STRAIGHT LINES

1 11^{th} Maths - Chapter 10

This is Problem 5 from Exercise-10.4

1. Find perpendicular distance from the origin to the line joining the points$(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$.
   Solution: Let

$$\mathbf{A} = \begin{pmatrix} \cos\theta \\ sin\theta \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \cos\phi \\ sin\phi \end{pmatrix} \tag{1}$$

$$\mathbf{m} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} \cos\phi - \cos\theta \\ \sin\phi - \sin\theta \end{pmatrix} \tag{2}$$

$$\tag{3}$$

The normal vector is,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \cos\phi - \cos\theta \\ \sin\phi - \sin\theta \end{pmatrix} = \begin{pmatrix} \sin\phi - \sin\theta \\ \cos\theta - \cos\phi \end{pmatrix} \tag{4}$$

$$\|\mathbf{n}\| = \sqrt{(\sin\phi - \sin\theta)^2 + (\cos\theta - \cos\phi)^2} \tag{5}$$

$$= \sqrt{2 - 2(\sin\phi\sin\theta + \cos\phi\cos\theta)} \tag{6}$$

$$= \sqrt{2 - 2(\cos(\phi - \theta))} \tag{7}$$

$$= \sqrt{2(1 - \cos(\phi - \theta))} \tag{8}$$

$$= \sqrt{2\left(2\sin^2\left(\frac{\phi - \theta}{2}\right)\right)} \tag{9}$$

$$\implies \|\mathbf{n}\| = 2\sin\left(\frac{\phi - \theta}{2}\right) \tag{10}$$

The line equation is,

$$\mathbf{n}^\top(\mathbf{x} - \mathbf{A}) = 0 \tag{11}$$

$$\implies \begin{pmatrix} \sin\phi - \sin\theta & \cos\theta - \cos\phi \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}\right) = 0 \tag{12}$$

$$\implies \begin{pmatrix} \sin\phi - \sin\theta & \cos\theta - \cos\phi \end{pmatrix} \mathbf{x} = \begin{pmatrix} \sin\phi - \sin\theta & \cos\theta - \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{13}$$

$$= (\sin\phi - \sin\theta)\cos\theta + (\cos\theta - \cos\phi)\sin\theta \tag{14}$$

$$= \sin\phi\cos\theta - \sin\theta\cos\theta + \sin\theta\cos\theta - \sin\theta\cos\phi \tag{15}$$

$$\implies \begin{pmatrix} \sin\phi - \sin\theta & \cos\theta - \cos\phi \end{pmatrix} \mathbf{x} = \sin(\phi - \theta) \tag{16}$$

from (16)

$$c = \sin(\phi - \theta) \tag{17}$$

The perpendicular distance from the origin to the line is

$$d = \frac{|c|}{\|\mathbf{n}\|} \tag{18}$$

$$\implies d = \frac{\sin(\phi - \theta)}{2\sin\left(\frac{\phi - \theta}{2}\right)} \tag{19}$$

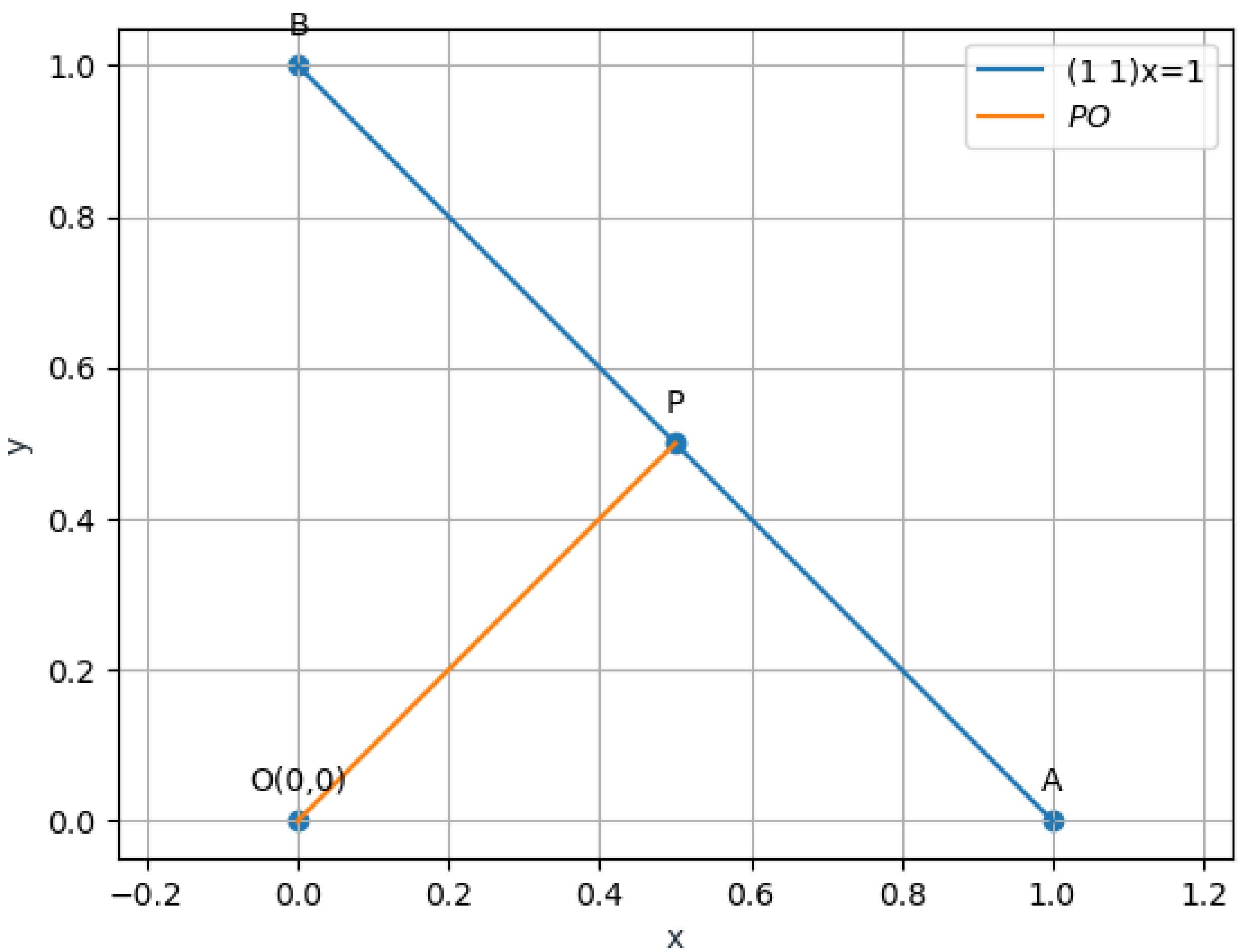

Figure 1