ADVANCED STATISTICS

Project Report

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Contents

	Problem 4	
4.1	What is the probability that a randomly chosen student gets a grade below 85 on this exam?	14
4.2	What is the probability that a randomly selected student score between 65 and 87?	14
4.3	What should be the passing cut-off so that 75% of the students clear the exam?	15
	Problem 5	
5.1	Earlier experience of Zingaro with this particular client is favorable as the stone surface was found to be of adequate hardness. However, Zingaro has reason to believe now that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?	17
5.2	Is the mean hardness of the polished and unpolished stones the same?	18
	Problem 6	
	Aquarius health club, one of the largest and most popular cross-fit gyms in the country has been advertising a rigorous program for body conditioning. The program is considered successful if the candidate is able to do more than 5 push-ups, as compared to when he/she enrolled in the program. Using the sample data provided can you conclude whether the program is successful? (Consider the level of Significance as 5%)	19
	Problem 7	
7.1	Test whether there is any difference among the dentists on the implant hardness. State the null and alternative hypotheses. Note that both types of alloys cannot be considered together. You must state the null and alternative hypotheses separately for the two types of alloys.?	22
7.2	Before the hypotheses may be tested, state the required assumptions. Are the assumptions fulfilled? Comment separately on both alloy types.?	22
7.3	Irrespective of your conclusion in 2, we will continue with the testing procedure. What do you conclude regarding whether implant hardness depends on dentists? Clearly state your conclusion. If the null hypothesis is rejected, is it possible to identify which pairs of dentists differ?	23

7.4	Now test whether there is any difference among the methods on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which pairs of methods differ?	25
7.5	Now test whether there is any difference among the temperature levels on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which levels of temperatures differ?	27
7.6	Consider the interaction effect of dentist and method and comment on the interaction plot, separately for the two types of alloys?	29
7.7	Now consider the effect of both factors, dentist, and method, separately on each alloy. What do you conclude? Is it possible to identify which dentists are different, which methods are different, and which interaction levels are different?	31

List of Figures

List of Tables

Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

	Striker	Forward	Attacking Midfielder	Winger	**Total**
Players Injured	45	56	24	20	**145**
Players Not Injured	32	38	11	9	**90**
Total	**77**	**94**	**35**	**29**	**235**

1.1 What is the probability that a randomly chosen player would suffer an injury?

Probability that a randomly chosen player would suffer an injury is given by

= No. of players injured / Total number of players.

= 145/235

= **0.6170**

1.2 What is the probability that a player is a forward or a winger?

Let us assume,

Event A =The selected player is a forward. P(A) = 94/235

Event B =The selected player is a winger. P(B)=29/235

Since events A and B are mutually exclusive events, we use the formula

$P(A \cup B) = P(A) + P(B)$

= 94/235 + 29/235

= **0.5234**

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

The number of players who play in a striker position AND has a foot injury is 45.

Hence, the probability of choosing a player who plays in striker position and has a foot injury is 45/235

= **0.1915**

1.4 What is the probability that a randomly chosen injured player is a striker?

Let,

Event A = The chosen player is striker.

Event B = The chosen player is injured. P(B)= 145/235

Event A ∩ B = The chosen player is a striker and injured. $P(A \cap B)$ = 45/235

The formula for the conditional probability is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{45/_{235}}{145/_{235}}$$

= 45/145

= **0.3103**

1.5 What is the probability that a randomly chosen injured player is either a forward or an attacking midfielder?

Let,

Event A = The chosen injured player is a forward. P(A) = 56/145.

Event B = The chosen injured player is an attacking midfielder. P(B)= 24/145.

Since the two events are mutually exclusive, we can use the formula

$$P(A \cup B) = P(A) + P(B)$$

= 56/145 + 24/145

= **0.5517**

Problem 2

An independent research organization is trying to estimate the probability that an accident at a nuclear power plant will result in radiation leakage. The types of accidents possible at the plant are, fire hazards, mechanical failure, or human error. The research organization also knows that two or more types of accidents cannot occur simultaneously.

According to the studies carried out by the organization, the probability of a radiation leak in case of a fire is 20%, the probability of a radiation leak in case of a mechanical 50%, and the probability of a radiation leak in case of a human error is 10%. The studies also showed the following;

The probability of a radiation leak occurring simultaneously with a fire is 0.1%.

The probability of a radiation leak occurring simultaneously with a mechanical failure is 0.15%.

The probability of a radiation leak occurring simultaneously with a human error is 0.12%.

On the basis of the information available, answer the questions below:

Let us define the events.

Event A = Occurrence of fire.

Event B = Occurrence of mechanical failure.

Event C = Occurrence of human error.

Event E = Occurrence of radiation leakage.

Given:

$P(E|A) = 0.2$

$P(E|B) = 0.5$

$P(E|C) = 0.1$

$P(E \cap A) = 0.001$

$P(E \cap B) = 0.0015$

$P(E \cap C) = 0.0012$

2.1 What are the probabilities of a fire, a mechanical failure, and a human error respectively?

We use the below formula to calculate the probabilities:

$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$$

Which implies that

$$P(Y) = \frac{P(X \cap Y)}{P(X|Y)}$$

Hence,

a. Probability of a fire = P(A) = P(E∩A) / P(E|A) = 0.001/0.2 = **0.005**
b. Probability of a mechanical failure = P(B) = P(E∩B) / P(E|B) = 0.0015/0.5 = **0.003**
c. Probability of a human error = P(C) = P(E∩C) / P(E|C) = 0.0012/0.1 = **0.012**

2.2 What is the probability of a radiation leak?

We can use the below formula to calculate the probability of radiation leak.

$$P(E) = P(E|A)P(A) + P(E|B)P(B) + P(E|C)P(C)$$

= (0.2) (0.005) + (0.5) (0.003) + (0.1) (0.012)

= **0.0037**

2.3 Suppose there has been a radiation leak in the reactor for which the definite cause is not known. What is the probability that it has been caused by:

- **A Fire.**
- **A Mechanical Failure.**
- **A Human Error.**

Probability that radiation leak is caused by fire =

P(A|E) = P(A∩E) / P(E) = 0.001/0.0037 = **0.2703**

Probability that radiation leak is caused by mechanical failure =

P(B|E) = P(B∩E) / P(E) = 0.0015/0.0037 = **0.4054**

Probability that radiation leak is caused by human error =

P(C|E) = P(C∩E) / P(E) = 0.0012/0.0037 = **0.3243**

<u>Problem 3:</u>

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information.

3.1 What proportion of the gunny bags have a breaking strength less than 3.17 kg per sq cm?

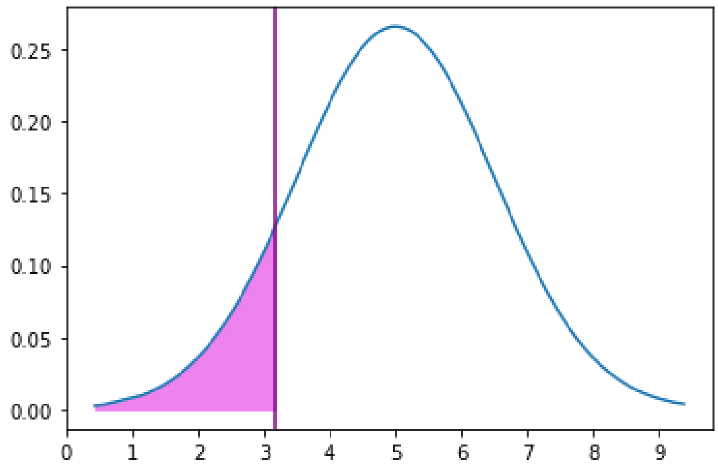

Fig. 1: Proportion of bags having a breaking strength less than 3.17 kg per sq cm

11.12% of the bags have a breaking strength less than 3.17 kg per sq cm.

