Network Analysis: Advisory, Friendship, and Work Relationships in a Law Firm

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Introduction

Network structures can be distinguished all around us, in nature, culture, economies, and societies - as a set of actors and the relations defined by them. Actors in a network can represent a single person, product, organisation, or anything else. For example, in social environments, these relations between entities could represent individuals and any form of existing relationship between them, whether friendly or professional, positive or negative, and others (*Great Learning, 2023*). Network analysis is a powerful tool to dive deeper and provide insights into these relationships, structures, and behaviours that networks represent (*Great Learning, 2023*). It considers a wide range of topics, such as understanding how and why a network is formed; the structure of the network; the characteristics of its members and how they influence the creation of links; and how these network connections in turn influence the behaviour of the entities in a network (*Diani, 1995*). In this report, we will focus specifically on social network analysis (SNA), in which we will try to analyse and understand all of the above in relation to a social professional environment.

This report will focus specifically on the interactions within a law firm, utilising network analysis to unravel the complex network among attorneys of a corporate law firm in the Northeastern US, referred to as *SG&R*. The networks that will be used are part of a dataset called Lazega. To delve into the firm's structure, and dynamics, a variety of approaches and methodologies will be used in R Studio. Visualisations to understand the overall structure of our networks will be conducted, followed by a local-level analysis to understand the properties of an attorney due to their position in the network and relationships. A global-level analysis will then allow us to understand the interactions inside the law firm and their directionality - such as whether they are one-sided or reciprocal, and what that entails about the cohesiveness of our group. At this level, we will also aim to understand the network's hierarchical structure. Further, at dyadic-level analysis, we will be looking at the pairwise relationships within the networks, assessing any similarities, modularity, and assortativity - all of which will be explained further in the 'Analysis section' of our report.

Dataset & Approaches

The dataset that will be analysed is Lazega's dataset and it has recorded the network among 71 attorneys (partners and associates) of *SG&R*. The dataset consists of three different networks:

1. Strong coworkers network: focusing on professional work shared between members of the law firm, such as having shared at least one case, having been assigned to one case, or reading/using each other's work product.

2. Basic advice network: consists of lawyers who have asked for professional advice from another coworker, in regard to a decision or handling a case.

3. Friendship network: consists of attorneys who have socialised outside of work functions.

All of the three networks contain the following attributes: status, gender, office, years with the firm, age, practice, and law school, which will help understand how specific attributes of employees affect the creation of links. Another dataset of the Advice-learning network of the employees will be considered, which uses the links of the advice network as base, and has generated a set of attributes to

those links called *weight*, a numerical factor (0-1) based on the tenure and status of the lawyers. This will determine the importance that the actor gives to the opinions of another actor, so lawyer.

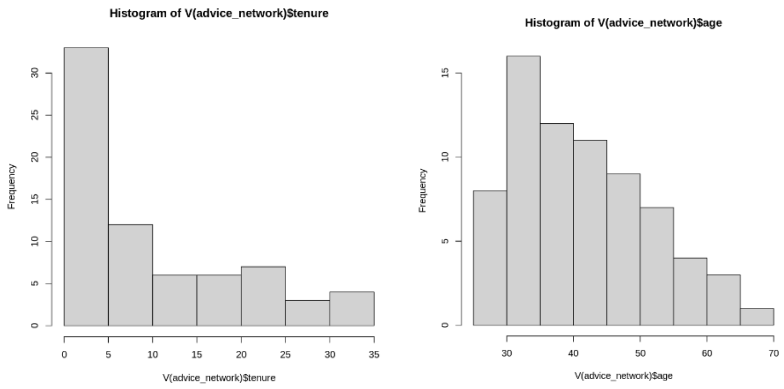
Ultimately, all of the analyses will aim to provide an understanding of the social and professional interactions between the attorneys at *SG&R*. By examining the structure and dynamics of the networks of Lazega, which are: '***ADVICE***', '***FRIENDSHIP***', and '***WORK***', we expect to identify the key players in the firm, uncovering their roles and influence within the networks. We aim to look into the cooperation among colleagues and the more dominant types of relationships. This will also help highlight any existent patterns of knowledge sharing, power balance, and the impact of social structures on professional interactions. Another aspect we will concentrate on is understanding how individual employees' attributes such as gender or seniority influence the formation of links. In general, our analyses will revolve around getting a better understanding of the firm's internal dynamics and how the relationships shape and are shaped by the firm's culture and policies.

