer demand among all the customers in $\Phi_{i,k}$.[1] Then, the customer generates a noisy version of the demand report, i.e., $p^l_{i,j}[t] + \gamma_{j,1}[t] - \gamma_{j,2}[t]$. Through an aggregation protocol based on homomorphic encryption, A_i obtains an aggregated demand for the kth privacy group denoted by $P_{i,k}[t]$:

$$P_{i,k}[t] = \sum\nolimits_{j\in\Phi_{i,k}} p^l_{i,j}[t] + \sum\nolimits_{j\in\Phi_{i,k}} \gamma_{j,1}[t] - \gamma_{j,2}[t]. \quad (5)$$

From Lemma 1 and the infinite divisibility property of the Laplace distribution (see Section III-A), the scheme above gives ϵ_k-DP for the kth privacy group at each bus.

V. Total Cost of Differential Privacy

In this section, we first define the total cost of DP in the DR-EDC. Then, we derive its analytic expression and show that it is always non-negative.

A. Definition of Total Privacy Cost

At the beginning of an EDC cycle, if given the true demand vector $\mathbf{p}^l$, the optimal economic dispatch (denoted by $\mathring{\mathbf{p}}^g$) is given by Eq. (4), i.e., $\mathring{\mathbf{p}}^g = \mathrm{EDC}(\mathbf{p}^l)$. However, under the demand reporting scheme in Section IV to ensure DP, the ISO obtains a noisy demand vector $\widehat{\mathbf{p}}^l = \mathbf{p}^l + \mathbf{n}$, where the ith element of $\mathbf{n}$ (denoted by n_i) that corresponds to the load bus i is given by $n_i = \sum_{k=1}^{K} \sum_{j\in\Phi_{i,k}} \gamma_{j,1} - \gamma_{j,2}$. Based on the inaccurate demand vector $\widehat{\mathbf{p}}^l$, the EDC solution $\widehat{\mathbf{p}}^g_0 = \mathrm{EDC}(\widehat{\mathbf{p}}^l)$ will be a generation dispatch that generally

[1] To guarantee ϵ-DP, the historical peak load of the bus can be used as a conservative and loose upper bound. The privacy cost analysis in this paper does not rely on accurate setting of this upper bound. We refer the reader to [10] for managing large noises resulting from conservative settings.

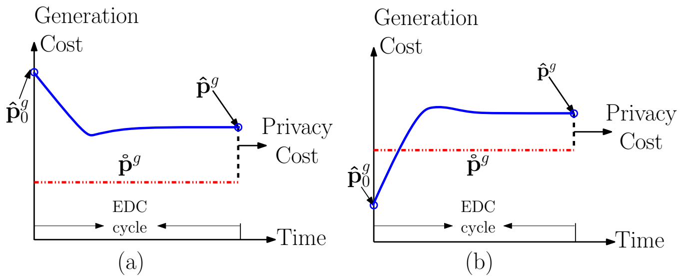

Fig. 1. Illustration of the total privacy cost in DR-EDC. Each customer adds a positive noise in (a) and a negative noise in (b), respectively.

cannot balance the total generation and the total demand. As discussed in Section III-B, the EDC solution is used to update the setpoints of the LFC. Starting from the initial setting $\widehat{\mathbf{p}}_0^g$, the LFC will adjust the power outputs of the generators, via a closed-loop control based on real-time measurements of $\mathbf{p}^l$, to exactly meet the demand and regulate the system frequency at the required nominal value. Thus, under the LFC, the actual generator outputs will converge to a new state, which we denote as $\widehat{\mathbf{p}}^g$. We note that the control cycle of the LFC is often two to four seconds and the convergence from $\widehat{\mathbf{p}}_0^g$ to $\widehat{\mathbf{p}}^g$ often takes a few LFC cycles. Denote by $\mathring{p}_i^g$ and $\widehat{p}_i^g$ the ith element of $\mathring{\mathbf{p}}^g$ and $\widehat{\mathbf{p}}^g$, respectively. For the tth DR-EDC cycle, the total privacy cost, denoted by $c[t]$, is defined as

$$c[t] = \sum\nolimits_{i\in\mathcal{G}} C_i(\widehat{p}_i^g[t]) - C_i(\mathring{p}_i^g[t]). \tag{6}$$

Thus, the accumulated total privacy cost up to the tth DR-EDC cycle is given by $\sum_{t'=1}^{t} c[t']$.