3.2 What proportion of the gunny bags have a breaking strength at least 3.6 kg per sq cm.?

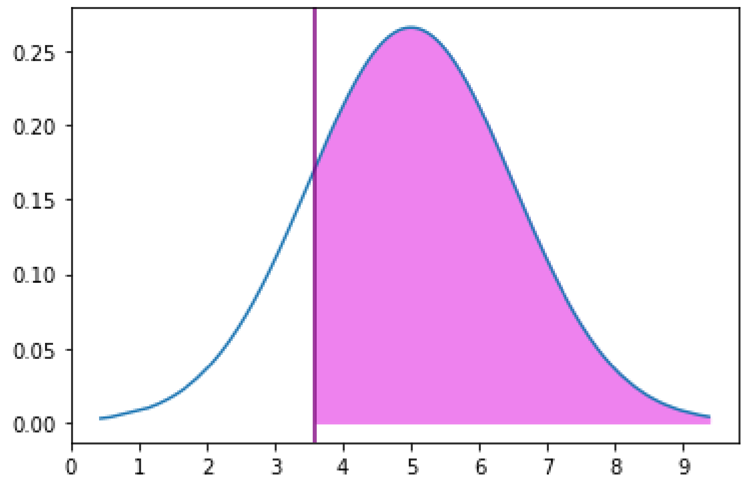

Fig. 2: Proportion of bags having a breaking strength at least 3.6 kg per sq cm

82.47% of the bags have a breaking strength of at least 3.6 kg per sq cm.

3.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

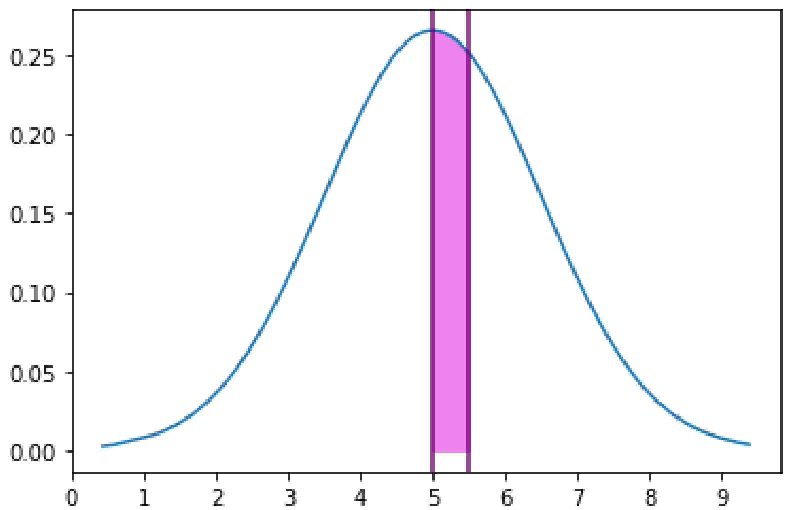

Fig. 3: Proportion of bags having a breaking between 5 and 5.5 kg per sq cm

13.06% of the bags have a breaking strength between 5 and 5.5 kg per sq cm.

3.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?

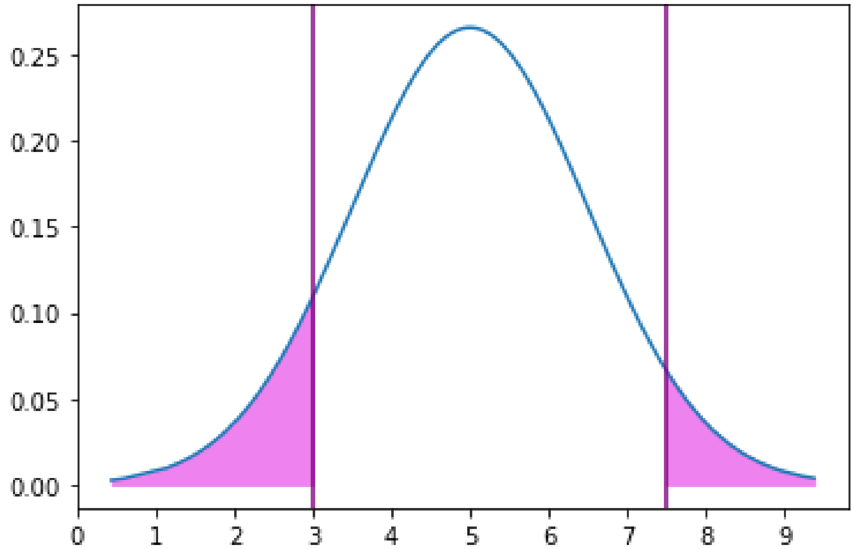

Fig. 4: Proportion of bags having a breaking not between 3 and 7.5 kg per sq cm

13.90% of the bags have a breaking strength less than 3 and more than 7.5 kg per sq cm.

Problem 4:

Grades of the final examination in a training course are found to be normally distributed, with a mean of 77 and a standard deviation of 8.5. Based on the given information answer the questions below.

4.1 What is the probability that a randomly chosen student gets a grade below 85 on this exam?

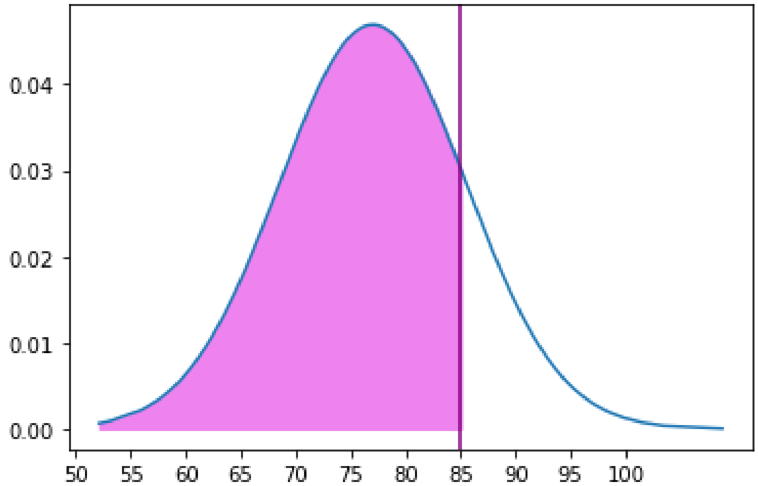

Fig. 5: Proportion of students scoring below 85.

82.67% of students score below 85 in this exam.

4.2 What is the probability that a randomly selected student score between 65 and 87?

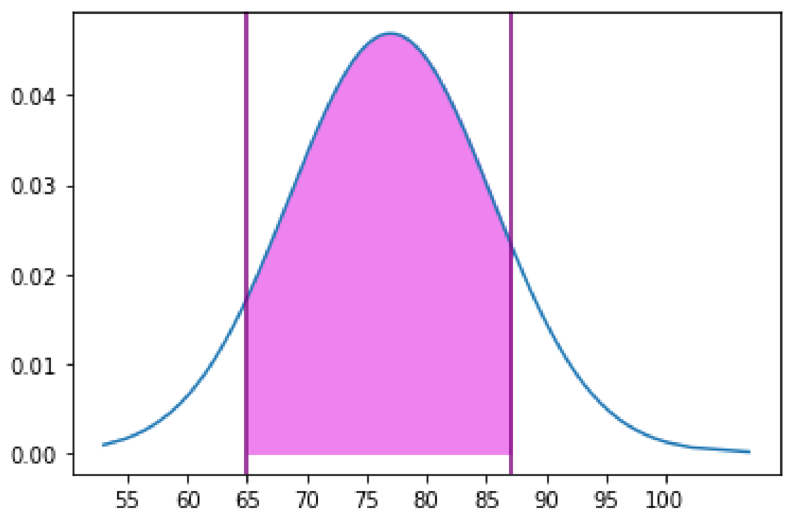

Fig. 6: Proportion of students scoring between 65 and 87.

80.13% of students score between 65 and 87.

4.3 What should be the passing cut-off so that 75% of the students clear the exam?

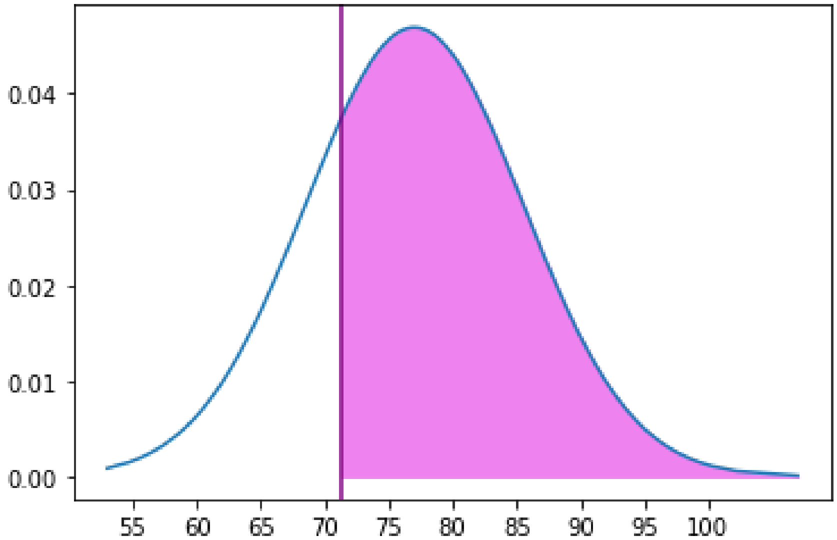

Fig. 7: Proportion showing 75% of the students passing.

