

CIRCLES

1 9th Maths - Chapter 10

This is Problem 2 from Exercise-10.6

Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

2 construction

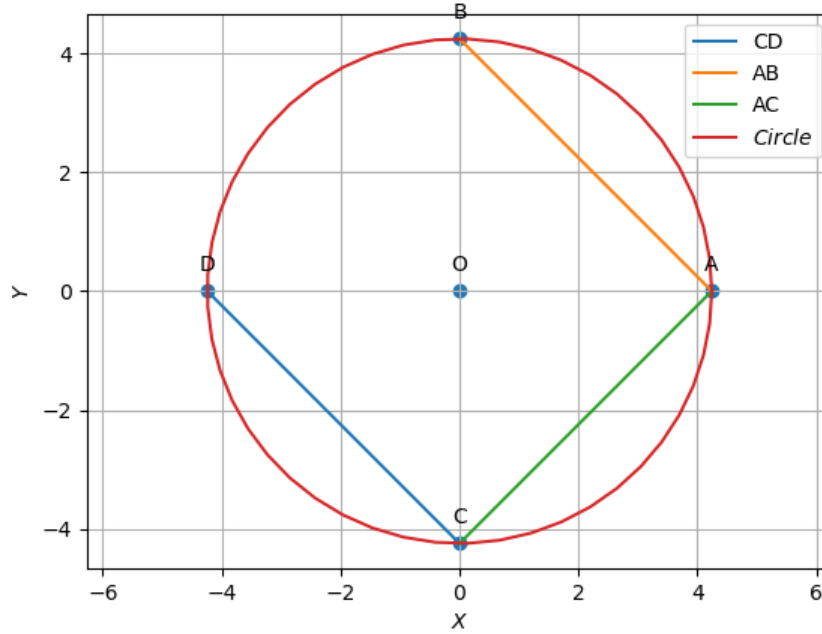


Figure 1

The input parameters for this construction are

Symbol	Value	Description
θ_1	0°	Assumed angle
θ_2	90°	Assumed angle
θ_3	270°	Assumed angle
θ_4	180°	Assumed angle
d	6	distance between AB and CD

$$\mathbf{A} = r \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix}, \mathbf{B} = r \begin{pmatrix} \cos \theta_2 \\ \sin \theta_2 \end{pmatrix}, \mathbf{C} = r \begin{pmatrix} \cos \theta_3 \\ \sin \theta_3 \end{pmatrix}, \mathbf{D} = r \begin{pmatrix} \cos \theta_4 \\ \sin \theta_4 \end{pmatrix} \quad (1)$$

Solution: Lines AB and CD are parallel.

Therefore,

$$\mathbf{m}_1 = \mathbf{m}_2 \quad (2)$$

$$\begin{pmatrix} \cos \theta_1 - \cos \theta_2 \\ \sin \theta_1 - \sin \theta_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_3 - \cos \theta_4 \\ \sin \theta_3 - \sin \theta_4 \end{pmatrix} \quad (3)$$

$$\Rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (4)$$

from (4) the normal vector is given by

$$\mathbf{n} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad (5)$$

The line equation of AB is

$$r \begin{pmatrix} -1 & -1 \end{pmatrix} \mathbf{x} = r^2 \begin{pmatrix} -1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (6)$$

$$\begin{pmatrix} -1 & -1 \end{pmatrix} = -r \quad (7)$$

The line equation of CD is

$$r \begin{pmatrix} -1 & -1 \end{pmatrix} \mathbf{x} = r^2 \begin{pmatrix} -1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad (8)$$

$$\begin{pmatrix} -1 & -1 \end{pmatrix} = r \quad (9)$$

from (7) and (9)

$$c_1 = -r, c_2 = r \quad (10)$$

The distance between parallel lines is

$$d = \frac{|c_1 - c_2|}{\|\mathbf{n}\|} \quad (11)$$

$$\Rightarrow 6 = \frac{2r}{\sqrt{2}} \quad (12)$$

$$\Rightarrow r = 4.24 \quad (13)$$