We now use Fig. 1 to illustrate the total privacy cost for a certain DR-EDC cycle. The two subfigures of Fig. 1 show the trajectories of the total generation cost during the DR-EDC cycle. To simplify the illustration, each customer adds a positive noise in the demand reporting in Fig. 1(a) and a negative noise in Fig. 1(b), respectively. In both the subfigures, the horizontal straight dotted lines represent $\sum_{i\in\mathcal{G}} C_i(\mathring{p}_i^g)$, i.e., the minimized total generation cost when the ISO is given the true demand vector $\mathbf{p}^l$. In Fig. 1(a), as the customers report demand values that are larger than their actual loads during the DR-EDC cycle, by following the generation dispatch $\widehat{\mathbf{p}}_0^g$, the generators will generate more power than actually demanded. As a result, the total generation cost will be high (as illustrated by the starting point of the blue curve) and over-frequency will be observed. After a transient LFC process, at the end of the DR-EDC cycle, the system frequency is restored back to the nominal value and the generators' outputs converge to $\widehat{\mathbf{p}}^g$. The associated total generation cost is $\sum_{i\in\mathcal{G}} C_i(\widehat{p}_i^g)$, as illustrated by the end point of the blue curve. The height of the vertical dashed line represents the total privacy cost. In Fig. 1(b), as the customers report demand values that are smaller than their actual loads, the generators will generate less power than actually demanded, resulting in low total generation cost and under-frequency. After a transient LFC process, the total generation cost will be higher than $\sum_{i\in\mathcal{G}} C_i(\mathring{p}_i^g)$.

B. Analytic Expression of Total Privacy Cost

An analytic expression of the total privacy cost is a prerequisite for fairly attributing shares of this cost to the customers. The ISO can compute $\mathring{p}_i^g$ in Eq. (6) right after it measured the actual bus loads $\mathbf{p}^l$. We note that the total power draw at each load bus is often directly measured in real time (e.g., every second) by power flow meters. However, the ISO has to wait until the convergence of the LFC to measure the $\widehat{p}_i^g$ in Eq. (6). In this section, through analysis based on a dynamic model of LFC that is widely adopted in power engineering, we obtain analytic expressions of $\widehat{p}_i^g$ and the total privacy cost defined in Eq. (6). The analytic expressions will be needed to compute the privacy cost shares of buses using the Shapley value approach in Section VI-A. Moreover, the ISO can use them to compute the total privacy cost once the $\mathbf{p}^l$ is measured. This improves the timeliness of the ISO's knowledge of the privacy cost.

Denote by $\widehat{p}_{0,i}^g$ the ith element of the DR-EDC solution $\widehat{\mathbf{p}}_0^g$ that corresponds to a generator bus i, and n_i the ith element of $\mathbf{n}$ that corresponds to a load bus i (i.e., the total noise in the aggregated demand report for load bus i). We note that $\mathbf{n}$ and n_i can be measured by the ISO once $\mathbf{p}^l$ is measured, since $\mathbf{n} = \widehat{\mathbf{p}}^l - \mathbf{p}^l$. We have the following lemma. The proof can be found in the appendix.

Lemma 2. *For a certain DR-EDC cycle,*

$$c = \sum_{i\in\mathcal{G}} C_i\left(\widehat{p}_{0,i}^g + \frac{G_i}{\sum_{i\in\mathcal{G}} G_i}\cdot\sum_{i\in\mathcal{L}} n_i\right) - C_i(\mathring{p}_i^g), \tag{7}$$

where G_i is the LFC gain for the generator at the bus i.

Note that the LFC gain G_i is a constraint known to the ISO. We refer to the proof in the appendix for details of this constant gain. Note that Eq. (7) is not a closed-form formula for c, because both $\widehat{p}_{0,i}^g$ and $\mathring{p}_i^g$ are obtained through solving the constrained optimization problem in Eq. (4). The following lemma gives an important property of the total privacy cost c.

Lemma 3. *For any DR-EDC cycle, $c \geq 0$.*

Proof. After the LFC converges, the generators' outputs $\widehat{\mathbf{p}}^g$ must satisfy the steady-state constraints in Eqs. (1), (2), and (3) with $\mathbf{p}^g$ replaced by $\widehat{\mathbf{p}}^g$ and p_i^g replaced by $\widehat{p}_i^g$. Since $\mathring{\mathbf{p}}^g$ is the optimal solution that minimizes the total generation cost subject to the same steady-state constraints that $\widehat{\mathbf{p}}^g$ satisfies, we must have $\sum_{i\in\mathcal{G}} C_i(\widehat{p}_i^g[t]) \geq \sum_{i\in\mathcal{G}} C_i(\mathring{p}_i^g[t])$ and $c \geq 0$. □

We now use a numeric example to illustrate the above non-negative property. The example is based on an IEEE 14-bus test system [21] that has two generators at bus 1 and bus 2, respectively. (There are three synchronous condensers that output reactive power only.) The following results are for a certain DR-EDC cycle, where the loads are fixed to their initial values specified in the system model. In Fig. 2, the blue curve illustrates all the possible values of $\sum_{i\in\mathcal{G}} C_i(\widehat{p}_i^g[t])$ when the $\widehat{\mathbf{p}}^g$ satisfies the steady-state constraints in Eqs. (1), (2), and (3). To help illustration, we use a flat plane in Fig. 2