71.27 should be the cut off marks so that 75% of the students clear the exam.

Problem 5:

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

When we load the data into the notebook, we observe the following:

a. There are two columns ('Unpolished' and 'Treated and Polished') and 75 records.
b. The columns contain the Brinell's hardness index of the unpolished and polished stones that Zingaro received respectively.
c. There are no missing values in the data set.
d. There are no duplicates in the data set.

Table 1: First five records of the Zingaro data.

	Unpolished	Treated and Polished
0	164.481713	133.209393
1	154.307045	138.482771
2	129.861048	159.665201
3	159.096184	145.663528
4	135.256748	136.789227

Table 2: Statistical description of the Zingaro data.

	count	mean	std	min	25%	50%	75%	max
Unpolished	75.0	134.110527	33.041804	48.406838	115.329753	135.597121	158.215098	200.161313
Treated and Polished	75.0	147.788117	15.587355	107.524167	138.268300	145.721322	157.373318	192.272856

5.1 Earlier experience of Zingaro with this particular client is favorable as the stone surface was found to be of adequate hardness. However, Zingaro has reason to believe now that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

We will find out if Zingaro is justified in thinking that the unpolished stones are not suitable for printing by framing and testing a hypothesis.

<u>Step 1:</u> Framing null and alternate hypothesis:

Null Hypothesis = H_0: The mean hardness μ of unpolished stones is greater than or equal to 150

Alternate hypothesis=H_A: The mean hardness μ of unpolished stones is less than 150

H_0: $\mu \geq 150$

H_A: $\mu < 150$

<u>Step 2:</u> Decide the level of significance:

It is given that the level of significance is 5%.

$\alpha = 0.05$

<u>Step 3:</u> Identify the test statistic:

a. We have only one sample data with respect to the unpolished stones.
b. This is a one tailed test. Furthermore, a left tailed test.
c. We do not know the standard deviation of whole population.

Hence, we are going for the **ONE SAMPLE T TEST**.

<u>Step 4:</u> Calculating the test statistic and p-value:

From the one sample t-test, we find that,

a. **Test statistic = -4.16463**
b. **p-value = 4.17129×10^{-5}**

<u>Step 5:</u> Decision to reject or fail to reject the null hypothesis:

Since p-value is less than α, we have enough evidence to reject the null hypothesis in favor of the alternate hypothesis. We can conclude that the mean hardness of unpolished stones is less than 150. Hence, the unpolished stones are not suitable for printing.

5.2 Is the mean hardness of the polished and unpolished stones the same?

We will answer this question again by framing and testing a hypothesis.

<u>Step 1:</u> Framing null and alternate hypothesis:

Null Hypothesis = H_0: The mean hardness of polished stones (μ_p) is equal to the mean hardness of unpolished stones (μ_u).

Alternate hypothesis=H_A: The mean hardness of polished stones (μ_p) is not equal to the mean hardness of unpolished stones (μ_u).

H_0: $\mu_p = \mu_u$

H_A: $\mu_p \neq \mu_u$

<u>Step 2:</u> Decide the level of significance:

It is given that the level of significance is 5%.

$\alpha = 0.05$

<u>Step 3:</u> Identify the test statistic:

a. We have two sample data, one of unpolished stones, other of treated and polished stones.
b. The two samples are independent.
c. This is a two tailed test.

Hence, we are going for the **INDEPENDENT TWO SAMPLE T TEST**.

<u>Step 4:</u> Calculating the test statistic and p-value:

From the independent two sample t test, we find that,

a. Test statistic = -3.24223

b. p-value = 1.46552×10^{-3}

<u>Step 5:</u> Decision to reject or fail to reject the null hypothesis:

Since p-value is less than α, we have enough evidence to reject the null hypothesis in favor of the alternate hypothesis. We can conclude that the mean hardness of polished stones is not equal to the mean hardness of the unpolished stones.

Problem 6:

Aquarius health club, one of the largest and most popular cross-fit gyms in the country has been advertising a rigorous program for body conditioning. The program is considered successful if the candidate is able to do more than 5 push-ups, as compared to when he/she enrolled in the program. Using the sample data provided can you conclude whether the program is successful? (Consider the level of Significance as 5%)¶

We will decide if the program is successful or not by testing the hypothesis. Before we jump into hypothesis testing, we will try to understand the data.

a. The data set has 3 columns and 100 records.

b. The three columns are ‘Sr. No.’, ’Before’ and ‘After’.

c. The ‘Before’ and ‘After’ columns contain the number of pushups the candidates used to do before and after the body conditioning program.

d. There are no missing values.

e. There are no duplicates.

Table 3: First 5 records of the Aquarius health club data

	Sr no.	Before	After
0	1	39	44
1	2	25	25
2	3	39	39
3	4	6	13
4	5	40	44

f. We are dropping the ‘Sr. No.’ column, since it is of no use to us.

Table 4: Statistical description of the data.

	count	mean	std	min	25%	50%	75%	max
Before	100.0	26.94	8.806357	3.0	21.75	28.0	32.25	47.0
After	100.0	32.49	8.779562	10.0	26.00	34.0	39.00	51.0

Now, that we have understood the data, we will proceed to frame and test the hypothesis to check whether the program has been successful or not.

Step 1: Framing null and alternate hypothesis:

Null Hypothesis = H_0: The candidate is unable to do more than 5 push-ups, as compared to when he/she enrolled in the program.

Alternate hypothesis=H_A: The candidate is able to do more than 5 push-ups, as compared to when he/she enrolled in the program.

Let μ_b and μ_a be the mean number of push-ups the candidate does before and after the program respectively.

H_0: $\mu_a \leq \mu_b + 5 \quad \rightarrow \quad H_0$: $\mu_a - \mu_b \leq 5$

H_A: $\mu_a > \mu_b + 5 \quad \rightarrow \quad H_0$: $\mu_a - \mu_b > 5$

Step 2: Decide the level of significance:

It is given that the level of significance is 5%.

$\alpha = 0.05$

Step 3: Identify the test statistic:

d. We have two sample data, one of the number of push-ups before the program and other of the number of push-ups after the program.

e. The two samples are related to each other.

f. This is a one tailed test, furthermore this is a left tailed test.

Hence, we are going for the **PAIRED T-TEST.**

Step 4: Calculating the test statistic and p-value:

From the independent two sample t test, we find that,

a. **Test statistic = -19.32262**

b. **p-value = 1.146020 x 10^{-35}**

Step 5: Decision to reject or fail to reject the null hypothesis:

Since p-value is less than α, we have enough evidence to reject the null hypothesis in favor of the alternate hypothesis. We can conclude that the body conditioning program is successful.

Problem 7:

Dental implant data: The hardness of metal implant in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as on the dentists who may favor one method above another and may work better in his/her favorite method. The response is the variable of interest.

First, we will take a look at the data before answering the below questions.

Table 5: First five records of the dental implant data.

	Dentist	Method	Alloy	Temp	Response
0	1	1	1	1500	813
1	1	1	1	1600	792
2	1	1	1	1700	792
3	1	1	2	1500	907
4	1	1	2	1600	792

- The given data has 5 columns, namely ‘Dentist’, ‘Method’, ‘Alloy’, ’Temp’ and ‘Response’.
- As given above, ‘Response’ is the variable of interest, which depends on the multiple factors.
- There are 90 records in the data frame.
- There are no missing values.
- There are no duplicates.
- The works of five dentists are recorded in the data.
- Three different methods are being followed for the dental implants.
- We find there are two alloys that are being used.
- Three different temperatures have been recorded in the data. 1500, 1600 and 1700.

Since it is given that both alloys cannot be considered together, we have divided the data into sub-sets on basis of alloy 1 and alloy 2. Now both the data frames have 45 records each.

Table 6: First five records of the divided data.

	Dentist	Method	Temp	Response
0	1	1	1500	813
1	1	1	1600	792
2	1	1	1700	792
3	1	2	1500	782
4	1	2	1600	698

	Dentist	Method	Temp	Response
0	1	1	1500	907
1	1	1	1600	792
2	1	1	1700	835
3	1	2	1500	1115
4	1	2	1600	835

Before we move on, we will fix the level of significance to be **α = 5% = 0.05** throughout the problem 7.