Analysis

Basic Network Characteristics

In each of the networks we can find 71 nodes representing the firm's lawyers and the existing links between each other. All networks apart from '***WORK'*** are directed, since they contain the descriptions of directed actions like advice or friendship, which as proven here does not have to be mutual. Work is the only mutual relation, ie. "if I worked with you, you worked with me". A fully connected company would exist with 4,970 links (directed case). When it comes to the number of links in each of the corresponding networks we can see the biggest number of them in the ***'ADVICE'*** network with the number of 892 links, after that, we have the ***'WORK'*** network which consists of only bi-directional links, making the graph defacto undirected, it has 756 links (378 - undirected). After that, we have the network with the least links which is the ***'FRIEND'*** graph with L = 575. This implies that advice is a much more frequent relation between lawyers inside the company than friendship or even working together. What is interesting working together is less common than giving advice inside the company structures. For all graphs without ***'LEARN ADVICE'*** the average path length is around 2, which means that everyone is reachable more or less within 2 steps or one intermediary.

Graphs 1 & 2 - Distribution of actors attributes.



The first histogram shows skewed-right distribution, that is, more individuals for shorter tenure and decreases as the tenure increases. The highest frequency in the first bin (0-5 years) would mean that most of the individuals have the duration of tenure of fewer than 5 years. The second histogram (age) presents a frequency distribution of a variable called V(advice_network)$age. This

histogram draws approximately normal distribution with small right skew. The most frequent age range seems to be from 40 to 45.

Within these networks, the status attribute is categorised into two groups which are "1" and "2". In this case, the distribution is consistent because 36 individuals have status "1" whereas 35 have status "2". This communicates that most of them are balanced to display an equitable representation of these statuses within the network. In terms of sex representation, categorised as "1" and "2", there's a noticeable disparity. Whereby 53 individuals representative of sex "1" and 18 sex "2", it clearly portrays that the network, and by extension, the firm, is majorly composed of individuals corresponding to sex "1", which denotes a higher men to women ratio. With the clearly divided categories of 1, 2 and 3 of the attribute office, there's clearly a preference. Office 1 compiles the majority with 48 people, followed by office 2 with 19, and the least populated is office 3 which has just 4 individuals. From this pattern, it can be drawn that office "1" is a preferred option by the largest percentage of the group. By observing the distribution of practices labelled "1" and "2", we can see that practice "1" and "2" attracted forty-one individuals, with additional thirty in practice "1". This shows a fairly even spread across these practices, with a slight edge to practise "1" in terms of participation. Finally, for the attribute named "school" divided into categories "1", "2" and "3", we can observe a relatively even repartition between all schools, mostly "2" and "3" with 28 people each compared to school "1" that is with lesser representation of 15. This distribution suggests a more balanced affiliation with schools "2" and "3" among the members of our network.

From the analysis of the attributes, it can be inferred that the age of the examined networks is relatively young and predominantly within a span of up to 10 years. The vast majority are in their 30s with only a couple below this age. Male dominates the gender distribution with 36 partners and 35 associates noted. The group is highly focused geographically in Boston and has a major orientation towards litigation. The corporation also features a strong presence which accounts for its larger numbers comparatively. Regarding the educational backgrounds, it presented 15 alumni who had this distinction from Harvard, 28 from Yale, and another 28 from various other eminent institutions. Such demographic and professional profiles paint the profile of a predominantly young, dynamic, male group, with strong academic credentials, and significant focus on legal practice in the Boston area.

Reciprocity in the *'**WORK**'* graph is equal to 1 since it is undirected. From other reciprocity metrics we can infer that around 61% of all links are bi-directional in the *'**FRIEND**'* network, so after all friendship, even though vastly not always, is rather a mutual relationship. That cannot be said for the *'**ADVICE**'* network, since only 39% of links were reciprocal. We can see advice is very rarely given both ways, it rather spreads along the company's hierarchy, with likely more experienced members more frequently giving advice.

In terms of the degree distribution all of the networks follow a mostly normal distribution, though they all seem to be slightly right skewed and exhibit high kurtosis.

Graph 3 - Degree Distribution in Each of the Given Networks.

