

Vector Algebra

1 12th Maths - Chapter 10

This is Problem 11 from Exercise-10.3

1. Show that $|\vec{a}| \vec{b} + |\vec{b}| \vec{a}$ is perpendicular to $|\vec{a}| \vec{b} - |\vec{b}| \vec{a}$, for any two nonzero vectors \vec{a} and \vec{b}

Solution: From the given information

$$(\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a})^\top (\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a}) \quad (1)$$

$$\Rightarrow \|\mathbf{a}\|^\top \mathbf{b}^\top \|\mathbf{a}\| \mathbf{b} - \|\mathbf{a}\|^\top \mathbf{b}^\top \|\mathbf{b}\| \mathbf{a} + \|\mathbf{b}\|^\top \mathbf{a}^\top \|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\|^\top \mathbf{a}^\top \|\mathbf{b}\| \mathbf{a} \quad (2)$$

$$\Rightarrow \|\mathbf{a}\|^2 \mathbf{b}^\top \mathbf{b} - \|\mathbf{a}\| \|\mathbf{b}\| \mathbf{b}^\top \mathbf{a} + \|\mathbf{a}\| \|\mathbf{b}\| \mathbf{a}^\top \mathbf{b} - \|\mathbf{b}\|^2 \mathbf{a}^\top \mathbf{a} \quad (3)$$

we know that

$$\mathbf{a}^\top \mathbf{a} = \|\mathbf{a}\|^2 \quad (4)$$

$$\mathbf{b}^\top \mathbf{b} = \|\mathbf{b}\|^2 \quad (5)$$

$$\mathbf{a}^\top \mathbf{b} = \mathbf{b}^\top \mathbf{a} \quad (6)$$

By using (4) and (5) and (6)

$$\Rightarrow \|\mathbf{a}\|^2 \|\mathbf{b}\|^2 - \|\mathbf{b}\|^2 \|\mathbf{a}\|^2 = 0 \quad (7)$$