7.1 Test whether there is any difference among the dentists on the implant hardness. State the null and alternative hypotheses. Note that both types of alloys cannot be considered together. You must state the null and alternative hypotheses separately for the two types of alloys.?

Case 1: Using alloy 1.

Null hypothesis: The mean response(hardness) of the dental implant is same for all five dentists while using alloy 1.

Alternate hypothesis: There is at least one dentist whose mean response(hardness) of the dental implant is different than others while using alloy 1.

Case 2: Using alloy 2.

Null hypothesis: The mean response(hardness) of the dental implant is same for all five dentists while using alloy 2.

Alternate hypothesis: There is at least one dentist whose mean response(hardness) of the dental implant is different than others while using alloy 2.

7.2 Before the hypotheses may be tested, state the required assumptions. Are the assumptions fulfilled? Comment separately on both alloy types.?

Assumptions

• Populations are normally distributed

• Populations have equal variances

• Samples are randomly and independently drawn.

7.3 Irrespective of your conclusion in 2, we will continue with the testing procedure. What do you conclude regarding whether implant hardness depends on dentists? Clearly state your conclusion. If the null hypothesis is rejected, is it possible to identify which pairs of dentists differ?

Case 1: Using alloy 1.

To check the hypotheses stated in 7.1, we have performed the ANOVA test, the result is as given below.

Table 7: ANOVA table for the factor 'Dentist' – Case1.

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	1.977112	0.116567
Residual	40.0	539593.555556	13489.838889	NaN	NaN

From the table above, we see that the F-value is 1.977. The corresponding p-value is **0.116567**, which is greater than α. Hence, **we fail to reject the null hypothesis.** We conclude the mean responses are same for all dentists while using alloy 1.
In other words, we can say that we are only about 89% sure about the impact of dentists on the hardness of dental implant, which is statistically not enough to reject the null hypothesis.

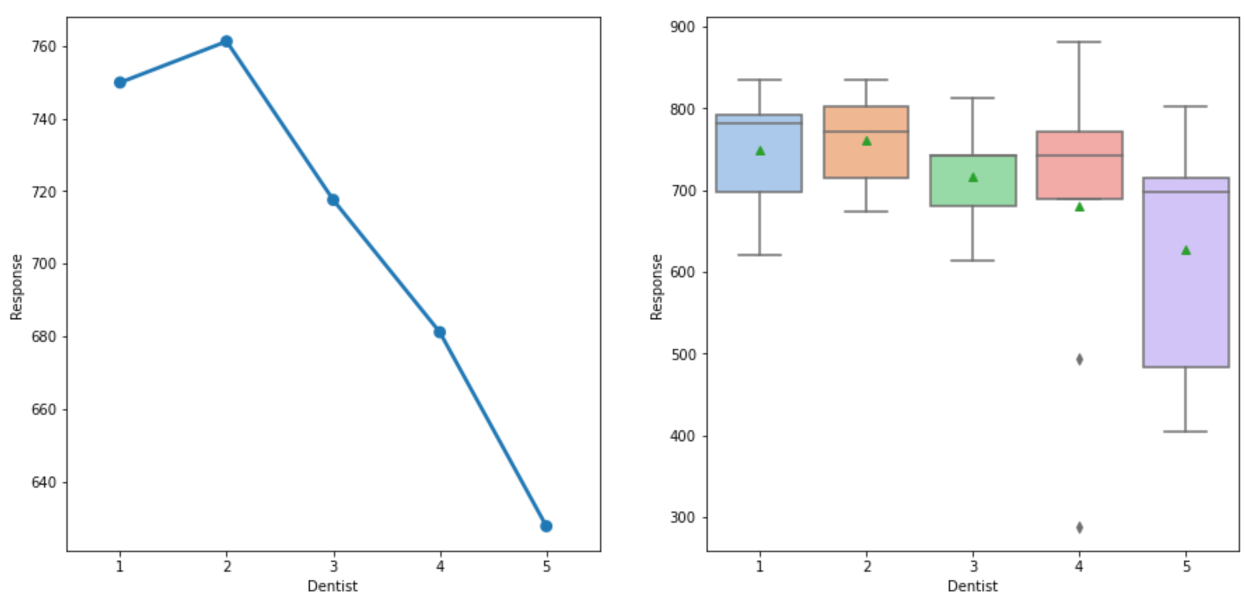

Fig. 8: Distribution of Response v/s Dentists – Case 1.

From the graphs above we see that mean response for the dentist no. 5 is lesser than the means of others, but we do not have enough statistical evidence to say so.

<u>Case 2</u>: Using alloy 2.

Similar to case 1, we have performed ANOVA test for the data in which alloy 2 is used, the result of the ANOVA test is given below.

Table 8: ANOVA table for the factor 'Dentist' – Case2.

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	5.679791e+04	14199.477778	0.524835	0.718031
Residual	40.0	1.082205e+06	27055.122222	NaN	NaN

In this case, F-value is 0.524835 and the corresponding p-value is **0.718031**, which is quite high compared to α. Hence, **we fail to reject the hypothesis**. We conclude the mean responses are same for all dentists while using alloy 2.

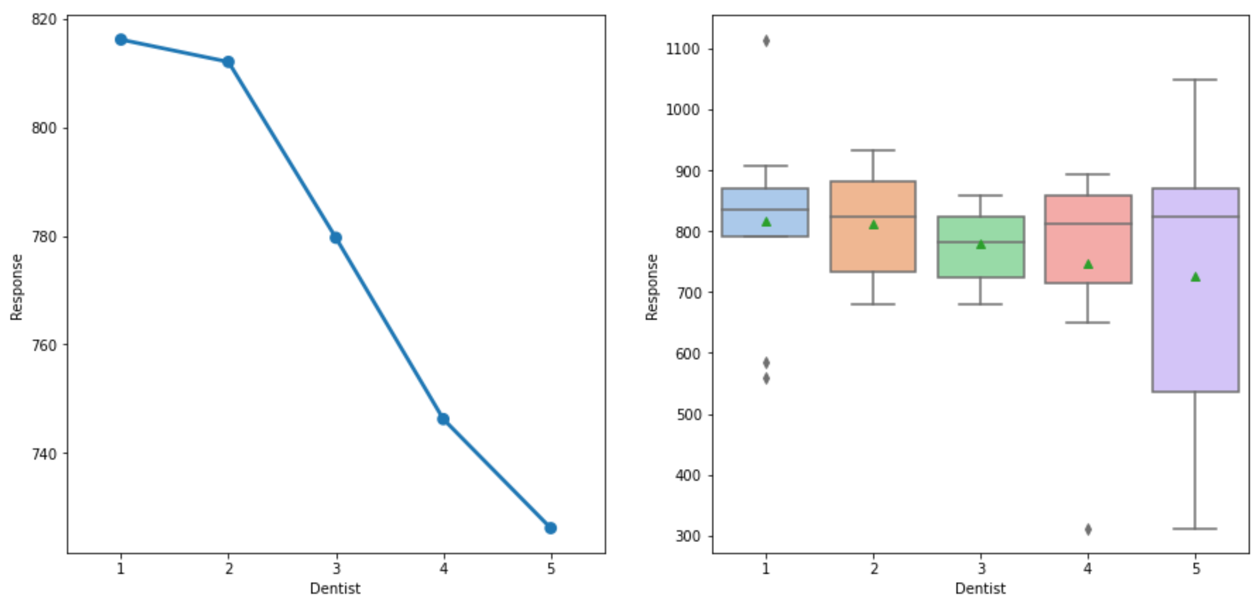

Fig. 9: Distribution of Response v/s Dentists – Case 2.

From the box plot above, we see that mean responses for different dentists are pretty same. We may say that the hardness of the dental implant for different dentists while using alloy 2 lie in the same range.

7.4 Now test whether there is any difference among the methods on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which pairs of methods differ?

<u>**Case 1**</u>**:** Using alloy 1.

<u>Null hypothesis:</u> The mean response(hardness) of the dental implant is same for all three temperatures, while using alloy 1.

<u>Alternate hypothesis:</u> There is at least one temperature in which mean response(hardness) of the dental implant is different than others, while using alloy 1.

We have fixed the level of significance α = 0.05.
Now, we shall perform ANOVA test on the above hypotheses.

Table 9: ANOVA table for the factor 'Temperature – Case1.

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	148472.177778	74236.088889	6.263327	0.004163
Residual	42.0	497805.066667	11852.501587	NaN	NaN

From the ANOVA test, we find that the F statistic is 6.263327 and the corresponding p-value is **0.004163**, which is less than α. Hence, we **reject the null hypothesis** in favor of the alternate hypothesis. There is at least one method, whose mean response is different from others.

