

* Manual Calculations for two iterations:-

$$f(x) = x^4 + 3x^2 + 10$$

1) Let $x = 10$ (initializing a Random variable value)

epochs = 2 (No of iterations)

$\eta = 0.001$ (learning rate)

initially consider $i = 1$ for iteration

$$\begin{aligned} 2) \frac{\partial f}{\partial x} \Big|_{x=10} &= 4x^3 + 6x \\ &= 4(10)^3 + 6(10) \\ &= 4060 \end{aligned}$$

$$\begin{aligned} 3) \Delta x &= -\eta \frac{\partial f}{\partial x} \Big|_{x=10} \\ &= -(0.001)(4060) \end{aligned}$$

$$\boxed{\Delta x = -4.06}$$

$$\begin{aligned} 4) x &= x + \Delta x \\ &= 10 + (-4.06) \end{aligned}$$

$$\boxed{x = 5.94}$$

increment $i = i + 1 = 1 + 1 = 2 \leq 2 \checkmark$

\therefore We have to go to Step 2 by taking x value in step 4.

$$\begin{aligned}\left. \frac{\partial f}{\partial x} \right|_{x=5.94} &= 4x^3 + 6x \\ &= 4(5.94)^3 + 6(5.94) \\ &= 873.978336.\end{aligned}$$

$$3) \Delta x = -\eta \left| \frac{\partial f}{\partial x} \right|_{x=5.94}$$

$$= -(0.001)(873.978336)$$

$$\boxed{\Delta x = -0.87397}$$

$$4) x = x + \Delta x$$

$$= 5.94 - 0.87397$$

$$\boxed{x = 5.06603}$$

$$\text{increment } i = i + 1 = 2 + 1 = 3 \leq 2x$$

\therefore We have to come out of the loop.

$$\therefore x = 5.06603.$$

$$f(x) = x^4 + 3x^2 + 10$$

$$= (5.06603)^4 + 3(5.06603)^2 + 10$$

$$\boxed{f(x) = 745.6687}$$