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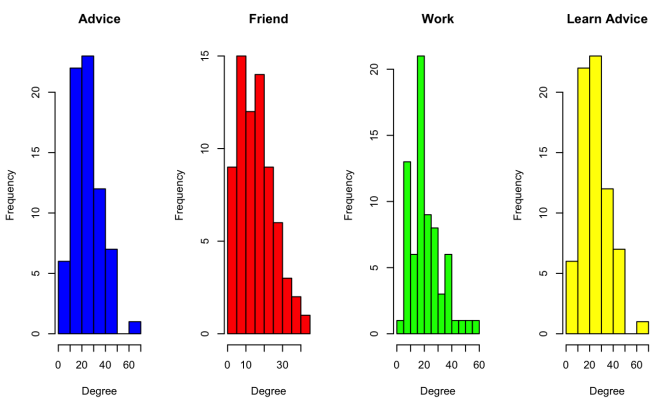


Cohesion of Coworkers

A global level of analysis will help us evaluate the cohesion of coworkers in the law firm. The global metrics *average degree*, *local clustering coefficient* (LCC), *global clustering coefficient* (GCC), and *density* show the number of connections per attorney, the likelihood of attorneys being connected, clustering over the entire network, and the proportion of possible relationships in the network, respectively. The higher the values of these metrics, the more interconnected the attorneys are, and therefore more cohesive. On the other hand, a low value of average path length (APL) entails that members of a team are closely connected to one another. *Reciprocity* measures the likelihood of mutual connections. All of the metrics can be seen in the output below:

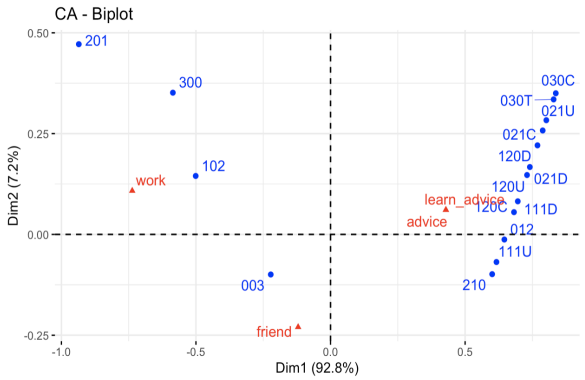
Table 1 - Global Metrics.

	Av. Degree	APL	LCC	GCC	Density	Reciprocity
Advice	25.12676	2.2432669	0.5217772	0.4787826	0.1794769	0.3923767
Friend	16.19718	2.5051258	0.5050982	0.4486229	0.1156942	0.6121739
Work	21.29577	2.1035197	0.3907728	0.3069738	0.1521127	1.0000000
Learn_advice	25.12676	0.1516169	0.5217772	0.4787826	0.1794769	0.3923767

We can see in *Table 1* that '***ADVICE***' and '***LEARN ADVICE***' appear to be the most cohesive networks, with the highest results in average degree, LCC, GCC, and the lowest values in APL, in comparison to the other networks. The metrics of both advice networks were equally ~25.13 in average degree, LCC of 0.52, GCC of 0.48, and density of 0.18. They only differed in APL, in which '***LEARN ADVICE***' performed significantly better with a result of 0.15 in comparison to 2.24 for '***ADVICE***'. Overall, both networks show high cohesion that indicates a stronger interconnectedness and more frequent interactions in the context of asking and receiving advice from fellow attorneys. This entails that the law firm seems to foster an environment where sharing professional advice and learning from one another is highly encouraged.

'***WORK***' has the lowest values in average degree, LCC, and GCC (all of them being 21.29, 0.39, and 0.31 respectively) suggesting that while attorneys are collaborative in advice-giving, actual work is individualised or divided among smaller groups of attorneys. '***FRIEND***' has the lowest average degree (16.19) and density (0.12), suggesting that the average number of friendship connections per attorney is low, in addition to the proportion of possible friendships. However, the network showed the highest reciprocity out of all with a value of 0.61 (almost double the result of our advice networks), which shows that mutual social connections within the law firm are valued, though less common per attorney, pointing toward a professional but socially cohesive environment.

Graph 4 - Triad Censuses.

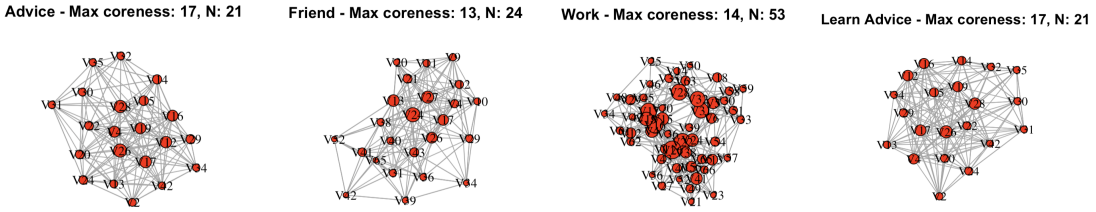


We can see from *Graph 4* that the '***ADVICE***' and '***LEARN ADVICE***' networks are closely associated with triads like 012 and 111D - showing a tendency for directional relationships and not as many closed triadic relationships. The '***WORK***' network is proximate to 300, complete closed triads) in the biplot - indicating a higher level of cohesion in some parts of the network. The '***FRIEND***' network's position near the 003 indicates a lack of triadic closure and low overall cohesion.

