Path-Planning for an Unmanned Aerial Vehicle with Energy Constraint in a Search and Coverage Mission

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***Abstract*--This paper describes the path planning algorithm for a search and coverage mission using a small UAV that optimizes the trajectory based on energy and maneuverability constraints. The proposed formulation has a high level of autonomy, without relying on the user to make the appropriate choice of optimization parameters. The computed trajectory maximizes spatial coverage while satisfying constraints such as the original flight plan of reaching a desired end state from the initial state for the initial available energy and the maneuverability limitations of the vehicle. Comparisons of this formulation to a path planning algorithm based on time constraint optimization show equivalent coverage performance but improvement in prediction of overall mission duration and terminal position of the vehicle.**

Keywords: Path Planning, UAV, Energy Constraint, Optimization, Guidance and Navigation

I. Introduction

The increased interest in UAVs has seen their implementation in military and civilian operations. Small inexpensive autonomous aerial vehicles are of great interest in search and coverage, surveillance, border patrol, and mapping missions [1]- [2]. These missions require repetitive aerial maneuvers in order to locate objects/targets as soon as possible in a region, or to generate a collage of a specified region. The repetitive nature allows for the automation of these mission, by using small UAV, that reduces cost of the mission and allow for a faster and more effective coverage of an area [1]- [3].

The primary challenge in implementing small UAVs for a search and coverage mission is planning the path of the vehicle that will effectively cover the specified region. Some algorithms used to generate the nonstandard search patterns are the A* and traveling salesmen, which are heuristic techniques [4]. A heuristic technique optimizes the trajectory based on the cost to reach the current state and the cost to reach the goal from the current state.

The foundation for this paper is the algorithm defined in [5]- [6] for generating a trajectory that maximizes spatial-temporal coverage based on preset turning rates and preset mission duration. The algorithm calculates the cost of each of the possible path for the discrete turning rates, which calculates uncovered area remaining and time to reach the desired exit state. The selected path is the path with the lowest value of the sum of the two costs. The algorithm utilizes a receding horizon control (RHC) formulation to generate the optimal trajectory which includes a feedback to account for any disturbance that may deviate the vehicle from its predicted path. There several shortcomings of the algorithm presented in [5]. One of the problems is that the algorithm uses a set mission duration related to the assumption of constant power consumption by the vehicle during the mission. In reality, the power consumption of the vehicle is not constant. In addition, the algorithm selects the optimal path using a discrete set of turning rates that is user specified, and hence may not even be the optimal set of turning rates.

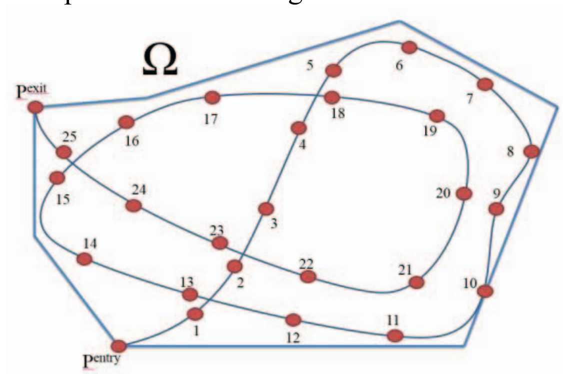

Figure 1: Problem of generating a trajectory that maximizes the area covered.

This paper proposes an optimization formulation for the path planning of a single UAV that maximizes the spatial coverage of an area under the constraints of limited energy and non-constant energy consumption. Additionally, the method operates over a range of turning rates rather than a discrete set. Finally, the formulation utilizes a Boolean operation based polygon area calculation to update the area covered by the UAV over a particular time interval to improve accuracy and overcome the limitation of choosing an appropriate discretization of the space by the user.

The rest of the paper is organized as follows: Section II provides the path planning of a UAV. Section III discusses the optimization formulation that is the primary contribution of the paper. Section IV provides the simulation results and discussion for optimization based on a time constraint and energy constraint. This is followed by conclusions in Section V.

