TRIANGLES

1 9^{th} Maths - Chapter 7

This is Problem 7 from Exercise-7.1

AB is a line segment and P is its mid-point.D and E are points on the same side of AB such that $\angle$ BAD = $\angle$ ABE and $\angle$ EPA = $\angle$ DPB.Show that

(i) $\triangle$ DAP $\cong$ $\triangle$ EBP

(ii) AD = BE

2 construction

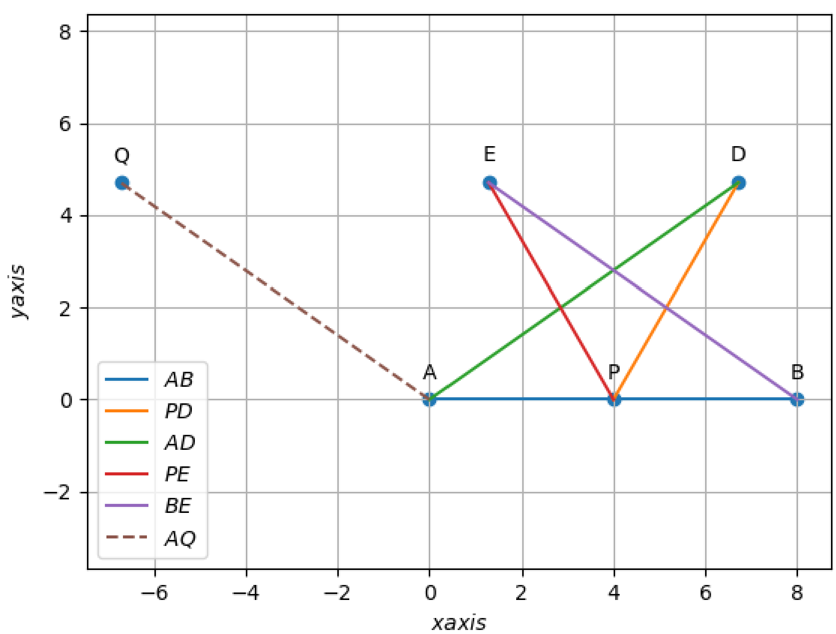

Figure 1

The input parameters for this construction are

Symbol	Value	Description
c	8	AB
b	8.2	AD
θ	35°	$\angle BAD$

Table 2

$$\mathbf{A} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{e_1} = \begin{pmatrix}1\\0\end{pmatrix}, \mathbf{B} = c \times \mathbf{e_1}, \mathbf{P} = \frac{\mathbf{A}+\mathbf{B}}{2}, \mathbf{D} = b\begin{pmatrix}\cos A\\ \sin A\end{pmatrix} \tag{1}$$

$$\text{Let } \mathbf{Q} - \mathbf{A} = \mathbf{E} - \mathbf{B} \tag{2}$$

$$\mathbf{Q} = b\begin{pmatrix}\cos(180-A)\\ \sin(180-A)\end{pmatrix} = b\begin{pmatrix}-\cos A\\ \sin A\end{pmatrix} \tag{3}$$

$$\mathbf{E} = \mathbf{B} + \mathbf{Q} - \mathbf{A} = c \times \mathbf{e_1} + b\begin{pmatrix}-\cos A\\ \sin A\end{pmatrix} \tag{4}$$

Solution: Given

$$\mathbf{A} - \mathbf{P} = \mathbf{P} - \mathbf{B} \tag{5}$$

$$\angle BAD = \angle ABE \tag{6}$$

$$\text{Assume } \mathbf{A} - \mathbf{D} = \mathbf{E} - \mathbf{B} \tag{7}$$

To Prove: $\angle EPA = \angle DPB$

$$\text{Let } \theta_1 = \angle EPA \tag{8}$$

$$\mathbf{m1} = \mathbf{D} - \mathbf{P} = \begin{pmatrix}2.7\\4.7\end{pmatrix}, \mathbf{m2} = \mathbf{B} - \mathbf{P} = \begin{pmatrix}4\\0\end{pmatrix} \tag{9}$$

$$\theta_1 = \cos^{-1}\frac{\mathbf{m1}^\top \mathbf{m2}}{\|\mathbf{m1}\|\,\|\mathbf{m2}\|} \tag{10}$$

$$\implies \theta_1 = \cos^{-1}\frac{\begin{pmatrix}2.7 & 4.7\end{pmatrix}\begin{pmatrix}4\\0\end{pmatrix}}{(5.42)(4)} = 60^\circ \tag{11}$$

$$\theta_2 = \angle DPB \tag{12}$$

$$\mathbf{n1} = \mathbf{E} - \mathbf{P} = \begin{pmatrix}-2.7\\4.7\end{pmatrix}, \mathbf{n2} = \mathbf{A} - \mathbf{P} = \begin{pmatrix}-4\\0\end{pmatrix} \tag{13}$$

$$\theta_2 = \cos^{-1}\frac{\mathbf{n1}^\top \mathbf{n2}}{\|\mathbf{n1}\|\,\|\mathbf{n2}\|} \tag{14}$$

$$\implies \theta_2 = \cos^{-1}\frac{\begin{pmatrix}-2.7 & -4.7\end{pmatrix}\begin{pmatrix}-4\\0\end{pmatrix}}{(5.42)(4)} = 60^\circ \tag{15}$$

from (11) and (15)

$$\angle \text{ EPA} = \angle \text{ DPB}$$