The largest cliques were then found to further examine the cohesion of our four networks. In regard to cohesion, a network with larger cliques indicates that many members are closely connected, therefore, a higher level of overall cohesion. The largest clique of each network was 10 for '***ADVICE***' and '***LEARN ADVICE***', 9 for '***FRIEND***', and 6 for '***WORK***', suggesting that the first two have the strongest cohesiveness out of the four networks, and '***FRIEND***' and '***WORK***' are the least cohesive in terms of tight-knit groups.

Coreness analysis allowed us to further analyse the connectivity of our network. The coreness of our networks can be seen below in *graphs 5-6* including max coreness, which represents the highest level of connectedness in the network (a higher value indicating a highly interconnected subgroup), and number of nodes (N) (a higher number suggesting a larger network). '***ADVICE***' and '***LEARN ADVICE***' have the highest max corenesses of 17, indicating each node is connected to at least 17 other nodes.

Graph 5-8 - Coreness of Advice, Friend, Work, and Learn Advice.



The networks with the highest max coreness of 17 are '***ADVICE***' and '***LEARN ADVICE***', indicating that each node is connected to at least 17 other nodes. '***FRIEND***' and '***WORK*** ' have the least cohesive cores among the four, of the values 13 and 14, respectively. Though '***WORK***' has the least cohesive core, it has the largest number of nodes (N: 53), which suggests that while the network is large, its cohesiveness is more well-distributed and not concentrated in groups or clusters.

Taking both the largest cliques and the coreness into consideration, the networks '***ADVICE***' and '***LEARN ADVICE***' appear to be the most cohesive due to their largest cliques and highest coreness, which indicates strong interconnectedness among the networks. The '***FRIEND***' network is slightly less cohesive, and '***WORK***' is the least cohesive potentially due to its larger size which can lead to an overall lower connectedness among the network. This information supports our previous assumptions that the most cohesive networks seem to be in the context of attorneys seeking/giving advice, while the least cohesive is in regard to working together given the individualistic nature of attorney work.

Overall, combining these insights, it appears that the law firm has a strong culture of collaboration, mentorship, and mutual friendship. The attorneys seem to frequently seek and exchange professional advice from one another, though the work remains more individualistic or focused on small groups given its relatively lower cohesion. This would make sense given that the attorneys mainly work with clients instead of group projects with one another. Friendship within the firm seems to be valued and mutual given its high performance in reciprocity, though there exist significantly fewer friendly relationships in comparison to those of advice sharing (as seen in our average degree results). This is also necessary in order to maintain professionalism in the workplace, especially in a law firm, where social mutual connections are important but boundaries need to be kept.

Most Common Relationship

Looking at the graph size of our networks, we have found that the '***ADVICE***' network is made of 892 links, in comparison to the '***WORK***' network with 756 links and the '***FRIEND***' network with 575 links. These values indicate that the most common relationship is that of *advice* and of *work*, which entails a company culture in which attorneys prioritise professional relationships over personal ones. Advice-sharing is also more common than working together, which we also discovered from our Global Analysis Metrics (specifically the LLC and GCC), which showed that attorneys are more collaborative in terms of advice and learning, while actual work is more individualised or divided among a smaller group of individuals. Overall, we can conclude that the firm *SG&R* has a more formal or traditional work culture, in which individual/smaller group work is prioritised (as we would assume in attorney work), an advice-seeking and learning culture is fostered, and socialisation outside of work is not as common.

Network Centralities and Most Important Actors

In our study we have conducted an extensive search into the most important nodes in each of the given networks. The first focus was on the degree, signifying the reach to others a node has inside the company's structure. In the *'**ADVICE**'* network most significant actors were node V26, followed by V13 and V28. These are the lawyers giving out the most advice, but also receiving the most advice, this node gave more advice than it received, with its in-degree being higher than the out-degree. Opposite rule applies to V13, with V28 being more of the advice giver than the taker, but not by much more. When it comes to who gave out the most advice those were nodes V19, V16, and again V28. The most notable advice takers are nodes V26, V13, and V17. Here we can say that the most active actor in the advice network was in fact node V28, but mostly in receiving advice. Node V13 is in a similar position being in second place in overall degree and in-degree. Node V26 is also an important actor in this network, but he mostly gives advice and takes a good portion of it as well. It is a generally a seeable trend that those who receive advice, also like to give it out. As for the position of those nodes in the hierarchy they are all partners, which means they are higher in it. With over 10 years of tenure, the ones mostly receiving advice are based in Boston and work on litigation, while the one giving advice, works in Hartford and works on corporate law.

