TRIANGLES

1 9^{th} Maths - Chapter 7

This is Problem 7 from Exercise-7.1

AB is a line segment and P is its mid-point.D and E are points on the same side of AB such that $\angle$ BAD = $\angle$ ABE and $\angle$ EPA = $\angle$ DPB.Show that

(i) $\triangle$ DAP $\cong$ $\triangle$ EBP

(ii) AD = BE

2 construction

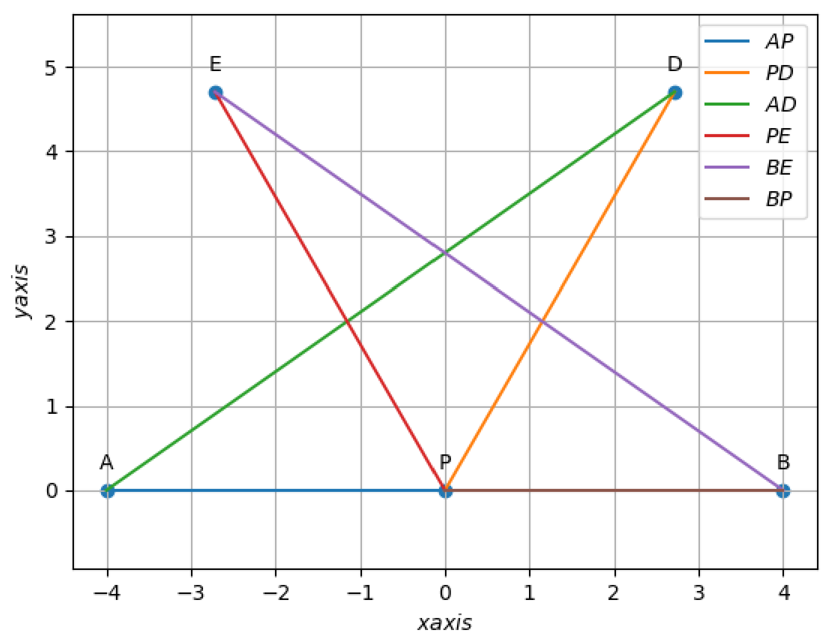

Figure 1

The input parameters for this construction are

Symbol	Value	Description
r	4	$AP = PB$
b	5.43	$EP = DP$
θ	$60°$	$\angle DPB$
θ_1	$120°$	$\angle EPB$

Table 2

$$\mathbf{A} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$$
$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$
$$\mathbf{D} = 5.43 \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix}$$
$$\mathbf{E} = 5.43 \begin{pmatrix} \cos 120^\circ \\ \sin 120^\circ \end{pmatrix}$$

Solution: Given

$$AP = BP \tag{1}$$
$$\angle BAD = \angle ABE \tag{2}$$

To Prove: $\triangle DAP \cong \triangle$ EPB
to prove $\angle DPA = \angle EPB$

$$\text{Let } \theta 2 = \angle DPA \tag{3}$$
$$\mathbf{m1} = \mathbf{P} - \mathbf{D} = \begin{pmatrix} -2.7 \\ -4.7 \end{pmatrix} \tag{4}$$
$$\mathbf{m2} = \mathbf{P} - \mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{5}$$
$$\theta 2 = \cos^{-1} \frac{\mathbf{m1}^\top \mathbf{m2}}{\|\mathbf{m1}\| \, \|\mathbf{m2}\|} \tag{6}$$
$$\implies \theta 2 = \cos^{-1} \frac{\begin{pmatrix} -2.7 & -4.7 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix}}{(5.42)(4)} = 120^\circ \tag{7}$$
$$\theta 3 = \angle EPB \tag{8}$$
$$\mathbf{n1} = \mathbf{P} - \mathbf{E} = \begin{pmatrix} 2.7 \\ -4.7 \end{pmatrix} \tag{9}$$
$$\mathbf{n2} = \mathbf{P} - \mathbf{B} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{10}$$
$$\theta 3 = \cos^{-1} \frac{\mathbf{n1}^\top \mathbf{n2}}{\|\mathbf{n1}\| \, \|\mathbf{n2}\|} \tag{11}$$
$$\implies \theta 3 = \cos^{-1} \frac{\begin{pmatrix} 2.7 & -4.7 \end{pmatrix} \begin{pmatrix} -4 \\ 0 \end{pmatrix}}{(5.42)(4)} = 120^\circ \tag{12}$$

from (7) and (12)

$$\angle DPA = \angle EPB \tag{13}$$

from $(1), (2)$ and (13)

$$\triangle DAP \cong \triangle EBP \tag{14}$$

from (14)

AD=BE