MACHINE LEARNING ASSIGNMENT

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November 2022

1 Singular Value Decomposition(SVD)

The singular value decomposition of a matrix A is the factorization of A into the product of three matrices A = U.$\sum .V^T$, *where the columns of U and V are orthonormal and the matrix D is diagonal with positive realentries. The SVD is useful in many tasks.*

A= $\begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$

First find the eigenvalues of A$^T.A$, *the roots of these values will be the singular values for A.*

$\det A^T.A - \lambda I = \begin{vmatrix} 8-\lambda & 2 & 0 \\ 2 & 5-\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} = 0$

$(8-\lambda)[(5-\lambda).(-\lambda)-0] - 2(-2\lambda - 0) = 0$

(8-λ)$[-5\lambda + \lambda^2] = 4\lambda = 0$

-40$\lambda + 8\lambda^2 + 5\lambda^2 - \lambda^3 + 4\lambda = 0$

-$\lambda^3 + 13\lambda^2 - 36\lambda = 0$

$\lambda^3 - 13\lambda^2 + 36\lambda = 0$

$\lambda(\lambda - 4)(\lambda - 9) = 0$

Thus,$\lambda_1 = 9, \lambda_2 = 4, \lambda_3 = 0$

So, the singular values are 3 , 2 *and* 0 *Ordering the singular values such that* $\sum$ *has increasing values along the diagonal to simplify notation.*

$\sum = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$V = \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 1/\sqrt{5} & -2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

find U by using the formula AV=U$\sum$

$$\text{A.V=}\begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix} . \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 1/\sqrt{5} & -2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4/\sqrt{5} & 2/\sqrt{5} & 0 \\ \sqrt{5} & 0 & 0 \\ -2/\sqrt{5} & 4/\sqrt{5} & 0 \end{bmatrix}$$

$$\text{U.}\sum = \begin{bmatrix} 4/3\sqrt{5} & 1/\sqrt{5} & 0 \\ \sqrt{5}/3 & 0 & 0 \\ -2/3\sqrt{5} & 2/\sqrt{5} & 0 \end{bmatrix} . \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$Now\ all\ the\ three\ components\ of\ the\ SVD-\ decomposition\ such\ that$

$$\text{A=U.}\sum .V^T = \begin{bmatrix} 4/3\sqrt{5} & 1/\sqrt{5} & 0 \\ \sqrt{5}/3 & 0 & 0 \\ -2/3\sqrt{5} & 2/\sqrt{5} & 0 \end{bmatrix} . \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} . \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 1/\sqrt{5} & -2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2 Linear Discriminant Analysis(LDA)

Linear Discriminant Analysis (LDA) is one of the commonly used dimensionality reduction techniques in machine learning to solve more than two-class classification problems. It is also known as Normal Discriminant Analysis (NDA) or Discriminant Function Analysis (DFA).This can be used to project the features of higher dimensional space into lower-dimensional space in order to reduce resources and dimensional costs.

Eg:Compute the Linear Discriminant projection for the following two-dimensional dataset:

$\text{X}_1 = (x_1, x_2) = [(4,1),(2,4),(2,3),(3,6),(4,4)]$

$\text{X}_2 = (x_1, x_2) = [(9,10),(6,8),(9,5),(8,7),(10,8)]$

The class statistics are:

$$\text{S}_1 = \begin{bmatrix} 0.80 & -0.40 \\ -0.40 & 2.60 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 1.84 & -0.04 \\ -0.04 & 2.64 \end{bmatrix}$$

$\mu_1 = \begin{bmatrix} 3.00 & 3.60 \end{bmatrix}$

$\mu_2 = \begin{bmatrix} 8.40 & 7.60 \end{bmatrix}$

The between-class scatter matrix is:

$S_B = \begin{bmatrix} 29.16 & 21.60 \\ 21.60 & 16.00 \end{bmatrix}$

The within-class scatter matrix is:

$S_W = \begin{bmatrix} 2.64 & -0.44 \\ -0.44 & 5.28 \end{bmatrix}$

The LDA projection is then obtained as the solution of the generalized eigenvalue problem.

$S_W^{-}1.S_B.V = \lambda.V => \left| S_W^{-}1.S_B - \lambda I \right| = 0$
$=> \begin{vmatrix} 11.89 - \lambda & 8.81 \\ 5.08 & 3.76 - \lambda \end{vmatrix} = 0 => \lambda = 15.65$

$\begin{bmatrix} 11.89 & 8.81 \\ 5.08 & 3.76 \end{bmatrix} . \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$
$= 15.65. \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$
$=> \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0.91 \\ 0.39 \end{bmatrix}$

Or directly by

$W^* = S_W^{-}1.(\mu_1 - \mu_2) = [-0.91 \quad -0.39]^T$