Vector Algebra

1 12^{th} Maths - Chapter 10

This is Problem 11 from Exercise-10.3

1. Show that $|\overrightarrow{a}|\ \overrightarrow{b} + \left|\overrightarrow{b}\right|\ \overrightarrow{a}$ is perpendicular to $|\overrightarrow{a}|\ \overrightarrow{b} - \left|\overrightarrow{b}\right|\ \overrightarrow{a}$, for any two nonzero vectors $\overrightarrow{a}$ and $\overrightarrow{b}$

   Solution: From the given information

$$\left(\|\mathbf{a}\|\,\mathbf{b} + \|\mathbf{b}\|\,\mathbf{a}\right)^{\top}\left(\|\mathbf{a}\|\,\mathbf{b} - \|\mathbf{b}\|\,\mathbf{a}\right) \tag{1}$$

$$\implies \|\mathbf{a}\|^{\top}\mathbf{b}^{\top}\|\mathbf{a}\|\,\mathbf{b} - \|\mathbf{a}\|^{\top}\mathbf{b}^{\top}\|\mathbf{b}\|\,\mathbf{a} + \|\mathbf{b}\|^{\top}\mathbf{a}^{\top}\|\mathbf{a}\|\,\mathbf{b} - \|\mathbf{b}\|^{\top}\mathbf{a}^{\top}\|\mathbf{b}\|\,\mathbf{a} \tag{2}$$

   we know that

$$\mathbf{a}^{\top}\mathbf{a} = \|\mathbf{a}\|^2 \tag{3}$$

$$\mathbf{b}^{\top}\mathbf{b} = \|\mathbf{b}\|^2 \tag{4}$$

$$\mathbf{a}^{\top}\mathbf{b} = \mathbf{b}^{\top}\mathbf{a} \tag{5}$$

   By using (**??**) and (4) and (5)

$$\implies \|\mathbf{a}\|^2\|\mathbf{b}\|^2 - \|\mathbf{b}\|^2\|\mathbf{a}\|^2 = 0 \tag{6}$$