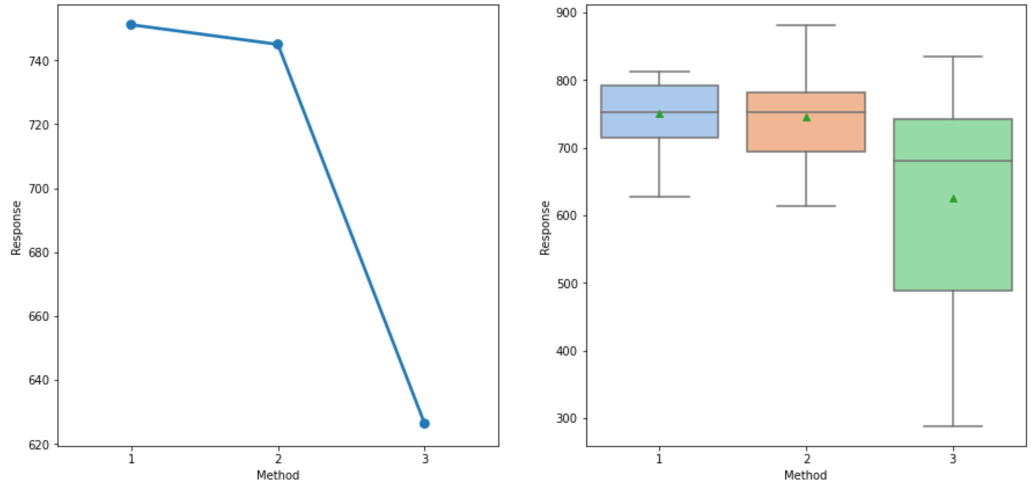

Fig. 10: Distribution of Response v/s Methods – Case 1.

From the graph, there is a visual difference in the mean response (hardness) while using method 3.

<u>Case 2</u>: Using alloy 2.

<u>Null hypothesis:</u> The mean response(hardness) of the dental implant is same for all three methods, while using alloy 2.

<u>Alternate hypothesis:</u> There is at least one method in which mean response(hardness) of the dental implant is different than others, while using alloy 2.

The result of the ANOVA test is as follows.

Table 10: ANOVA table for the factor 'Method' – Case2.

	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	499640.4	249820.200000	16.4108	0.000005
Residual	42.0	639362.4	15222.914286	NaN	NaN

The value of F-statistic is 16.4108, the corresponding p-value is **0.000005**, which is very less compared to α. Hence, we have strong evidence to **reject the null hypothesis** in favor of the alternate hypothesis. We can conclude that 'Method' is a significant factor which plays an important role in determining the hardness of the dental implant while using both alloy 1 and alloy 2.

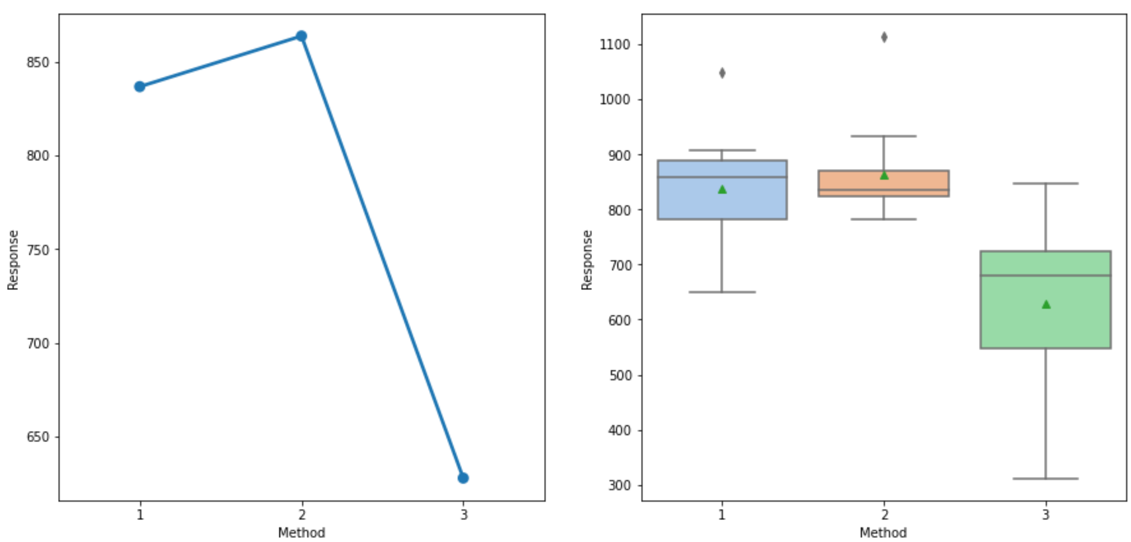

Fig. 11: Distribution of Response v/s Methods – Case 2.

Similar to case 1, we see that the hardness of the dental implant while using the method 3 is low compared to the other two methods.

7.5 Now test whether there is any difference among the temperature levels on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which levels of temperatures differ?

Case 1: Using alloy 1.

Null hypothesis: The mean response(hardness) of the dental implant is same for all three temperatures, while using alloy 1.

Alternate hypothesis: There is at least one temperature in which mean response(hardness) of the dental implant is different than others, while using alloy 1.

We have fixed the level of significance α = 0.05.
Now, we shall perform ANOVA test on the above hypotheses.

Table 11: ANOVA table for the factor 'Method' – Case1.

	df	sum_sq	mean_sq	F	PR(>F)
C(Temp)	2.0	10154.444444	5077.222222	0.335224	0.717074
Residual	42.0	636122.800000	15145.780952	NaN	NaN

From the ANOVA test, we find that the F statistic is 0.335224 and the corresponding p-value is **0.717074**, which is greater than α. Hence, we **fail to reject the null hypothesis** in favor of the alternate hypothesis. We will have to conclude that temperature is not a significant factor that affects the hardness of the dental implant.

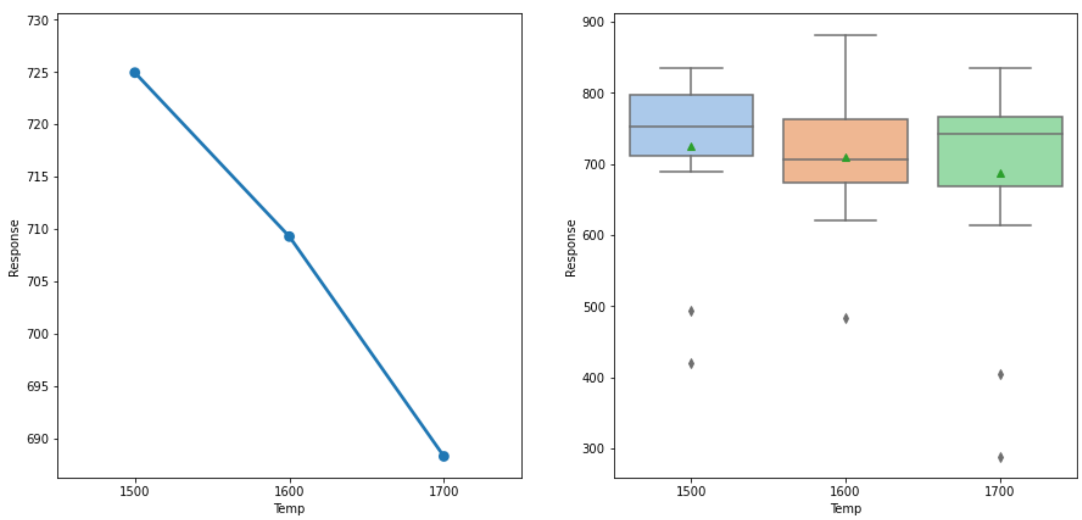

Fig. 12: Distribution of Response v/s Temperature – Case 1.

In the above graph, even though the point plot shows a variation in the response, the box plot gives more clarity. We see the means are lying in almost the same range.

<u>Case 2</u>: Using alloy 2.

<u>Null hypothesis:</u> The mean response(hardness) of the dental implant is same for all three temperatures, while using alloy 2.

<u>Alternate hypothesis:</u> There is at least one temperature in which mean response(hardness) of the dental implant is different than others, while using alloy 2.

We have fixed the level of significance α = 0.05.
Now, we shall perform ANOVA test on the above hypotheses.

