Homework #1
Introduction to Algorithms/Algorithms 1
600.363/463
Spring 2016

Due on: Thursday, February 4rd, 5pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On blackboard, under student assessment.
Otherwise, please bring your solutions to the lecture.

January 26, 2016

1 Problem 1 (25 points)

1.1 (15 points)

For each statement below explain if it is true or false and prove your answer. Be as precise as you can. The base of $\log$ is 2 unless stated otherwise.

1. $\frac{n}{\log n} = \Theta(n)$
2. $2^n = o(3^n)$
3. $2^n = \Theta(\left(\frac{3}{2}\right)^n)$
4. $2^n = O(e^{n+\log n})$
5. $\log\log n = O(\log(\frac{\sqrt{n}}{3\log n}) + \log(n - \log n))$
6. Let f, g, h be positive functions. Then $h(n)/(f(n)+g(n)) = O(h(n)/(f(n)g(n)))$
7. Let f, g be positive functions. Then $|f(n) - g(n)| = \Theta(\min(f(n), g(n)))$
8. Let f, g be positive functions. Then $f(n) + g(n) = \Theta(\max(f(n), g(n)))$
9. $n^{n\log n} = \Omega(e^{n^2 - n\log n})$

10. Let f, g, h be a positive functions. If $g(n) = o(f(n))$ and $f(n) = O(h(n))$, then $g(n) = o(h(n))$.

1.2 (10 points)

1. Prove that
$$\sum_{i=1}^{n} i^2 \ln\left(e + \frac{1}{i}\right) = \Theta(n^3).$$

2 Problem 2(25 Points)

2.1 (10 points)

Prove by induction that $\binom{2n}{n} > \frac{2^n}{\sqrt{\pi n}}$ for all positive n.

2.2 (15 points)

1. Let A, B, C be sets. Prove that
   (a) $A \setminus (A \setminus B) = A \cap B$
   (b) $A \cup B = (A \setminus B) \cup (A \cap B) \cup (B \setminus A)$
   (c) $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$
2. There are n identical apples in the basket. What is the number of different ways to divide the apples between Alice, Bob and John? What is the answer if we request that Alice and John get at most $k < n/2$ apples each?
3. Suppose that a random machine outputs each number from 1 to $x - 1$ with equal probability. What is probability that the output is coprime with x, where $x = 3^n 5^m 7^k$ and $n, m, k \in \mathcal{Z}$.
4. Assume there are n students in a class. How many different ways are there to form n/k teams with k students in each?