CHAPTER-10
VECTOR ALGEBRA

EXERCISE - 10.5

1. Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.

2. Find the scalar components and magnitude of the vector joining the points P(x_1, y_1, z_1) and Q $(x_2, y_2, z_2\ )$.

3. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

4. If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

5. Find the value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

6. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

7. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$,$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

8. Show that the points A (1, -2, -8), B (5, 0, -2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.

9. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in the ratio 1 : 2. Also, show that P is the mid point of the

line segment RQ.

10. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

11. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\pm[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}]$.

12. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$,$\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$,and $\vec{c}.\vec{d}$=15.

13. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

14. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{c}.\vec{d}$=15 is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

15. Prove that $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{a}|^2+|\vec{b}|^2$,if and only if $\vec{a}, \vec{b}$ are perpendicular, given $\vec{a} \neq \vec{0}$,$\vec{b} \neq \vec{0}$.

Choose the correct answer in Exercises 16 to 19.

16. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$,then $\vec{a}.\vec{b} \geq 0$ only when

    (a) $0 < \theta < \frac{\pi}{2}$
    (b) $0 \leq \theta \leq \frac{\pi}{2}$
    (c) $0 < \theta < \pi$
    (d) $0 \leq \theta \leq \pi$

17. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them.Then $\vec{a} + \vec{b}$ is a unit vector if

    (a) $\theta = \frac{\pi}{4}$

(b) $\theta = \frac{\pi}{3}$

(c) $\theta = \frac{\pi}{2}$

(d) $\theta = \frac{2\pi}{3}$

18. The value of $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{i} \times \hat{k}) + \hat{k}.(\hat{i} \times \hat{j})$ is

(a) 0

(b) -1

(c) 1

(d) 3

19. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$,then $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to

(a) 0

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) π