

TRIANGLES

1 9th Maths - Chapter 7

This is Problem 7 from Exercise-7.1

AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. Show that

- (i) $\triangle DAP \cong \triangle EBP$
- (ii) $AD = BE$

2 construction

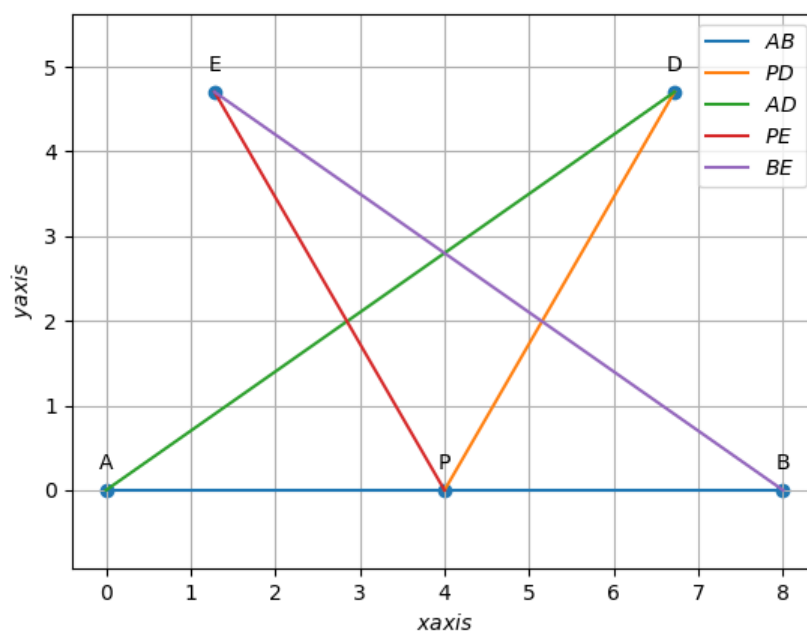


Figure 1

The input parameters for this construction are

Symbol	Value	Description
c	8	AB
b	8.2	AD
θ	35°	$\angle BAD$

Table 2

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1)$$

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2)$$

$$\mathbf{B} = c \times \mathbf{e}_1 \quad (3)$$

$$\mathbf{D} = b \begin{pmatrix} \cos A \\ \sin A \end{pmatrix} \quad (4)$$

$$\text{Let } \mathbf{Q} - \mathbf{A} = \mathbf{E} - \mathbf{B} \quad (5)$$

$$\mathbf{Q} = b \begin{pmatrix} \cos(180 - A) \\ \sin(180 - A) \end{pmatrix} \quad (6)$$

$$\mathbf{E} = \mathbf{B} + \mathbf{Q} - \mathbf{A} \quad (7)$$

$$\Rightarrow \mathbf{E} = c \times \mathbf{e}_1 + b \begin{pmatrix} -\cos A \\ \sin A \end{pmatrix} \quad (8)$$

Solution: Given

$$\mathbf{A} - \mathbf{P} = \mathbf{P} - \mathbf{B} \quad (9)$$

$$\angle BAD = \angle ABE \quad (10)$$

$$\text{Assume } \mathbf{A} - \mathbf{D} = \mathbf{E} - \mathbf{B} \quad (11)$$

To Prove: $\angle EPA = \angle DPB$

$$\text{Let } \theta_1 = \angle EPA \quad (12)$$

$$\mathbf{m}_1 = \mathbf{D} - \mathbf{P} = \begin{pmatrix} 2.7 \\ 4.7 \end{pmatrix} \quad (13)$$

$$\mathbf{m}_2 = \mathbf{B} - \mathbf{P} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (14)$$

$$\theta_1 = \cos^{-1} \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \quad (15)$$

$$\Rightarrow \theta_1 = \cos^{-1} \frac{(2.7 \ 4.7) \begin{pmatrix} 4 \\ 0 \end{pmatrix}}{(5.42)(4)} = 60^\circ \quad (16)$$

$$\theta_2 = \angle DPB \quad (17)$$

$$\mathbf{n}_1 = \mathbf{E} - \mathbf{P} = \begin{pmatrix} -2.7 \\ 4.7 \end{pmatrix} \quad (18)$$

$$\mathbf{n}_2 = \mathbf{A} - \mathbf{P} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad (19)$$

$$\theta_2 = \cos^{-1} \frac{\mathbf{n}_1^\top \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \quad (20)$$

$$\Rightarrow \theta_2 = \cos^{-1} \frac{(-2.7 \ -4.7) \begin{pmatrix} -4 \\ 0 \end{pmatrix}}{(5.42)(4)} = 60^\circ \quad (21)$$

from (16) and (21)

$$\angle EPA = \angle DPB$$