Table 12: ANOVA table for the factor 'Method' – Case2.

	df	sum_sq	mean_sq	F	PR(>F)
C(Temp)	2.0	9.374893e+04	46874.466667	1.883492	0.164678
Residual	42.0	1.045254e+06	24886.996825	NaN	NaN

From the ANOVA test, we find that the F statistic is 1.883492 and the corresponding p-value is **0.164678**, which is greater than α. Hence, we **fail to reject the null hypothesis** in favor of the alternate hypothesis.

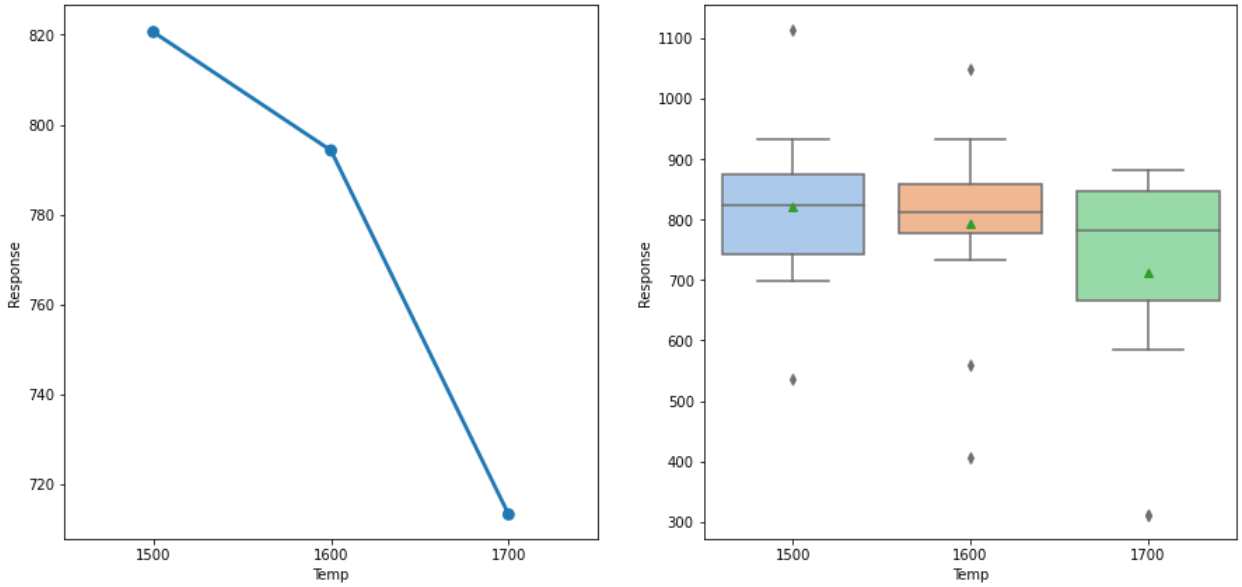

Fig. 13: Distribution of Response v/s Temperature – Case 2.

From the box plot above, we see that the mean response for the temperature 1700 is lesser when compared to other temperatures. But we are only about 84% sure of the impact of the temperature on the hardness of the implant. Hence, we do not have enough statistical evidence to reject the null hypothesis.

7.6 Consider the interaction effect of dentist and method and comment on the interaction plot, separately for the two types of alloys?

<u>Case 1</u>: Using alloy 1.

<u>Null hypothesis:</u> There is no interaction effect between the factors 'Dentist' and 'Method' with respect to the mean response, while using alloy 1.

<u>Alternate hypothesis:</u> There is interaction effect between the factors 'Dentist' and 'Method' with respect to the mean response, while using alloy 1.

We have fixed the level of significance α = 0.05.
Now, we shall perform ANOVA test on the above hypotheses.

Table 13: ANOVA table for the interaction effect – Case1.

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist):C(Method)	14.0	441097.244444	31506.946032	4.606728	0.000221
Residual	30.0	205180.000000	6839.333333	NaN	NaN

From the table above, we see that F value is 4.606728 and the p-value is **0.000221**, which is less than α. Hence, we **reject the null hypothesis** in favor of the alternate hypothesis.

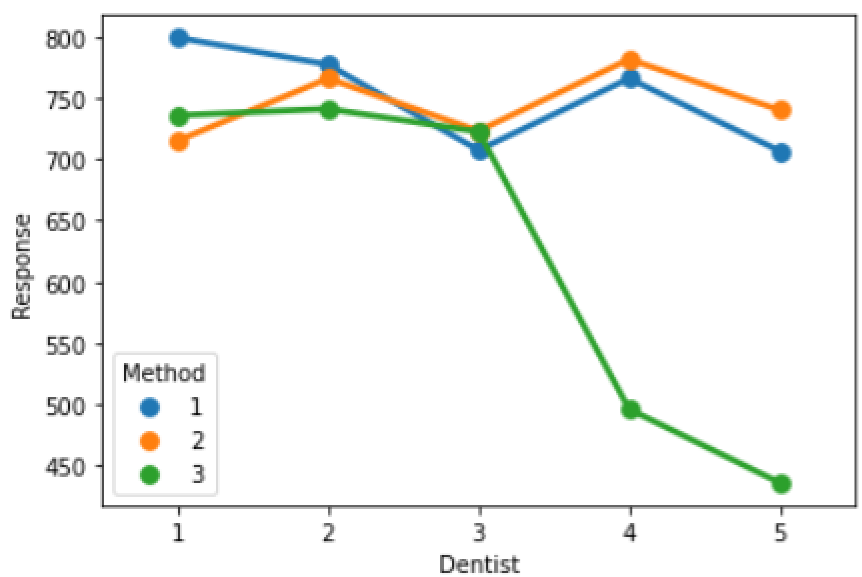

Fig. 14: Interaction between the factors 'Dentist' and 'Method' – Case 1.

We have statistical and visual evidence to conclude that there is an interaction effect between the two factors. The mean hardness of dentists 4 and 5, when using method 3 is on the lower side.

<u>Case 2</u>: Using alloy 2.

<u>Null hypothesis:</u> There is no interaction effect between the factors 'Dentist' and 'Method' with respect to the mean response, while using alloy 2.

<u>Alternate hypothesis:</u> There is interaction effect between the factors 'Dentist' and 'Method' with respect to the mean response, while using alloy 2.

We have fixed the level of significance α = 0.05.
Now, we shall perform ANOVA test on the above hypotheses.

Table 14: ANOVA table for the interaction effect – Case2.

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist):C(Method)	14.0	753898.133333	53849.866667	4.194953	0.000482
Residual	30.0	385104.666667	12836.822222	NaN	NaN

From the table above, we see that F value is 4.194953 and the p-value is **0.000482**, which is less than α. Hence, we **reject the null hypothesis** in favor of the alternate hypothesis.

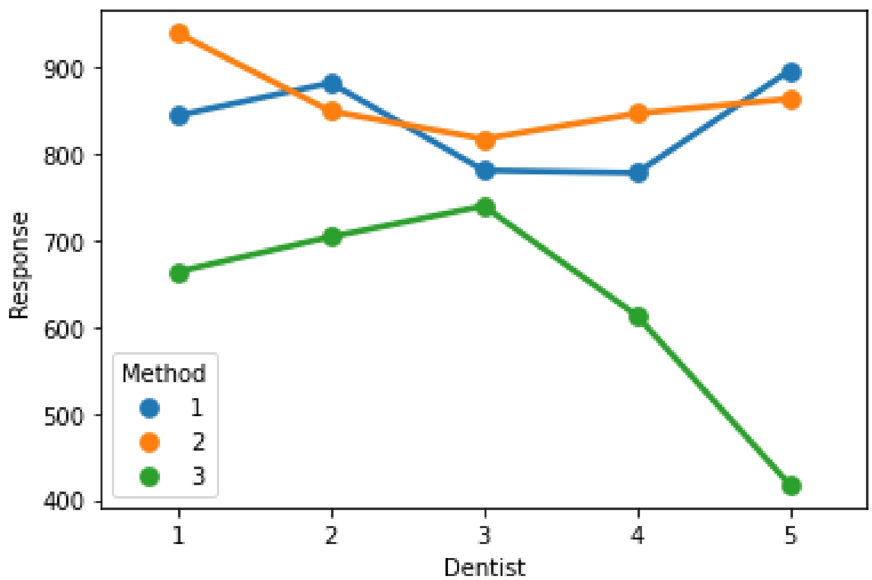

Fig. 15: Interaction between the factors 'Dentist' and 'Method' – Case 2.

We have statistical and visual evidence to conclude that there is a significant interaction effect between the two factors. In this case, we see that the mean hardness of all the dentists when using the method 3 is lesser, compared to other methods. The mean hardness appears to be the least for dentist 5, when using method 3, in both case 1 and case 2 (alloy 1 and alloy 2).

