Assignment -11 :-

Let consider a sample dataset have one i/p (x_i^q) & one o/p (y_i^q) & number of samples 4. Develop a simple linear regression model using Nesterov Accelerated Gradient (NAG) optimizer.

sample(i)	x_i^q	y_i^q
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

- Do manual calculations for 2 iterations with First two samples).

sol:-

sample(i)	x_i^q	y_i^q
1	0.2	3.4
2	0.4	3.8

Applying NAG + SGD.

step1:- $[x, y]$, $m=1$, $c=1$, $\eta=0.1$, epochs $=2$, ns $=2$
$\gamma=0.9$, $v_m=v_m=0$

step2:- iter $=1$

step3:- sample $=1$.

step4:- $\frac{dE}{dm} = \frac{-1}{\quad}\left(y_i - (m+\gamma v_m)x_i - (c+\gamma v_c)\right)$
(x_i)

$= -1\left[\left[3.4 - [1 + (0.9)(0)][0.2] - (1 + (0.9)(0)\right]\right.$ (0.2)

$= -1[(2.2)(0.2)]$

$= -1[-0.44]$

$\frac{dE}{dm} = -0.44$

$\frac{dE}{dc} = -[y_i - (m + \gamma v_m)x_i - (c + \gamma v_c)]$

$= -[3.4 - (1 + (0.9)(0))(0.2) - [1 + (0.9)(0)]$

$= -[3.4 - 0.2 - 1]$

$\frac{dE}{dc} = -2.2$

step 5:- $v_m = \gamma v_m - \eta g_m$

$= (0.9)(0) - (0.1)(-0.44)$

$v_m = 0.044$

$v_c = \gamma v_c - \eta g_c$

$= (0.9)(0) - (0.1)(-2.2)$

$v_c = 0.22$

step 6:- $m = m + v_m$ $\quad c = c + v_c$

$= 1 + 0.044$ $\quad = 1 + 0.22$

$m = 1.044$ $\quad C = 1.22$

Step 7:- sample = sample + 1

= 1 + 1

= 2

Step 8:- if (sample > ns)

2 > 2 ✗

False: go to step 4.

Step 4:- $\frac{\partial E}{\partial m} = -\left(y_i - (m+\gamma v_m)x_i - (c+\gamma v_c)\right)x_i$

$= -\{3.8 - (1.044 + 0.9 * 0.044)0.4 - (1.22 + 0.9 * 0.22)\}(0.4)$

$= -[3.8 - 0.43344 - 1.418](0.4)$

$= -(1.94856)(0.4)$

$\frac{\partial E}{\partial m} = -0.779424$

$\frac{\partial E}{\partial c} = -[y_i - (m+\gamma v_m)x_i - (c+\gamma v_c)]$

$= -\{3.8 - (1.044 + 0.9 * 0.044)0.4 - (1.22 + 0.9 * 0.22)\}$

$= -[3.8 - 0.43344 - 1.418]$

$\frac{\partial E}{\partial c} = -1.94856$

Step 5:- $v_m = \gamma v_m - \eta \frac{\partial E}{\partial m}$

$= (0.9)(0.044) - (0.1)(-0.779424)$

$V_m = 0.1175424$

$V_c = \gamma V_c - \eta \frac{dE}{dc} = (0.9)(0.22) - (0.1)(-1.94856)$

$V_c = 0.392856$

step6:- $m = m + V_m$

$= 1.044 + 0.1175424$

$m = 1.1615424$

$c = c + V_c$

$= 1.22 + 0.392856$

$c = 1.612856$

step7:- sample = sample + 1

2 + 1

$= 3$

step8:- if (sample > ns)

$3 > 2$ ✓

True: go to next step

step9:- iter = iter + 1

1 + 1

$= 2$

step10:- if (iter > epochs)

$2 > 2$

False: go to step 3.