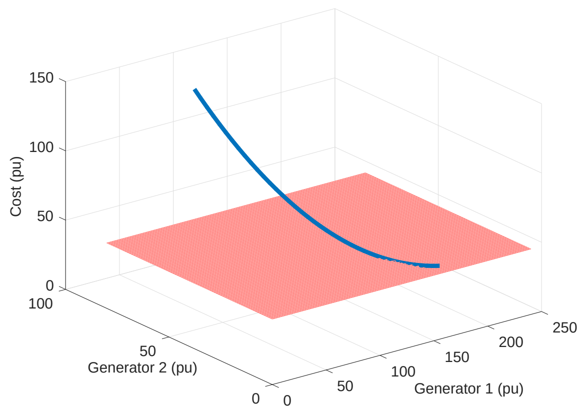

Fig. 2. All possible values of $\sum_{i\in\mathcal{G}} C_i(\widehat{p}_i^g[t])$ after LFC converges (represented by the blue curve) and the minimized generation cost $\sum_{i\in\mathcal{G}} C_i(\mathring{p}_i^g[t])$ (represented by the red flat plane) for the IEEE 14-bus test system.

to represent $\sum_{i\in\mathcal{G}} C_i(\mathring{p}_i^g[t])$. We can see that $\sum_{i\in\mathcal{G}} C_i(\widehat{p}_i^g[t])$ is lower-bounded by $\sum_{i\in\mathcal{G}} C_i(\mathring{p}_i^g[t])$.

VI. Differential Privacy Cost Sharing

Given the total cost of DP, a fundamental question is how to fairly attribute fair shares of the cost among customers requesting different privacy levels. In this section, we first consider the problem of sharing the cost among buses. Then, we consider the problem of further distributing the per-bus privacy cost to customers attached on the bus.

A. Bus-Level Privacy Cost Sharing

As discussed in Section V-B, the aggregated noise in the demand report of each bus, i.e., n_i, can be measured. However, the complex and non-linear relationship between c and each n_i will render fitting c as $c = \sum_{i\in\mathcal{L}} f_i(n_i)$ extremely difficult, where f_i's are the fitting functions. (We note that if we could find such a fitting, $f_i(n_i)$ would be a fair cost share for bus i.) Indeed, from our regression-based fitting trials, the aggregated noise amounts n_i are deeply coupled with one another in determining the c. To address this issue, in Section VI-A1 we apply the principle of Shapley value [22] for attributing privacy costs with several desirable properties. However, the computation of Shapley value has poor scalability to a large number of buses. Thus, we additionally propose two heuristic but efficient cost sharing approaches in Section VI-A2, and compare their performance with that of the Shapley value approach.

1) Shapley value-based approach: The Shapley value is a solution concept for fairly distributing a total value generated by a coalition of players in cooperative games [22]. It reflects a player's marginal contribution to all possible coalitions, and has several desirable properties such as efficiency, symmetry, and linearity [22]. The solution concept can be similarly applied for attributing cost, which is a dual concept of value. To facilitate presentation, in this paper we use the term "Shapley cost" to refer to the cost share of a "player" (i.e., a bus in the current context) as determined by the Shapley value principle. Specifically, the Shapley cost of a bus is the marginal increase in generation cost due to the bus in the coalition.

We now describe how to compute the Shapley costs of buses. For bus i, if it is in the coalition, it reports its *noisy* aggregated demand (i.e., $p_i^l + n_i$) to the aggregator; otherwise, it reports its *actual* aggregated demand (i.e., p_i^l). Denote by $\mathcal{S}$ the coalition of buses. From Lemma 2, the total privacy cost given a coalition of buses $\mathcal{S}$ is $c(\mathcal{S}) = \sum_{i\in\mathcal{G}} C_i\left(\widehat{p}_{0,i}^g(\mathcal{S}) + \frac{G_i}{\sum_{i\in\mathcal{G}} G_i} \cdot \sum_{i\in\mathcal{S}} n_i\right) - C_i(\mathring{p}_i^g)$, where $\widehat{p}_{0,i}^g(\mathcal{S})$ is the initial power output of the generator at bus i as solved by the EDC based on noisy demand reports from the buses in $\mathcal{S}$ and actual demand values from the remaining buses. By the Shapley's principle, the Shapley cost of a load bus i, denoted by c_i^s, is

$$c_i^s = \frac{1}{\|\mathcal{L}\|!} \sum_{\pi\in\mathrm{perm}(\mathcal{L})} c(\mathcal{S}(\pi, i)) - c(\mathcal{S}(\pi, i)\backslash i), \tag{8}$$

where π is a permutation of the load buses in $\mathcal{L}$, $\mathcal{S}(\pi, i)$ is a subset of π that includes the buses in π no later than i in order. From the efficiency property of Shapley value, the total privacy cost is shared among all the load buses, i.e., $c = \sum_{i\in\mathcal{L}} c_i^s$. We note that the Shapley cost of a load bus can be negative (see the numeric results in Section VII-C). For example, when the demand report of the bus has negative noise and all the other buses use positive noise, the negative noise may offset in part the positive noise and hence reduce the total generation cost. In this case, the bus in point should be rewarded with a negative Shapley cost. For the load bus i, the accumulated Shapley cost up to the tth DR-EDC cycle is $\sum_{t'=1}^{t} c_i^s[t']$.