7.7 Now consider the effect of both factors, dentist, and method, separately on each alloy. What do you conclude? Is it possible to identify which dentists are different, which methods are different, and which interaction levels are different?

<u>Case 1</u>: Using alloy 1.

<u>Null hypothesis:</u>
H_{01}: The mean response(hardness) of the dental implant is same for all five dentists, while using alloy 1.
H_{02}: The mean response(hardness) of the dental implant is same for all three dentists, while using alloy 1.
H_{03}: There is no interaction effect between the factors 'Dentist' and 'Method' with respect to the mean response, while using alloy 1.

<u>Alternate hypothesis:</u>
H_{A1}: The mean response(hardness) of the dental implant is not same for all five dentists, while using alloy 1.
H_{A2}: The mean response(hardness) of the dental implant is not same for all three dentists, while using alloy 1.
H_{A3}: There is interaction effect between the factors 'Dentist' and 'Method' with respect to the mean response, while using alloy 1.

We have fixed the level of significance α = 0.05.
Now, we shall perform ANOVA test on the above hypotheses.

Table 15: Two-way ANOVA with interaction – Case1.

	sum_sq	df	F	PR(>F)
Intercept	1.915203e+06	1.0	280.027732	9.251850e-17
C(Dentist)	2.120173e+04	4.0	0.774993	5.501582e-01
C(Method)	1.149422e+04	2.0	0.840303	4.414857e-01
C(Dentist):C(Method)	1.859414e+05	8.0	3.398383	6.792747e-03
Residual	2.051800e+05	30.0	NaN	NaN

<u>Factor – 'Dentist'</u>: The p-value for this factor is **0.55015**, which is greater than α. Hence, we **fail to reject the null hypothesis, H_{01}.** We may say, this factor doesn't play an important role in determining the mean hardness of the dental implant.

<u>Factor – 'Method'</u>: The p-value for this factor is **0.44148**, which is greater than α. Hence, we **fail to reject the null hypothesis, H_{02}.** This factor doesn't play an important part in determining the hardness of the dental implant.

<u>Interaction effect</u>: The p-value is **0.006793**, which is less than α. Hence, we **reject the null hypothesis, H_{03}**. We conclude that there is a significant interaction between the above two factors.

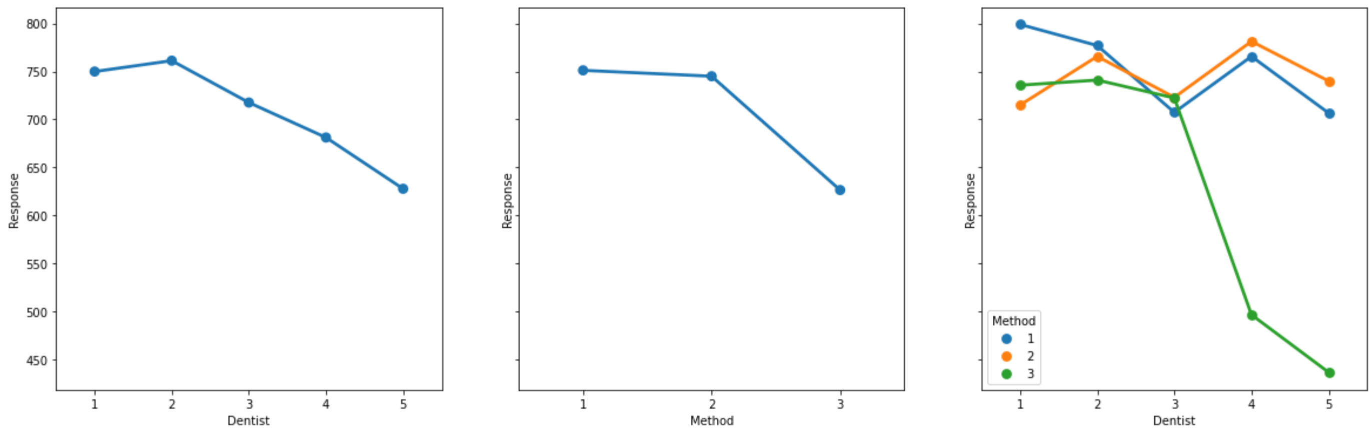

Fig. 16: Two-way ANOVA with interaction – Case 1.

The above graph shows us where the hardness of dental implant is falling drastically low. We may see that dentists 4 and 5 have lesser mean response compared to other three dentists. Also, we can see a drop in the mean response in method 3. When we see their interaction, we clearly see that the response is dropping in method 3, especially for dentists 4 and 5. The other three dentists seem to have maintained the mean response in the range of 700-800. We may also see that, dentists 4 and 5, maintaining higher mean response, while using method 1 and 2.

Case 2: Using alloy 2.

Null hypothesis:
H_{01}: The mean response(hardness) of the dental implant is same for all five dentists, while using alloy 2.
H_{02}: The mean response(hardness) of the dental implant is same for all three dentists, while using alloy 2.
H_{03}: There is no interaction effect between the factors 'Dentist' and 'Method' with respect to the mean response, while using alloy 2.

Alternate hypothesis:
H_{A1}: The mean response(hardness) of the dental implant is not same for all five dentists, while using alloy 2.
H_{A2}: The mean response(hardness) of the dental implant is not same for all three dentists, while using alloy 2.
H_{A3}: There is interaction effect between the factors 'Dentist' and 'Method' with respect to the mean response, while using alloy 2.

We have fixed the level of significance $\alpha = 0.05$.
Now, we shall perform ANOVA test on the above hypotheses.

Table 16: Two-way ANOVA with interaction – Case2

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	sum_sq	df	F	PR(>F)
Intercept	2.140385e+06	1.0	166.737943	8.745286e-14
C(Dentist)	3.664200e+04	4.0	0.713611	5.891730e-01
C(Method)	1.179049e+05	2.0	4.592448	1.819861e-02
C(Dentist):C(Method)	1.974598e+05	8.0	1.922787	9.323404e-02
Residual	3.851047e+05	30.0	NaN	NaN

<u>Factor – 'Dentist'</u>: The p-value for this factor is **0.58917**, which is greater than α. Hence, we **fail to reject the null hypothesis, H_{01}.** Here, we conclude that this factor is not significant in determining the mean hardness of the dental implant.

<u>Factor – 'Method'</u>: The p-value for this factor is **0.01819**, which is less than α. Hence, we have evidence to **reject the null hypothesis, H_{02}.** This factor plays an important part in determining the hardness of the dental implant.

<u>Interaction effect</u>: The p-value is **0.093234**, which is greater than α. Hence, we **fail to reject the null hypothesis, H_{03}**. We conclude that there is no significant interaction between the above two factors.

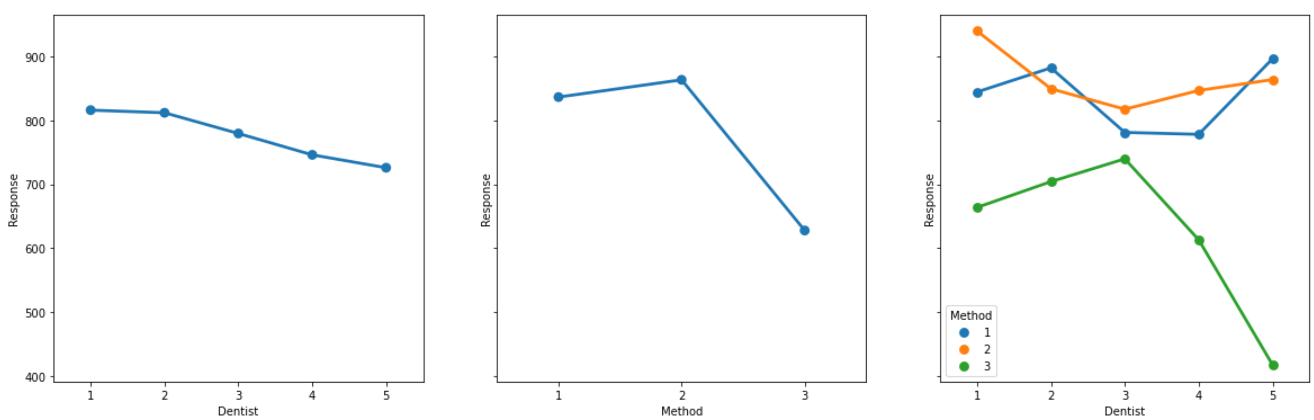

Fig. 17: Two-way ANOVA with interaction– Case 2.

