Vector Algebra

12^{th} Maths - Chapter 10

Exercise 10.2 Problem-13

1. Find the direction cosines of the vector joining the points A (1, 2, 3) and B(1, 2, 1), directed from A to B.

   Solution: The direction cosines are the cosines of the angles formed by the given vector with the respective axes, given vectors are **A** and **B**

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} \tag{1}$$

   The direction vector m of the line joining two points A, B is given by

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix} \tag{2}$$

$$\mathbf{m} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix} \tag{3}$$

$$\|\mathbf{m}\| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = 6 \tag{4}$$

   The Directional vectors of x, y and z axes are given respectively

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{5}$$

The direction cosines are the elements of direction vector m.Therefore using direction vector m,direction cosines are calculated.
The magnitudes for $\mathbf{m}$ and directional vectors $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ are

$$\|\mathbf{m}\| = 6, \|\mathbf{e_1}\| = 1, \|\mathbf{e_2}\| = 1, \|\mathbf{e_3}\| = 1 \tag{6}$$

$$\cos\theta_i = 1, 2, 3 \tag{7}$$

So for different values of $\cos\theta_i$ the direction cosines of vector $\mathbf{A}$ and $\mathbf{B}$ are

$$\cos\theta_1 = \frac{\begin{pmatrix}-2 & -4 & 4\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}}{6} = \frac{-1}{3} \tag{8}$$

$$\cos\theta_2 = \frac{\begin{pmatrix}-2 & -4 & 4\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}}{6} = \frac{-2}{3} \tag{9}$$

$$\cos\theta_3 = \frac{\begin{pmatrix}-2 & -4 & 4\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}}{6} = \frac{2}{3} \tag{10}$$