II. Path Planning for UAV

Consider the UAV modeled as a non-holonomic point mass moving in a two-dimensional plane at a constant velocity [5]. The following equations represent the vehicle's dynamics.

$$\dot{x} = v\cos(\theta)\ ;\ \dot{y} = v\sin(\theta)\ ;\ \dot{\theta} = \omega \qquad (1)$$

where (x,y) are the coordinates of the vehicle in a two dimensional space and θ is the turn angle of the vehicle. The vehicle has a constant velocity *v*. The state of the vehicle is defined by $z = \{x, y, \theta\} \in Z$. The vehicle is expected to search the bounded region defined by $\Omega \in \Re^2$and the vehicle has a sensor footprint defined by $\psi: Z \to 2^{\Re^2}$. The vehicle starts the mission at point $p^{entry}(x,y)$ and is expected to end the mission at point $p^{exit}(x,y)$. E_{total} is the amount of energy available at the start of the mission. The exact solution $\{x^*(t), y^*(t), \theta^*(t)\}$ of the problem satisfies

$$\{x^*(t), y^*(t), \theta^*(t)\} = \arg_{z(t)} max \cup_t \big(\Psi\big(z(t)\big) \cap \Omega\big) \qquad (2)$$

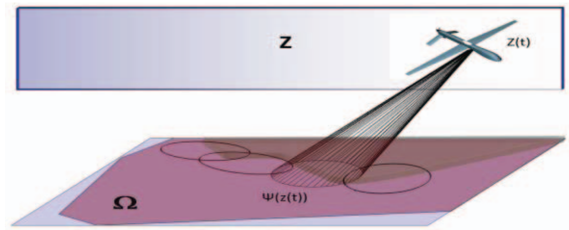

Figure 2: Area covered by sensor footprint for generated trajectory throughout the mission

The trajectory $z^*(t)$ maximizes the sensor footprint coverage of the search region. Figure 1 demonstrates a visual representation of the problem of generating a trajectory that maximizes the area covered and satisfies the exit stated for energy available. Figure 2 demonstrates the generated trajectory along with the sensor footprint area throughout the mission.

III. Range of Turning Rates and Available Energy Optimization

The problem of interest is of a UAV operating at a fixed altitude in a closed, bounded region. The vehicle searches the bounded region equipped with a non-fixed camera and a refresh rate of τ (sec), which can also be termed as the turn duration and the length of the execution horizon. The goal is to find a feasible trajectory, defined by $z(t)$, to get maximum coverage of the bounded region in each time interval. The problem requires time discretization because a continuous solution is computationally exhaustive.

A. Turning rate range

The proposed formulation optimizes the maximum coverage trajectory for a range of turning rates. Equation (3) provides the equation to calculate the range of turning rates by calculate the maximum turning rate that the vehicle can achieve [7] [8].

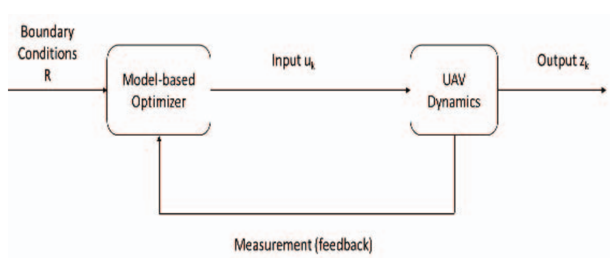

Figure 3: Block diagram of path-planning algorithm execution

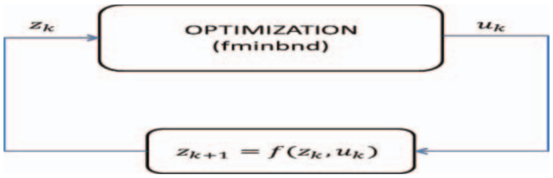

Figure 4: Model-based optimization

$$\omega_{\max} = \frac{g\sqrt{n_{\max}{}^2 - 1}}{v} \quad (3)$$

The maximum load factor, defined by n_{max} [7], determines the vehicle's turn capability. The range of turning rates is therefore defined as $\{-\omega_{max}, \omega_{max}\}$.

B. Optimization Formulation

The algorithm generates the optimal trajectory for a range of turning rates by minimizing the sum of the terminal function and the product of the priority function and area function. Equation (4) details the cost function that is used to generate the optimal trajectory of the vehicle for the time interval [(k-1)τ, kτ] for $k = 1,2,..N$, where τ is the execution horizon (turn duration) and k is the maneuver count. In Equation (4) S is the priority function, A is the area function, and C is the terminal function. The optimization of Equation (4) is done in the MATLAB programming environment using the ***fminbnd*** function [9].