In the first graph, we see almost a straight line, which tells us that the difference in the mean responses, among different levels of dentists, is negligible. In the second graph, we can see a drop in the mean response in method 3. Changing the method, will change the mean response. Hence, we can say that method is a significant factor in case 2(using alloy 2). Now, when we look at the third graph, we do see some interaction between the factors. But we are only 91% sure about this interaction affecting the response. Even though visually we see the interaction, statistically we do not have enough evidence to conclude the same.

ALTERNATE METHOD

Without passing 'typ' parameter.

<u>Case 1</u>: Using alloy 1.

<u>Null hypothesis:</u>
H_{01}: The mean response(hardness) of the dental implant is same for all five dentists, while using alloy 1.
H_{02}: The mean response(hardness) of the dental implant is same for all three dentists, while using alloy 1.
H_{03}: There is no interaction effect between the factors 'Dentist' and 'Method' with respect to the mean response, while using alloy 1.

<u>Alternate hypothesis:</u>
H_{A1}: The mean response(hardness) of the dental implant is not same for all five dentists, while using alloy 1.
H_{A2}: The mean response(hardness) of the dental implant is not same for all three dentists, while using alloy 1.
H_{A3}: There is interaction effect between the factors 'Dentist' and 'Method' with respect to the mean response, while using alloy 1.

We have fixed the level of significance $\alpha = 0.05$.
Now, we shall perform ANOVA test on the above hypotheses.

Table 17: Two-way ANOVA with interaction – Case1. (Alternate method)

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist):C(Method)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

<u>Factor – 'Dentist'</u>: The p-value for this factor is **0.011484**, which is lesser than α. Hence, we **reject the null hypothesis, H_{01}.** This factor plays an important role in determining the mean hardness of the dental implant.

<u>Factor – 'Method'</u>: The p-value for this factor is **0.000284**, which is lesser than α. Hence, we **reject the null hypothesis, H_{02}.** This factor does play an important part in determining the hardness of the dental implant.

<u>Interaction effect</u>: The p-value is **0.006793**, which is less than α. Hence, we **reject the null hypothesis, H_{03}**. We conclude that there is a significant interaction between the above two factors.

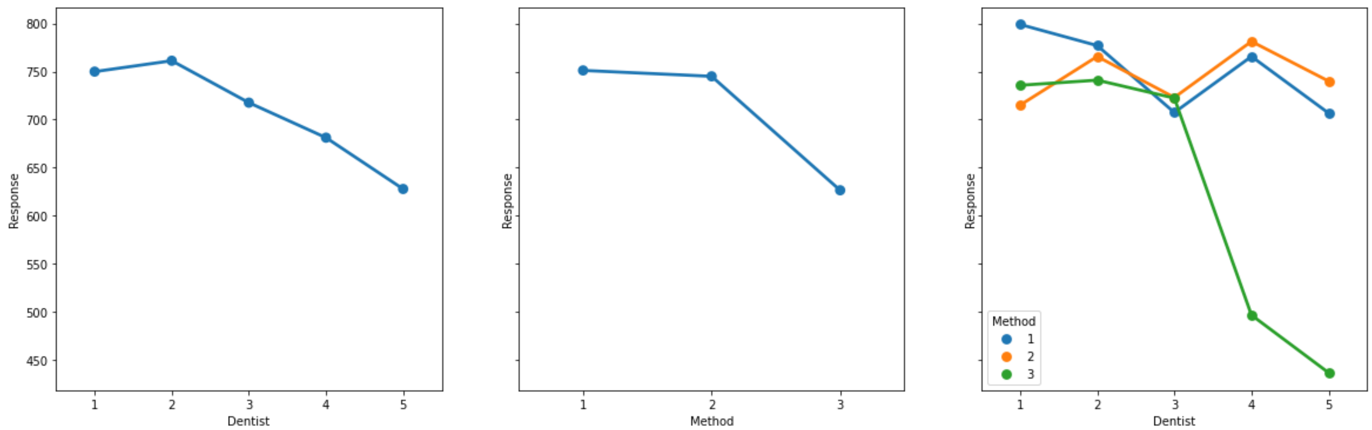

Fig. 18: Two-way ANOVA with interaction – Case 1. (Alternate method)

The above graph shows us where the hardness of dental implant is falling drastically low. We may see that dentists 4 and 5 have lesser mean response compared to other three dentists. Also, we can see a drop in the mean response in method 3. When we see their interaction, we clearly see that the response is dropping in method 3, especially for dentists 4 and 5. The other three dentists seem to have maintained the mean response in the range of 700-800. We may also see that, dentists 4 and 5, maintaining higher mean response, while using method 1 and 2.

<u>Case 2</u>: Using alloy 2.

<u>Null hypothesis:</u>
H_{01}: The mean response(hardness) of the dental implant is same for all five dentists, while using alloy 2.
H_{02}: The mean response(hardness) of the dental implant is same for all three dentists, while using alloy 2.
H_{03}: There is no interaction effect between the factors ‘Dentist’ and ‘Method’ with respect to the mean response, while using alloy 2.

<u>Alternate hypothesis:</u>
H_{A1}: The mean response(hardness) of the dental implant is not same for all five dentists, while using alloy 2.
H_{A2}: The mean response(hardness) of the dental implant is not same for all three dentists, while using alloy 2.
H_{A3}: There is interaction effect between the factors ‘Dentist’ and ‘Method’ with respect to the mean response, while using alloy 2.

We have fixed the level of significance α = 0.05.
Now, we shall perform ANOVA test on the above hypotheses.

Table 18: Two-way ANOVA with interaction – Case2 (Alternate method)

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist):C(Method)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

<u>Factor – 'Dentist'</u>: The p-value for this factor is **0.371833**, which is greater than α. Hence, we **fail to reject the null hypothesis, H_{01}.** Here, we conclude that this factor is not significant in determining the mean hardness of the dental implant.

<u>Factor – 'Method'</u>: The p-value for this factor is **0.000004**, which is less than α. Hence, we have evidence to **reject the null hypothesis, H_{02}.** This factor plays an important part in determining the hardness of the dental implant.

<u>Interaction effect</u>: The p-value is **0.093234**, which is greater than α. Hence, we **fail to reject the null hypothesis, H_{03}**. We conclude that there is no significant interaction between the above two factors.

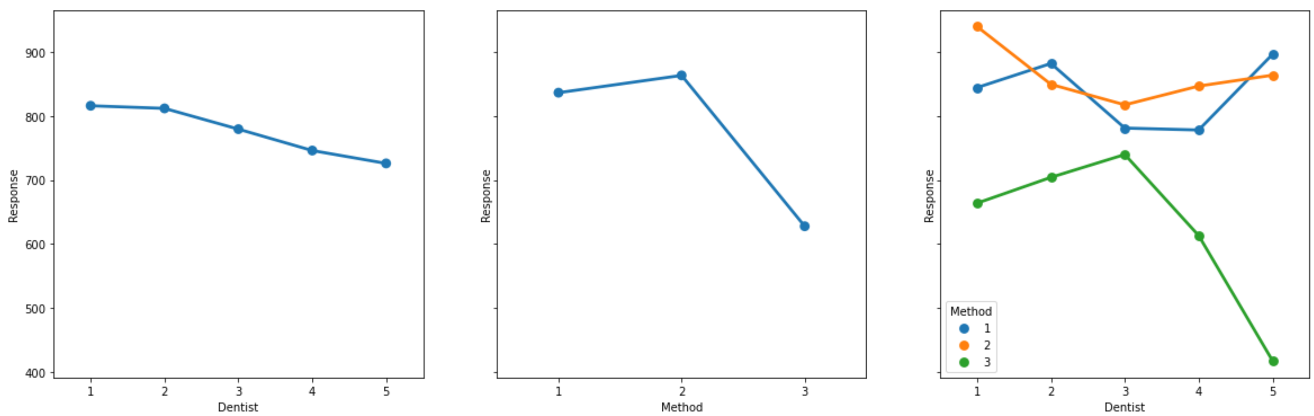

Fig. 19: Two-way ANOVA with interaction – Case 2. (Alternate method)

In the first graph, we see almost a straight line, which tells us that the difference in the mean responses, among different levels of dentists, is negligible. In the second graph, we can see a drop in the mean response in method 3. Changing the method, will change the mean response. Hence, we can say that method is a significant factor in case 2(using alloy 2). Now, when we look at the third graph, we do see some interaction between the factors. But we are only 91% sure about this interaction affecting the response. Even though visually we see the interaction, statistically we do not have enough evidence to conclude the same.