2) Heuristic approaches: Despite several desirable properties of the Shapley cost [22], Eq. (8) has exponential complexity in the number of load buses due to the permutation. This renders the Shapley cost infeasible to compute for large power grids. To manage the computational cost, a Monte Carlo method can be applied to approximate Eq. (8) [23]. However, the method does not guarantee exact answers and often it still requires high compute overhead for good results [23]. Therefore, we now present two heuristic cost sharing approaches based on assigning privacy cost as a function of the corresponding prescribed noise. Specifically, in the two approaches we use respectively the magnitude and variance of n_i as weight for proportional sharing of the privacy cost. Formally, the cost shares of load bus i, denoted respectively by c_i^n and c_i^σ for the two approaches, are given by $c_i^n = \frac{|n_i|}{\sum_{i\in\mathcal{L}} |n_i|} \cdot c$ and $c_i^\sigma = \frac{\sigma_i^2}{\sum_{i\in\mathcal{L}} \sigma_i^2} \cdot c$, where σ_i^2 is the variance of n_i. In the definition of c_i^n, the rationale of using the magnitude of n_i as weight is that either a positive or negative n_i should incur a positive privacy cost (see Fig. 1). The σ_i^2 in c_i^σ can be estimated based on historical trace of n_i. In Section VII-C, we will evaluate the effectiveness of the above two heuristic approaches by comparing their results with the corresponding Shapley costs.

B. Customer-Level Sharing of Privacy Cost

This section discusses how to further distribute the cost share of bus i, i.e., c_i, to customers attached on the bus. We assume that the customers do not report their actual power consumption due to privacy protection. Thus, the noise introduced by an individual customer j, i.e., $\gamma_{j,1} - \gamma_{j,2}$, cannot be measured. As a result, the Shapley cost described in Section VI-A1 cannot be applied to customer-level cost sharing. Instead, our proposed approach first divides c_i among the privacy groups, and then further divides a group's share to the group's customers. We adopt a heuristic approach similar to those proposed in Section VI-A2 and use $\|\Phi_{i,k}\| \cdot 2 \cdot (S(\mathcal{F}_{i,k})/\epsilon_k)^2$ as weight for the kth privacy group in dividing c_i among the privacy groups. Note that $2 \cdot (S(\mathcal{F}_{i,k})/\epsilon_k)^2$ is the variance of the kth privacy group's Laplacian noise, which follows $\mathrm{Lap}(S(\mathcal{F}_{i,k})/\epsilon_k)$. Thus, our solution considers jointly the privacy group size $\|\Phi_{i,k}\|$ and the noise variance. A privacy group's cost share is further divided equally among all the customers in the group. In the above approach, the customers who do not require DP (i.e., $\epsilon = \infty$) share no privacy cost.

The power distribution network attached to a bus often adopts a tree topology, in which the customers are the leaf nodes of the tree. Some intermediate nodes of the tree may have power meters installed. Therefore, the noise amounts for demand reports of subtrees rooted at these intermediate nodes can be measured. Our future work will investigate how to apply Shapley costs to these subtrees in order to refine the fairness of the customer-level cost sharing.

C. Implementation

The bus-level privacy cost sharing approaches can be readily integrated with various ex-post real-time pricing schemes that are popular in wholesale markets such as New England ISO, PJM, and Midwest ISO. Ex-post electricity prices are determined based on load measurements of the buses, which are often obtained at the end of each EDC cycle. The load measurements can also be used to compute the privacy cost shares of the buses using the methods in Section VI-A.

Relaying upstream real-time prices to end customers is generally considered a desirable feature of smart grids. It has been implemented by utilities such as ComEd [24] and Ameren [25]. The customer-level privacy cost sharing in Section VI-B can be readily integrated with such real-time pricing for end customers.

VII. Simulations

Our simulations are based on the 5-bus power system model shown in Fig. 3 ([17], Chapter 6, pp.327). The two generators' cost functions are quadratic functions of different parameters. We simulate a total of 200 customers in the system. For DP, each customer chooses an ϵ value from $\{0.5, 1, 1.5, 2.5, \infty\}$. The EDC cycle is five minutes. Hourly load data traces of U.S. domestic customers in January 2015 [26] are interpolated and used to set the loading of buses in our simulations. Unless otherwise specified, all the power values in this section are normalized to per unit (pu) values.

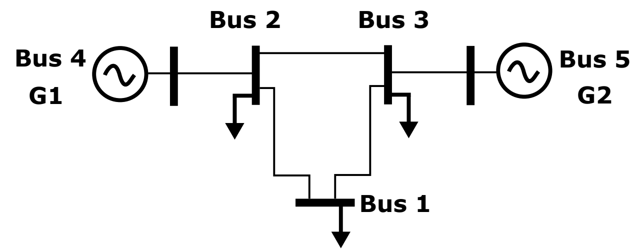

Fig. 3. 5-bus system ([17], Chapter 6, pp.327).

