

CHAPTER-10  
VECTOR ALGEBRA

## EXERCISE - 10.5

1. Write down a unit vector in XY-plane, making an angle of  $30^\circ$  with the positive direction of x-axis.
2. Find the scalar components and magnitude of the vector joining the points P( $x_1, y_1, z_1$ ) and Q ( $x_2, y_2, z_2$ ).
3. A girl walks 4 km towards west, then she walks 3 km in a direction  $30^\circ$  east of north and stops. Determine the girl's displacement from her initial point of departure.
4. If  $\vec{a} = \vec{b} + \vec{c}$ , then is it true that  $|\vec{a}| = |\vec{b}| + |\vec{c}|$ ? Justify your answer.
5. Find the value of x for which  $x(\hat{i} + \hat{j} + \hat{k})$  is a unit vector.
6. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors  $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ .
7. If  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$  and  $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$ , find a unit vector parallel to the vector  $2\vec{a} - \vec{b} + 3\vec{c}$ .
8. Show that the points A (1, -2, -8), B (5, 0, -2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.
9. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are  $(2\vec{a} + \vec{b})$  and  $(\vec{a} - 3\vec{b})$  externally in the ratio 1 : 2. Also, show that P is the mid point of the

line segment RQ.

10. The two adjacent sides of a parallelogram are  $2\hat{i}-4\hat{j}+5\hat{k}$  and  $\hat{i}-2\hat{j}-3\hat{k}$ . Find the unit vector parallel to its diagonal. Also, find its area.
11. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are  $\pm[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}]$ .
12. Let  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$  and  $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ . Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$ , and  $\vec{c} \cdot \vec{d} = 15$ .
13. The scalar product of the vector  $\hat{i} + \hat{j} + \hat{k}$  with a unit vector along the sum of vectors  $2\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$  is equal to one. Find the value of  $\lambda$ .
14. If  $\vec{a}, \vec{b}, \vec{c}$  are mutually perpendicular vectors of equal magnitudes, show that the vector  $\vec{c} \cdot \vec{d} = 15$  is equally inclined to  $\vec{a}, \vec{b}$  and  $\vec{c}$ .
15. Prove that  $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$ , if and only if  $\vec{a}, \vec{b}$  are perpendicular, given  $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$ .

Choose the correct answer in Exercises 16 to 19.

16. If  $\theta$  is the angle between two vectors  $\vec{a}$  and  $\vec{b}$ , then  $\vec{a} \cdot \vec{b} \geq 0$  only when
  - (a)  $0 < \theta < \frac{\pi}{2}$
  - (b)  $0 \leq \theta \leq \frac{\pi}{2}$
  - (c)  $0 < \theta < \pi$
  - (d)  $0 \leq \theta \leq \pi$
17. Let  $\vec{a}$  and  $\vec{b}$  be two unit vectors and  $\theta$  is the angle between them. Then  $\vec{a} + \vec{b}$  is a unit vector if
  - (a)  $\theta = \frac{\pi}{4}$

(b)  $\theta = \frac{\pi}{3}$

(c)  $\theta = \frac{\pi}{2}$

(d)  $\theta = \frac{2\pi}{3}$

18. The value of  $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$  is

(a) 0

(b) -1

(c) 1

(d) 3

19. If  $\theta$  is the angle between any two vectors  $\vec{a}$  and  $\vec{b}$ , then  $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$  when  $\theta$  is equal to

(a) 0

(b)  $\frac{\pi}{4}$

(c)  $\frac{\pi}{2}$

(d)  $\pi$