$$\min_{[-\omega_{max}<\omega<\omega_{max}]} \left[\mathrm{A}\left(z(\mathrm{k}\tau), \Omega\big((\mathrm{k}-1)\tau\big)\right) S + \mathrm{C}\right] \quad (4)$$

The algorithm in Equation (4) operates in the model-based optimizer block in Figure 3. The model-based optimizer simulates the motion of the vehicle for a planning horizon for k^{th} step. Its input includes the boundary conditions such as exit and entry point, velocity of the vehicle, entry heading angle and available energy together defines as R. The output of the model-based optimizer is the optimal turning rate sequence *u(t)* for the planning horizon of the predicted optimal trajectory. The UAV only executes the first maneuver of the optimal turning rate sequence determined by the model-based optimizer. This is repeated after updating the state of the UAV, *z(t)*. This continues until the boundary conditions are met. The algorithm only applies the first optimal turning rates and feeds back the actual position of the vehicle at the end of the execution horizon to account for disturbances. Figure 4 provides the block diagram of the model-based optimizer. As previously stated, the model-based optimizer plans the path for a particular planning horizon that covers the most area of the search region not previously covered.

C. Priority function

The priority function determines the immediate objective of the vehicle during the mission. The priority function either determines if the vehicle continues searching the area or heads to the exit point. The function can be based on the time constraint as in [9] or can be based on the energy constraint as in this study.

Equation (5) is the priority function with an energy constraint. If the energy remaining in the vehicle at the end of the interval is greater than the energy required to reach the desired exit state, using a direct path a zero turning rate, then the priority function directs the vehicle to continue search as much area as possible. However, if the amount of energy remaining at the end of the interval is less than the energy required to reach the desired exit, using a direct path, then the priority function directs the vehicle to reach the exit immediately. Moreover, the priority function takes into consideration that the power consumption of the vehicle is not constant.

$$\mathrm{S(E)} = \begin{cases} 1 & \text{if } E_{remaining} > \mathrm{P_{required}t_{exit}} \\ 0 & \text{if } 0 \le E_{remain} \le E_{exit} \;\; or \;\; 0 \le E_{remain} \end{cases} \quad (5)$$

D. Area Function – A(z, τ)

The area function determines the amount of area covered in each time interval by the possible paths. If the sensor covers area that was not previously covered then the area function is equal to the inverse of the new area covered. However, if the sensor doesn't cover any new area then the area function is equal to the distance from the current state to the centroid of uncovered area. The second condition in the area function ensures that the algorithm always searches for uncovered area not previously covered. The amount of uncovered area is obtained using *polybool* – a Boolean operation on polygons in MATLAB.

$$A\left(z(k\tau), \Omega((k-1)\tau)\right) = \begin{cases} \frac{1}{\psi(z(k\tau)) \cap \Omega((k-1)\tau)} & \text{if } \psi(z(k\tau)) \cap \Omega(k-1) > 0 \\ \sqrt{\bar{x}^2 + \bar{y}^2} & \text{if } \psi(z(k\tau)) \cap \Omega(k-1) = 0 \end{cases} \quad (6)$$

where $(\bar{x}, \bar{y})$ represents the centroid of the uncovered area

E. Terminal Function

The cost function of a receding horizon optimization problem estimates the cost-to-go from a selected terminal state to the final goal. Again, this terminal function can be based on time constraint [9] or the energy constraint as in this study.

Equation (7) provides the terminal function for the energy constraint. If the remaining energy at the end of the interval is greater than the energy required to directly head to the desired exit state, the cost is zero, then the objective of the vehicle is to continue covering. If the remaining energy at the end of the interval is less than the energy required to head directly to the desired exit state, then the cost function is equal to the inverse of the remaining energy required. However, if the remaining energy is less than zero then the cost function is equal to infinity. The energy available at the current state of the terminal function accounts for a non-constant energy consumption.

$$C(E) = \begin{cases} 0 & \text{if } E_{available} > \mathrm{P_{required}t_{exit}} \\ \frac{1}{E_{available}} & \text{if } 0 \le E_{available} \le \mathrm{P_{required}t_{exit}} \\ \infty & \text{if } E_{available} < 0 \end{cases} \quad (7)$$

F. Power required

In order to optimize the trajectory that satisfies the energy constraint requires calculating the power requirements of the vehicle during a maneuver and the power required to reach the desired exit states, using a direct path [7] [8]. Equation (8) calculates the power required by the vehicle to reach the exit state using a direct path. Equation (9) calculates the power required by the vehicle for a particular maneuver. The power required by a maneuver is a function of the load factor of the maneuver. Equation (10) calculates the load factor of the maneuver, which in turn is a function of the turn rate, $\dot{\theta}$.

