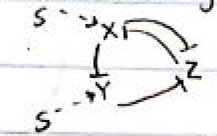
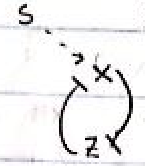
2. AC-DC: nonlinear dynamics



(a)



$$\frac{\partial \tilde{x}}{\partial \tilde{t}} = \frac{\tilde{\alpha}_x + \tilde{\beta}_x S}{1+S+(\tilde{z}/\tilde{z}_x)^{n_{zx}}} - \tilde{\delta}_x \tilde{x}$$

$$\frac{\partial \tilde{z}}{\partial \tilde{t}} = \frac{\tilde{\alpha}_z}{1+(\tilde{x}/\tilde{x}_z)^{n_{xz}}} - \tilde{\delta}_z \tilde{z}$$

(b)

$$\frac{\partial x}{\partial t} = \frac{\alpha_x + \beta_x S}{1+S+(z/z_x)^{n_{zx}}} - x$$

$$\frac{\partial z}{\partial t} = \frac{1}{1+(x/x_z)^{n_{xz}}} - \delta_z z$$

Correction: $t = \tilde{t}\,\tilde{\delta}_x$

$$\frac{\partial \tilde{x}}{\partial \tilde{t}} = \frac{\partial[x\tilde{\alpha}_z/\tilde{\delta}_x]}{\partial[t/\tilde{\delta}_x]} = \frac{\alpha_x\tilde{\alpha}_z + \beta_x\tilde{\alpha}_z S}{1+S+\left[\left(\frac{z\tilde{\alpha}_z}{\tilde{\delta}_x}\right)\Big/\left(\frac{z_x\tilde{\alpha}_z}{\tilde{\delta}_x}\right)\right]^{n_{zx}}} - \tilde{\delta}_x\left(\frac{x\tilde{\alpha}_z}{\tilde{\delta}_x}\right)$$

$$= \frac{\partial x}{\partial t} = \left[\left[\frac{\alpha_x + \beta_x S}{1+S+\left[\frac{z}{z_x}\right]^{n_{zx}}}\right] - x\right] = \frac{\alpha_x + \beta_x S}{1+S+\left[\frac{z}{z_x}\right]^{n_{zx}}} - x$$

$$\frac{\partial \tilde{z}}{\partial \tilde{t}} = \frac{\partial[z\tilde{\alpha}_z/\tilde{\delta}_x]}{\partial[t/\tilde{\delta}_x]} = \frac{\cancel{\tilde{\alpha}_z}}{1+\left[\left(\frac{x\tilde{\alpha}_z}{\tilde{\delta}_x}\right)\Big/\left(\frac{x_z\tilde{\alpha}_z}{\tilde{\delta}_x}\right)\right]^{n_{xz}}} - \tilde{\delta}_z\left[\frac{z\tilde{\alpha}_z}{\tilde{\delta}_x}\right]$$

$$= \frac{\partial z}{\partial t} = \frac{1}{1+\left[\frac{x}{x_z}\right]^{n_{xz}}} - \delta_z z$$