step3:- sample = 1

Step4:- $\frac{dE}{dm} = -[y_i - (m + \gamma V_m) x_i - [c + \gamma V_c]] x_i$

$$= -\{3.4 - (1.1615424 + [(0.9)(0.1175424)](0.2)$$
$$* [1.612856 + (0.9)(0.392856)]\}(0.2)$$

$$= -[3.4 - (1.26733056)0.2 - 1.96642340.2$$

$$= -[3.4 - 0.253466112 - 1.96642340.2$$

$$\frac{\partial E}{\partial m} = -[1.1801104880.2$$

$$\boxed{\frac{\partial E}{\partial m} = -0.23602209760}$$

$$\frac{\partial E}{\partial c} = -(y_i - (m + \gamma v_m)x_1 - (c + \gamma v_c)]$$

$$= -\{3.4 - [1.1615424 + (0.9)(0.1175424)]0.2$$
$$- (1.612856 + (0.9)(0.392856)]\}$$

$$= -\{3.4 - 0.253466112 - 1.96642340.2$$

$$\boxed{\frac{\partial E}{\partial c} = -1.180110488}$$

Steps:- $v_m = \gamma v_m - \eta \frac{\partial E}{\partial m}$

$$= (0.9)(0.1175424) - (0.1)(-0.2360220976)$$

$$\boxed{v_m = 0.1293903698}$$

~~$v_m = 0.08218594024$~~

$$v_c = \gamma v_c - \eta \frac{\partial E}{\partial c}$$

$$= (0.9)(0.392856) - (0.1)(-1.180110488)$$

$$\boxed{v_c = 0.4715814488}$$

step6:- $m = m + \Delta m$

$= 1.1615424 + 0.1293903698$

$m = 1.2909327 7$

$c = c + \Delta c$

$= 1.612856 + 0.4175814488$

$c = 2.0304377449$

step7:- sample = sample + 1

$= 1 + 1$

$= 2$

step8:- if (sample > ns)

$2 > 2$ ✗

False: go to step 4

step4:- $\frac{\partial E}{\partial m} = -[y_i - (m + \delta \Delta m) x_i - (c + \delta \Delta c)] x_i$

$= -\{(3.8) - [1.2909327 7 + (0.9)(0.1293903698)(0.4)$

$- [2.0304377449 + (0.9)(0.4715814488)](0.4)$

$= -[3.8 - 0.5629536411 - 2.4548607 53]0.4$

$= -(0.7821856059)(0.4)$

$\frac{\partial E}{\partial m} = -0.3128742424$

$$\frac{\partial E}{\partial c} = -[y_i - (m + \gamma v_m)x_i - (c + \gamma v_c)]$$

$$= -[3.8 - [1.290093277 + (0.9)(0.1293903698)](0.4) - [2.0304437449 + (0.9)(0.4715814488)]]$$

$$= -[3.8 - 0.5629536411 - 2.4548860753]$$

$$\frac{\partial E}{\partial c} = -0.7821856059$$

Steps:- $v_m = \gamma v_m - \eta \frac{\partial E}{\partial m}$

$$= (0.9)(0.1293903698) - (0.1)(-0.3128742424)$$

$$v_m = 0.1477387571$$

$$v_c = \gamma v_c - \eta \frac{\partial E}{\partial c}$$

$$= (0.9)(0.4715814488) - (0.1)[-0.7821856059)$$

$$v_c = 0.5026378645$$

Step6:- $m = m + v_m$

$$= 1.290093277 + 0.1477387571$$

$$m = 1.4386671527$$

$$c = c + v_c$$

$$= 2.0304437449 + 0.5026378645$$

$$c = 2.5330815314$$

step7:- sample = sample + 1
= 2 + 1
= 3

step8:- if (sample > ns)
3 > 2 ✓

True ; go to next step

step9 :- iter = iter + 1
= 2 + 1
= 3

step10: if (iter > epochs)
3 > 2 ✓

True :- go to next step.

step11:- print m and c values.

m = 1.4386715 27, c = 2.5330755314