TABLE I
DEMAND REPORTING VS. DEMAND FORECASTING

Scheme	Additional cost (%)
DR-EDC with 5% customers choosing $\epsilon = 0.5$	3.28
DR-EDC with no customers choosing $\epsilon = 0.5$	2.83
EDC based on demand forecasting	5.02

A. Demand Forecasting vs. Demand Reporting

This set of simulations compares the privacy cost of DR-EDC with the additional generation cost caused by inaccuracy of traditional demand forecasting. The demand forecasting is based on the widely adopted persistence model [27], in which the immediate past load is used as the current demand forecast for each bus. To evaluate the DR-EDC, we simulate the system for 100 rounds, where each round corresponds to the load traces on one day. At the beginning of each round, the customers randomly choose their ϵ values. The average aggregated total privacy cost over the 100 rounds is presented.

Table I shows the results for DR-EDC under two settings, as well as traditional EDC based on demand forecast. The additional generation costs due to either DP-induced noise or inaccurate forecast are in percentages of the total generation cost. In the first DR-EDC setting, ten out of 200 (i.e., 5%) customers choose the highest privacy level, i.e., $\epsilon = 0.5$, resulting in a 3.28% privacy cost; in the second setting, no customers choose this highest privacy level, resulting in a 2.83% privacy cost. This result is consistent with intuition that the privacy cost decreases with the customers' required privacy. By comparison, the results based on demand forecast show an additional cost of around 5%, which is higher than the privacy cost of DR-EDC in either setting. Note that this 5% additional cost is significant and comparable to line loss (e.g., 7% in U.S. [28]). Furthermore, it would increase with higher demand uncertainty. In contrast, as our results show, DR-EDC with DP protection can reduce the additional cost by up to 43%, which is substantial.

B. Total Privacy Cost over a Day

This set of results shows how the total privacy cost changes with load and customers' DP-induced noise over time. At the beginning of the simulation, each customer randomly chooses a privacy level. The distribution of customers choosing the five ϵ values from 0.5 to ∞ is 18.5%, 17%, 21%, 24%, and 19.5%. Fig. 4(a) shows the actual total load (red curve) and aggregate of the demand reports (blue curve) over a day. Fig. 4(b) shows

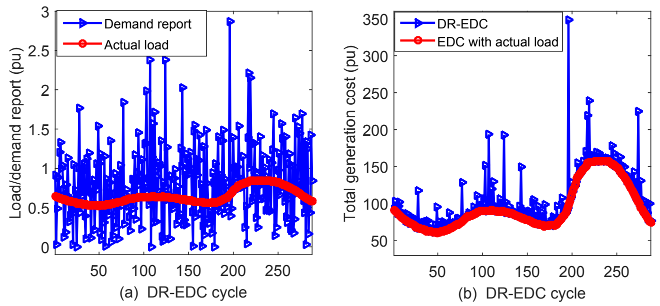

Fig. 4. Load, demand report, and generation cost over a day. (a) Total load and the demand report aggregate. (b) Total generation costs of the EDC given actual load and the DR-EDC.

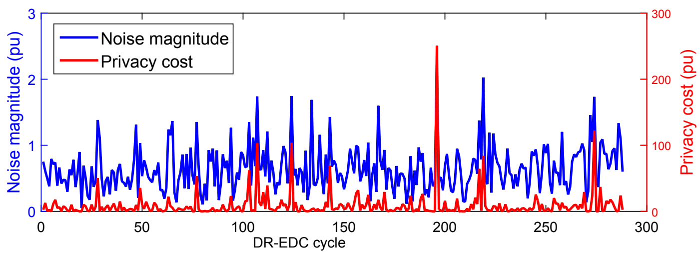

Fig. 5. Total noise magnitude and total privacy cost over a day.

the total generation costs of the EDC given the actual load (red curve) and the DR-EDC (blue curve). We can see that the total generation cost of DR-EDC is always higher than that of EDC given the actual load. This result is consistent with Lemma 3.

Fig. 5 shows the magnitude of total noise (i.e., absolute difference between the two curves in Fig. 4(a)) and the total privacy cost (i.e., difference between the two curves in Fig. 4(b)). We can observe that the total privacy cost is not always proportional to the magnitude of the total noise. This is because the total privacy cost is also affected by the distribution of noise among the buses. Intuitively, if a generator with a low generation cost curve $C_i(\cdot)$ is close to a load bus with a high noise for its demand report, the noise will not cause a significant increase in the generation cost.

