Assignment - 03 :

Let consider a sample dataset have one input (x_i^a) and one output (y_i^a), and number of samples 4. Develop a simple linear regression model using stochastic gradian optimizer.

Sample(i)	x_i^a	y_i^a
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

- Do manual calculations for two iterations with first two samples.
- Write the python code to build simple linear regression model using SGD optimizer (consider all 4 samples)

Manual calculations for two iterations with first two samples :-

sample(i)	x_i^a	y_i^a
1	0.2	3.4
2	0.4	3.8

No of samples = 2 (ns). Initilize the model parameters randomly

step1:- [x, y], m = 1, c = 1, η = 0.1, epochs = 2

step2:- iter = 1

step3:- sample = 1

step4:- $\because$ ns = 1 (sample by sample)

$$E = \frac{1}{2n_s}[y_i - mx_i - c]^2$$

$$\frac{\partial E}{\partial m} = \frac{2}{2(1)}(y_i - mx_i - c)(-x_i)$$

$= -(3.4 - 1(0.2) - 1)(0.2)$

$= -(2.2)(0.2)$

$\frac{\partial E}{\partial m} = -0.44$

$\frac{\partial E}{\partial c} = \frac{2}{2\eta_s}(y_i - mx_i - c)(-1)$

$= \frac{-1}{1}(y_i - mx_i - c)$

$= -(3.4 - 1(0.2) - 1)$

$\frac{\partial E}{\partial c} = -2.2$

step5:- $\Delta m = -\eta \frac{\partial E}{\partial m}$

$= -(0.1)(-0.44)$

$\Delta m = 0.044$

$\Delta c = -\eta \frac{\partial E}{\partial c}$

$= -(0.1)(-2.2)$

$\Delta c = 0.22$

step6:- $m = m + \Delta m$

$= 1 + 0.044$

$m = 1.044$

$c = c + \Delta c$

$= 1 + 0.22$

$c = 1.22$

step7:- sample = sample + 1

$= 1 + 1 = 2$

step8:- if (sample > η_s)

(2 > 2)

true: go to step9

false: go to step4 ✓

step4:-

$\frac{\partial E}{\partial m} = -x_i(y_i - mx_i - c)$

$= -0.4[3.8 - (1.044)(0.4) - 1.22]$

$\frac{\partial E}{\partial m} = -0.86496$

$$\frac{\partial E}{\partial c} = -(y_i - mx_i - c)$$

$$= -[3.8 - (1.044)(0.4) - 1.22]$$

$$\boxed{\frac{\partial E}{\partial c} = -2.1624}$$

Step5:-

$$\Delta m = -\eta \frac{\partial E}{\partial m} \qquad \Delta c = -\eta \frac{\partial E}{\partial c}$$

$$\Delta m = -0.1(-0.86496) \qquad = -0.1(-2.1624)$$

$$\boxed{\Delta m = 0.086496} \qquad \boxed{\Delta c = 0.21624}$$

Step6:- $m = m + \Delta m$ $\qquad c = c + \Delta c$

$= 1.044 + 0.086496$ $\qquad c = 1.22 + 0.21624$

$\boxed{m = 1.130496}$ $\qquad \boxed{c = 1.43624}$

Step7:- sample = sample + 1

$= 2 + 1$

$= 3$

Step8:- if (sample > ns)

$3 > 2$ ✓

True: goto next step ✓ (step9)

False: goto step 4

Step9:- iter = iter + 1 = 1 + 1 = 2

Step10:- if (iter > epochs)

$2 > 2$

True: go to nextstep (step11)

False: go to step 3. ✓

step3:- sample = 1

step4:- $\frac{\partial E}{\partial m} = -x_i(y_i - mx_i - c)$

$= -(0.2)[3.4 - (1.130496)(0.2) - 1.43624]$

$\boxed{\frac{\partial E}{\partial m} = -0.34753216}$

$\frac{\partial E}{\partial c} = -(y_i - mx_i - c)$

$= -[3.4 - (1.130496)(0.2) - 1.43624]$

$\boxed{\frac{\partial E}{\partial c} = -1.7376608}$

step5:- $\Delta m = -\eta \frac{\partial E}{\partial m}$

$= -(0.1)(-0.34753216)$

$\boxed{\Delta m = 0.034753216}$

$\Delta c = -\eta \frac{\partial E}{\partial c}$

$= -(0.1)(-1.7376608)$

$\boxed{\Delta c = 0.17376608}$

step6:- $m = m + \Delta m$

$= 1.130496 + 0.034753216$

$\boxed{m = 1.165249216}$

$c = c + \Delta c$

$= 1.43624 + 0.17376608$

$\boxed{c = 1.61000608}$

step 7:- sample = sample + 1
= 1 + 1
= 2

step 8:- if (sample > ns)
2 > 2 ✗
True : go to ~~step 9~~ next step (step 9)
False : go to step 4 ✓

step 4:-

$$\frac{\partial E}{\partial m} = -x_i(y_i - mx_i - c)$$
$$= -0.4\,[3.8 - (1.165249216)(0.4) - 1.61000608]$$
$$= -0.4\,(1.723894234)$$
$$\boxed{\frac{\partial E}{\partial m} = -0.6895576934}$$

$$\frac{\partial E}{\partial c} = -(y_i - mx_i - c)$$
$$= -[3.8 - (1.165249216)(0.4) - 1.61000608]$$
$$\boxed{\frac{\partial E}{\partial c} = -1.723894234}$$

step 5:- $\Delta m = -\eta \frac{\partial E}{\partial m}$
$= -0.1\,(-0.6895576934)$
$\boxed{\Delta m = 0.06895576934}$

$\Delta c = -\eta \frac{\partial E}{\partial c}$
$= -0.1\,(-1.723894234)$
$\boxed{\Delta c = 0.1723894234}$

step6:- $m = m + \Delta m$

$= 1.165249216 + 0.0689557 6934$

$\boxed{m = 1.234204985}$

$c = c + \Delta c$

$= 1.61000608 + 0.1723894234$

$\boxed{c = 1.7823955503}$

step7:- sample = sample + 1

$= 2 + 1$

$= 3$

step8:- if (sample > ns)

$3 > 2$ ✓

True : go to next step ✓

False : go to step 4

step9:- iter = iter + 1 = 2 + 1 = 3

step10:- if (iter > epochs)

$3 > 2$ ✓

True : go to next step ✓

False : go to step 3.

step11:- print m and c values

$m = 1.234204985$

$c = 1.7823955503.$