$$\mathrm{P_{required}} = \mathrm{vD} = \mathrm{v}\left(\frac{1}{2}\rho \mathrm{v}^2 \mathrm{SC_{D_0}} + \frac{2(\mathrm{mg})^2}{\rho \mathrm{v}^2 \mathrm{S}\pi \mathrm{ARe}}\right) = \frac{1}{2}\rho S v^3 \left(C_{D_0} + 3C_{D_0}\right) \quad (8)$$

$$\mathrm{P_{traj}} = \mathrm{vD} = \mathrm{v}\left(\frac{1}{2}\rho \mathrm{v}^2 \mathrm{SC_{D_0}} + \frac{2(\mathrm{mg})^2 \mathrm{n}^2}{\rho \mathrm{v}^2 \mathrm{S}\pi \mathrm{ARe}}\right) = \frac{1}{2}\rho S v^3 \left(C_{D_0} + 3n^2 C_{D_0}\right) \quad (9)$$

$$\mathrm{n} = \sqrt{\left(\frac{\dot{\theta}\mathrm{v}}{\mathrm{g}}\right)^2 + 1} \quad (10)$$

IV. Simulation Results

The simulation performed considers a single UAV navigating a specified region. The region is defined from the maximum area that the vehicle can observe, based on the vehicle specifications, assuming ideal conditions. The UAV operates in a bounded region of 312129.5 m^2. This can be represented by a square region of approximately 558 m side. Table 1 provides the vehicle specifications for the simulation. The camera is assumed to always pointing straight to the ground during the maneuvers. In addition, the simulation considers that the vehicle is operating at altitude of 121.92 meters.

The planning horizon is three times the execution horizon. The results provided are for a path-planning optimization for a range of turning rates. The results presented are for two case, time and energy constraint.

Table 1: Properties of the Vehicle for Simulation

Property	**Value**
Vehicle specification	
Wing Area (m^2)	.5
Wing Span (m)	1
Mass (kg)	5
Zero-lift coefficient	.08
Oswald efficiency	1
Motor efficiency	.9
Max load factor	1.5
Battery specification	
Electric charge (mAh)	2200
Voltage (V)	11.1
Sensor specification	
Camera radius (m)	50

A. Time Constraint Based Optimization with Constant Power Consumption

Figures 5 - 6 provide the performance of the path-planning optimization for the optimization with time constraint and constant power consumption. Figure 5 describes the coverage performance of the path-planning optimization with a time constraint for different execution horizons. However, as stated earlier, this constraint assumes constant energy consumption. If the power required for each maneuver is calculated individually and the position of the vehicle is updated, then the actual coverage can be identified as shown in Figure 5. It is evident that the actual coverage of the search region is not always the same as the predicted coverage for varying execution horizons. This is because the vehicle may exhaust its entire available energy before completing the generated path for an optimization based on time constant power consumption.

Figure 6 provides the performance for the time constrained based optimization. Figure 6(a) provides the predicted and actual duration of mission for the vehicle. It is evident that the vehicle exhausts its total initial available energy prior to completing a trajectory optimized with a time constraint. Figure 6(b) provides additional insight into the disadvantages of a constant power based time constraint optimization by charting the distance of the vehicle at the end of the mission from the desired exit location. It shows that the vehicle ends the mission very far from the desired exit state. The distance from the desired exit state at the end of the mission is crucial since it facilitates recovery of the UAV at the end of the mission.

B. Energy Constraint Based Optimization

Figure 7(a) addresses the coverage of the search region by the vehicle. Comparing Figure 7(a) confirms that there is no significant degradation in the coverage performance of the energy constraint based algorithm as compared to the time constraint based algorithm. Figure 7(b) provides the mission duration of the vehicle for the total available energy. The mission duration provided in this figure considers varying power consumption by the vehicle, since the energy constraint optimization calculates the power requirement by the vehicle depending on the turn rate chosen for the maneuver. Figure 7(c) provides the distance of the vehicle from the desired exit state at the end of the mission and