C. Evaluation of Privacy Cost Sharing Approaches

1) Bus-level cost sharing: We compare the three bus-level cost sharing approaches discussed in Section VI-A, namely the Shapley cost (SC) and the two heuristic approaches using respectively noise magnitude (NM) and noise variance (NV) as weight to split the privacy cost among the buses. Each simulation lasts for 300 DR-EDC cycles. Fig. 6 shows the demand-report noises of the three load buses in four selected DR-EDC cycles, and the corresponding cost shares under the SC and NM approaches. In the first DR-EDC cycle, the buses use similar noises and the total privacy cost is almost equally shared between them. In the second cycle, the buses use different noises and their cost shares are roughly proportional to the noise magnitudes. In these two cycles, the two approaches of SC and NM yield similar results. In the

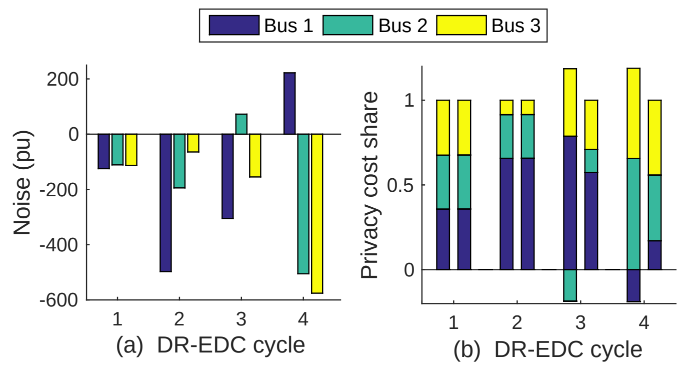

Fig. 6. Total noise amounts of buses in four DR-EDC cycles and the corresponding privacy cost shares under the SC and NM approaches. In each bar group of (b), the first and second bars are respectively the results of SC and NM.

third cycle, bus 2 uses positive noise, while the other two buses use negative noises. As a result, the Shapley cost of bus 2 is negative. This is because bus 2's presence in the coalition helps reduce the total generation cost, as its positive noise offsets the negative noises of the other buses. Interestingly, therefore, in this particular cycle bus 2's noise helps make the overall demand report more accurate. Under the Shapley's principle, bus 2 should be rewarded. We see a similar result in the fourth cycle, where bus 1 is rewarded because its noise counteracts the noises of the other buses. We note that, under the SC approach, the aggregated cost share of a bus over a long time period is almost surely positive, since it is extremely unlikely for the noise of any bus to help reduce the overall inaccuracy all the time. We next illustrate this observation.

We compare the aggregated privacy cost shares over the 300 DR-EDC cycles computed by the three approaches. Fig. 7 shows the results. Consistent with our previous discussion, the aggregated Shapley cost is positive for every bus. Both the NM and NV approaches yield similar aggregated privacy cost shares as the Shapley costs. Specifically, the maximum per-bus deviations from the Shapley costs are 6.2% and 12.1% respectively for the NM and NV approaches. From Fig. 7, therefore, the NM approach appears to perform better. Note that, to compute the Shapley costs for the 5-bus system, the EDC problem in Eq. (4) needs to be solved 24 times. Moreover, as discussed in Section VI-A2, this number will increase exponentially with the number of load buses, which renders the Shapley cost approach infeasible for large power systems. In comparison, the NM approach has much lower computational time complexity.

2) Customer-level cost sharing: We follow the approach described in Section VI-B to further distribute the per-bus Shapley cost among the customers. Fig. 8 shows, for each bus, the ratio of per-customer aggregated cost in a privacy group to the sum of privacy costs of all the privacy groups. We can see that on a certain bus, the per-customer cost increases with the customer's required privacy, which is consistent with intuition.

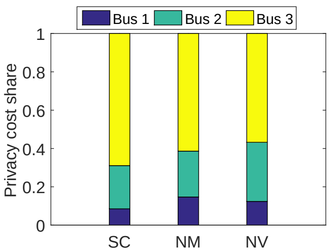

Fig. 7. Bus-level cost shares computed by the different approaches.

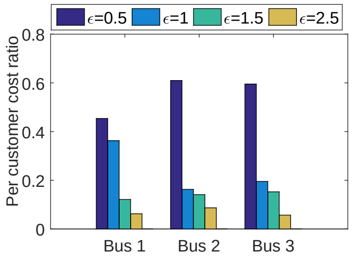

Fig. 8. Ratio of per-customer aggregated cost.

Note that the customers not requiring any DP protection share no privacy cost.

VIII. Conclusion and Future Work

We analyzed the cost of differential privacy as increase of a grid's total generation cost when its EDC is given noisy demand reports, rather than the customers' true demand data, for the sake of the privacy protection. We then applied the principle of Shapley value to distribute this total privacy cost among different buses in the power system. Since Shapley value is infeasible to compute for large systems, we additionally developed heuristic cost sharing algorithms that scale well to large grids, and compare their solutions to corresponding Shapley value solutions. Simulations based on a 5-bus power system model illustrated the privacy cost, its Shapley cost shares, and corresponding cost shares according to the heuristic algorithms.

