STRAIGHT LINES

1 11th Maths - Chapter 10

This is Problem 5 from Exercise-10.4

1. Find perpendicular distance from the origin to the line joining the points($\cos \theta$, $\sin \theta$) and ($\cos \phi$, $\sin \phi$). Solution: Let

$$\mathbf{A} = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \cos \phi \\ \sin \phi \end{pmatrix} \tag{1}$$

$$\mathbf{m} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} \cos \phi - \cos \theta \\ \sin \phi - \sin \theta \end{pmatrix}$$
 (2)

The normal vector is,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \cos \phi - \cos \theta \\ \sin \phi - \sin \theta \end{pmatrix} = \begin{pmatrix} \sin \phi - \sin \theta \\ \cos \theta - \cos \phi \end{pmatrix}$$
(3)

$$\|\mathbf{n}\| = \sqrt{\left(\sin\phi - \sin\theta\right)^2 + \left(\cos\theta - \cos\phi\right)^2} \tag{4}$$

$$= \sqrt{2 - 2\left(\sin\phi\sin\theta + \cos\phi\cos\theta\right)} \tag{5}$$

$$=\sqrt{2-2\left(\cos\left(\phi-\theta\right)\right)}\tag{6}$$

$$=\sqrt{2\left(1-\cos\left(\phi-\theta\right)\right)}\tag{7}$$

$$=\sqrt{2\left(2\sin^2\left(\frac{\phi-\theta}{2}\right)\right)}\tag{8}$$

$$\implies \|\mathbf{n}\| = 2\sin\left(\frac{\phi - \theta}{2}\right) \tag{9}$$

The line equation is,

$$\mathbf{n}^{\top} \left(\mathbf{x} - \mathbf{A} \right) = 0 \tag{10}$$

$$\implies \left(\sin\phi - \sin\theta \quad \cos\theta - \cos\phi\right) \left(\mathbf{x} - \begin{pmatrix} \cos\theta\\ \sin\theta \end{pmatrix}\right) = 0 \tag{11}$$

$$\implies (\sin \phi - \sin \theta \quad \cos \theta - \cos \phi) \mathbf{x} = (\sin \phi - \sin \theta \quad \cos \theta - \cos \phi) \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$
(12)

$$= (\sin \phi - \sin \theta) \cos \theta + (\cos \theta - \cos \phi) \sin \theta \tag{13}$$

 $= \sin \phi \cos \theta - \sin \theta \cos \theta + \sin \theta \cos \theta - \sin \theta \cos \phi$

(14)

$$\implies (\sin \phi - \sin \theta \quad \cos \theta - \cos \phi) \mathbf{x} = \sin (\phi - \theta) \tag{15}$$

from (15)

$$c = \sin\left(\phi - \theta\right) \tag{16}$$

The perpendicular distance from the origin to the line is

$$d = \frac{|c|}{\|\mathbf{n}\|} \tag{17}$$

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$$\implies d = \frac{\sin(\phi - \theta)}{2\sin\left(\frac{\phi - \theta}{2}\right)}$$
(17)

The input parameters are

Symbol	Value	Description
θ	0°	assumed angle
φ	90°	assumed angle
A	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	point A
В	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	point B
m	$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	directional vector
n	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	normal vector

Table 1

substituting the input parameters in (18)

$$\implies d = \frac{\sin 90^{\circ}}{2 \sin 45^{\circ}}$$

$$\implies d = \frac{1}{\sqrt{2}}$$
(20)

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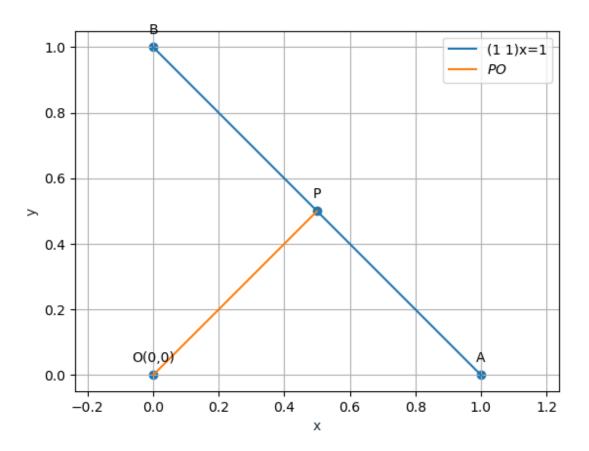


Figure 1