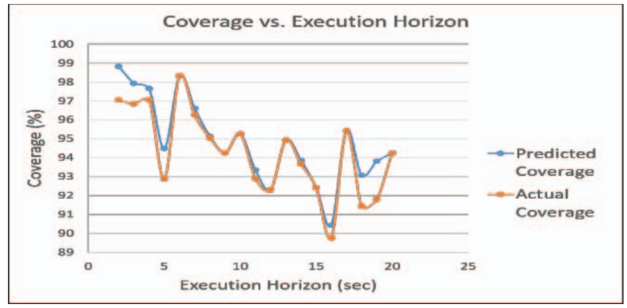

Figure 5: Coverage of search region using time constraint optimization

indicates the most important advantage of the energy constraint optimization. From Figure 7(c) and more importantly Figure 8, it is clear that the path-planning optimization with energy constraint ensures that the vehicle achieves very good coverage of the search region while approaching the desired exit state as close as possible by the end of the mission, especially for execution horizons of under 10 seconds as compared to the time constraint based optimization. The algorithm thus reduces distance of the vehicle at the end of the mission from the desired exit making recovery of the vehicle easier. More importantly, it enables the determination of a more precise location of the vehicle at end of the mission as compared to the time constraint optimization.

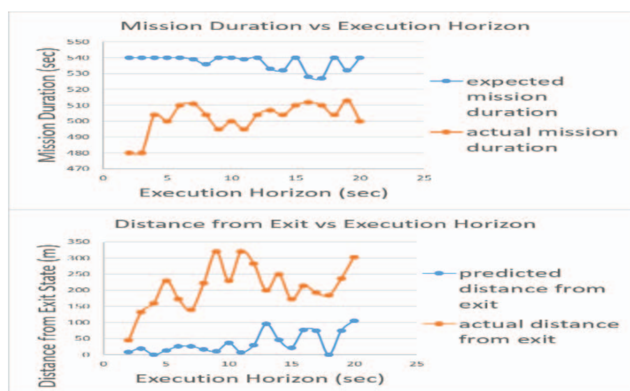

Figure 6 a) Mission duration using time constraint b) Distance from desired exit at the end of mission using time constraint

V. Conclusion

This paper presents a path planning algorithm that utilizes available energy as a constraint and updates the remaining energy after a maneuver based on the power consumed. Further, the maneuvers which are defined by turn rates are not restricted to a discrete set but chosen from a range of possible values. Simulation results show that the novel formulation of the optimization problem does not degrade the area covered as compared to the typical optimization using a time constraint. Evaluation of the overall mission duration assuming that the power required is not constant but dependent on the maneuver (turn rate) indicates that the optimization using time constraint calculates it incorrectly. This further causes the vehicle to complete its position at a location that is not at the one predicted by the algorithm. Contrary to that, the algorithm proposed in this paper that uses energy as a constraint and varies the power required based on the maneuver is more accurate in the prediction of the mission duration and final position. Further, direct comparison of the final position of the vehicle when comparing the two optimization formulation shows that the energy constraint allows the vehicle to be recovered from a location closer to the desired exit point. Potential future work includes understanding the effect of disturbances and model uncertainties on the performance of the algorithm and implementation using multiple cooperative vehicles.

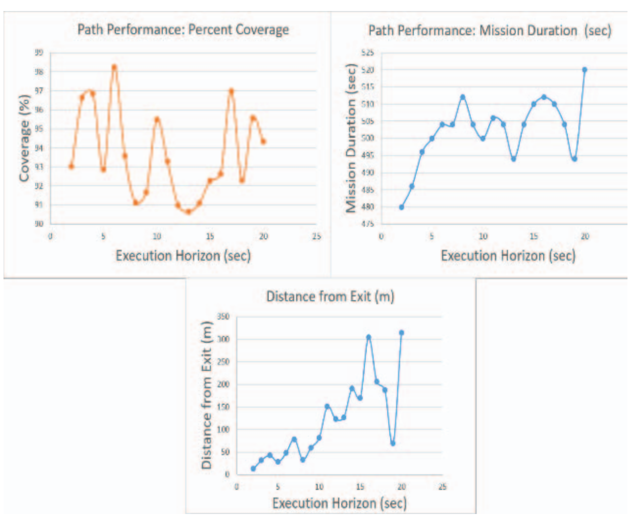

Figure 7 a) Coverage of with path-planning with energy constraint for varying execution horizon b) Mission Duration of path-planning with energy constraint for varying execution horizon c) Distance from desired exit state for energy constraint

Figure 8 Distance from exit for time constraint and energy constraint based optimization

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