To improve the accuracy of the DP cost analysis, it is interesting for future work to extend the analysis in Section V-B to address line loss and/or ac power flow models. It is also interesting to design incentive programs with effective mechanisms to motivate customers to participate in the demand reporting. For instance, an interesting question is how to design the set of offered ϵ values such that all the customers are sure to pay less irrespective of their choice of the ϵ value, compared with the choice of not reporting their demand at all.

Acknowledgement

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References

[1] "Illinois public act 094-0977," http://www.ilga.gov/legislation/publicacts/94/PDF/094-0977.pdf.
[2] A. J. Wood and B. F. Wollenberg, *Power generation, operation, and control*. John Wiley & Sons, 2012.
[3] N. Li, C. Zhao, and L. Chen, "Connecting automatic generation control and economic dispatch from an optimization view," *IEEE Trans. on Control of Network Systems*, 2015, DOI: 10.1109/TCNS.2015.2459451.
[4] C.-L. Su and D. Kirschen, "Quantifying the effect of demand response on electricity markets," *IEEE Trans. Power Syst.*, vol. 24, no. 3, 2009.
[5] J. Zhao, T. Jung, Y. Wang, and X. Li, "Achieving differential privacy of data disclosure in the smart grid," *Proc. of Infocom*, 2014.
[6] C. Dwork, "Differential privacy," *Proc. of ICALP*, 2006.
[7] E. Shi, R. Chow, T.-H. H. Chan, D. Song, and E. Rieffel, "Privacy-preserving aggregation of time-series data," *Proc. of NDSS*, 2011.
[8] G. Ács and C. Castelluccia, "I have a dream! (differentially private smart metering)," *Information Hiding*, vol. 6958, pp. 118–132, 2011.
[9] J. Won, C. Ma, D. Yau, and N. Rao, "Proactive fault-tolerant aggregation protocol for privacy-assured smart metering," *Proc. of Infocom*, 2014.
[10] V. Gulisano, V. Tudor, M. Almgren, and M. Papatriantafilou, "Bes-differentially private and distributed event aggregation in advanced metering infrastructures," *Proc. of ACM CPSS*, 2016.
[11] C. Zhao, E. Mallada, and F. Dorfler, "Distributed frequency control for stability and economic dispatch in power networks," *Proc. of ACC*, 2015.
[12] C. Dwork, F. McSherry, K. Nissim, and A. Smith, "Calibrating noise to sensitivity in private data analysis," *Proc. of the 3rd conference on Theory of Cryptography*, 2006.
[13] S. Kotz, T. J. Kozubowski, and K. Podgorski, *The Laplace Distribution and Generalizations*. Birkhäuser, 2001.
[14] P. Kundur, *power system stability and control*. McGraw-Hill, 1994.
[15] D. Cai, E. Mallada, and A. Wierman, "Distributed optimization decomposition for joint economic dispatch and frequency regulation," *Proc. of CDC*, 2015.
[16] X. Zhang, N. Li, and A. Papachristodoulou, "Achieving real-time economic dispatch in power networks via a saddle point design approach," *Proc. of IEEE PES General Meeting*, 2015.
[17] J. D. Glover, M. S. Sarma, and T. J. Overbye, *Power System Analysis and Design*, 5th ed. Cengage Learning, 2011.
[18] U.S. DoE, "The value of economic dispatch – a report to congress pursuant to section 1234 of the energy policy act of 2005," 2005, http://energy.gov/sites/prod/files/oeprod/DocumentsandMedia/value.pdf.
[19] R. R. Mohassel, A. Fung, F. Mohammadi, and K. Raahemifar, "A survey on advanced metering infrastructure," *International Journal of Electrical Power & Energy Systems*, vol. 63, pp. 473–484, 2014.
[20] F. Garcia and B. Jacobs, "Privacy-friendly energy-metering via homomorphic encryption," *Workshop on Security and Trust Manag.*, 2010.
[21] "IEEE 14-bus system," http://icseg.iti.illinois.edu/ieee-14-bus-system/.
[22] L. S. Shapley, "A value for n-person games," *Annals of Mathematical Studies*, vol. 28, 1953.
[23] S. S. Fatima, M. Wooldridge, and N. R. Jennings, "A linear approximation method for the shapley value," *Artificial Intellilgence*, vol. 172, no. 14, pp. 1673–1699, 2008.
[24] "Comed's hourly pricing program," https://hourlypricing.comed.com/.
[25] "Real-time pricing for residential customers," https://www.ameren.com/illinois/electric-choice/residential-real-time-pricing.
[26] "2015 static load profiles," http://on.sce.com/2aAHY3b.
[27] M. Roozbehani, M. Dahleh, and S. Mitter, "Volatility of power grids under real-time pricing," *IEEE Trans. Power Syst.*, vol. 27, no. 4, 2012.
[28] http://www.eia.gov/tools/faqs/faq.cfm?id=105&t=3.

Appendix: Proof of Lemma 2

Proof. Denote by $\mathcal{E}$ the set of transmission lines and (i,j) the line connecting the bus i and bus j. Let Δx represent the derivation of x from its past steady-state value. Generators' primary control is described by [16]:

$$2H_i\Delta\dot{\omega}_i+D_i\Delta\omega_i=\Delta P_{M_i}-\sum_{(i,j)\in\mathcal{E}} B_{ij}\Delta\theta_{ij},\ i\in\mathcal{G}, \quad (9)$$

$$D_i\Delta\omega_i = -\Delta P_{d_i} - \sum_{(i,j)\in\mathcal{E}} B_{ij}\Delta\theta_{ij},\ i\in\mathcal{L}, \quad (10)$$

$$\Delta\dot{\theta}_{ij} = \Delta\omega_i - \Delta\omega_j,\ (i,j)\in\mathcal{E}, \quad (11)$$

$$\Delta\dot{P}_{M_i} = K_i R_i (L_i - \Delta P_{M_i} - R_i^{-1}\Delta\omega_i), \quad (12)$$

where ω_i is the grid frequency, H_i is the generator inertia, D_i is the load damping constant, P_{M_i} is the mechanical power input, P_{d_i} is the load, B_{ij} is the susceptance of (i, j), θ_{ij} is the difference of voltage phases difference between bus i and bus j, L_i is the reference signal from the LFC, K_i is the primary control gain, and R_i is the droop [14]. The LFC control law in the Laplace domain is $\widetilde{\Delta L_i} = -\frac{G_i}{s}\widetilde{\Delta\omega_i}$ [14], where s is the Laplace operator and G_i is the LFC gain. To simplify the analysis, we ignore the differences between ω_i's and consider a globally identical grid frequency ω. This simplification is safe as the differences are often tiny. It is often adopted in power system analysis [14]. By replacing ω_i and ω_j in Eqs. (9)-(12) with ω, summing Eq. (9) and Eq. (10), and applying the Laplace transform, we have

$$\widetilde{\Delta\omega} = \frac{1}{2Hs + D} \cdot (\widetilde{\Delta P_M} - \widetilde{\Delta P_d}), \tag{13}$$

where $H = \sum_{i\in\mathcal{G}} H_i$, $D = \sum_{i\in\mathcal{G}\cup\mathcal{L}} D_i$, $P_M = \sum_{i\in\mathcal{G}} P_{M_i}$, and $P_d = \sum_{i\in\mathcal{L}} P_{d_i}$. The Laplace transform of Eq. (12) is

$$\widetilde{\Delta P_{M_i}} = -\frac{K_i}{s + K_i R_i} \cdot \widetilde{\Delta\omega} + \frac{K_i R_i}{s + K_i R_i} \cdot \widetilde{\Delta L_i}. \tag{14}$$

The $\widetilde{\Delta P_{M_i}}$ can be solved from Eqs. (13), (14), and the LFC control law $\widetilde{\Delta L_i} = -\frac{G_i}{s}\widetilde{\Delta\omega}$:

$$\widetilde{\Delta P_{M_i}} = \frac{\frac{K_i}{s+K_iR_i}\left(1 + \frac{R_iG_i}{s}\right)}{2Hs + D + \sum\limits_{i\in\mathcal{G}} \frac{K_i}{s+K_iR_i}\left(1 + \frac{R_iG_i}{s}\right)} \cdot \widetilde{\Delta P_d}. \tag{15}$$

At the beginning of an DR-EDC cycle, the generators' settings are updated such that they output $\widehat{\mathbf{p}}_0^g$ that is scheduled based on the noisy demand $\widehat{\mathbf{p}}^l = \mathbf{p}^l + \mathbf{n}$. Then, the LFC controls the generators to meet the actual load $\mathbf{p}^l$. This process is equivalent to that, the LFC controls the generators when there is a step change of load (i.e., $\mathbf{n}$) from $\widehat{\mathbf{p}}^l$ to $\mathbf{p}^l$ at the beginning of the EDC cycle. In the Laplace domain, this step change can be expressed by $\widetilde{\Delta P_d} = \sum_{i\in\mathcal{L}} n_i/s$. By replacing $\widetilde{\Delta P_d}$ in Eq. (15) with $\sum_{i\in\mathcal{L}} n_i/s$ and applying the Final Value Theorem, the steady-state value of ΔP_{Mi} in time domain is given by $\Delta P_{M_i}^{\infty} = \lim_{s\to 0} s \cdot \widetilde{\Delta P_{M_i}} = \frac{G_i}{\sum_{i\in\mathcal{G}} G_i} \cdot \sum_{i\in\mathcal{L}} n_i$. As discussed in Section V-A, LFC converges in a few LFC cycles. In the steady state after the convergence, generators' electricity power outputs equal their mechanical power inputs [14]. Thus, at the end of the EDC cycle, we have $\widehat{p}_i^g = \widehat{p}_{0,i}^g + \Delta P_{Mi}^{\infty}$. By applying this result to Eq. (6), we have Eq. (7). □