In the friendship network nodes V17, V24, and V31 are king, with the highest number of in and outgoing connections. What is interesting when it comes to the number of incoming edges, different nodes V26 and V13 show up again. This fact might be signalling their importance in the whole structure of the firm, as they control both advice and receive most friendship annotations. V17 is also proven to receive friendship. As for outgoing connections, nodes V31, V17, and V24 have the highest coefficient, signalling they designated the most actors as their friends. As for the lawyers with the most degree centralities, they are all partners, with over 10 years of tenure, males, above 40. Here we have another piece of evidence that the more senior partners were able to establish more friendships over time.

As for the *'**WORK**'* undirected network, the most relevant nodes by degree were V22, V26, and V16. These are the most important actors with the most working experience in the firm. As for their characteristics, they are partners aged above 40, high tenure, apart from one actor who had 8

years tenure, but he was over 40, so he might have had prior experience in the industry. They were all from Boston.

The most important actors in those networks finished either Yale, Harvard or UCONN, their practice was mixed between either corporate law or litigation. All of them were males. This study reflects the more experience and the higher the position inside the firm the more connections an actor has to the rest of the actors inside the network.

It is valuable to note that the betweenness centrality is low for all of the networks, which reflects the short average path length and overall short way of information flow inside the firm. As for the advisor network nodes V16, V17, and V30 were most important in the flow of advice. Again, all of them partners with experience inside the company. As for the flow of information inside the *'**FRIEND**'* graph, nodes V64, V31, and V17, were the most important. What is interesting again is all of them are partners apart from V64 who is an associate, a woman, with a low tenure, but a high age, who also finished Yale. She might be an important link for the flow of information along the friendship paths for some parts of the firm more weakly connected to others. For the '***WORK***' network, V22, V16, and V26 are the ones controlling the road of information spread. Here again, they are partners with higher tenure and more experience inside the firm.

When looking at the closeness centrality for the *'**ADVICE**'* graph, new nodes show up - V66, V65, and V42. They are all associates and they seem to be the closest to all other nodes in the network, they have a good well-connected position. The nodes closest to all other nodes by means of friendship are V65, V31, and V24. Only V65 is an associate, they seem to have good links to other actors. In the *'**WORK**'* network nodes V22, V26, and V16 are the ones closest to all the other lawyers in the firm, connected through the means of prior work experience. All are partners, with high tenure. Since, in case of working and friendship graphs not a single component, the closeness centralities are only computed on the single largest component, to illustrate the positions of actors over there.

The most important nodes when comparing the highest eigenvalue centrality coefficients for the *'**ADVICE**'* network, were: V26, V24, and V17. They are the nodes with the most connections to nodes with high degrees, making them truly interconnected to the whole company's structure. They have the biggest number of advice links, in and outgoing to other lawyers having a vast advisor experience as well. As for the friendship network, V24, V17, and V13 have most friends who have a lot of friends inside the firm's structure. Here again, they are all partners with a lot of experience inside the company. As for the last network *'**WORK**'*, nodes V26, V22, and V16, who are partners. They have the biggest number of work experience to people who have also a big number of work experience with others in the firm.

As to the point, the actor who seems to be the most important in the whole company, showing along all metrics and graphs is node V26, a partner with a 15-year tenure in the company. Diagnosing from the centrality metrics he has a very prestigious and central position in the whole company in all advice, friendship, and work experience. Other notable actors in those networks are V24, V17, and V13, all holding prestigious positions inside the firm. Some nodes have been found similar to the most popular nodes, with the use of Katz Similarity tests. For example, V66 is similar to node V26 in the advice network, in the friendship network V24 has been found similar to V26, and in the work network nodes V16 and V17 have been found similar.

Centralizations

Centralizations will show the distribution of the centralities of our four networks, which we saw above. The output on R can be seen below:

Table 2 - Centralizations.

	Degree	Closeness	Betweenness	Eigenvector
Advice	0.2636646	0.3617093	0.06433246	0.6261727
Friend	0.1822981	NaN	0.11789350	0.7069196
Work	0.2550725	NaN	0.10595574	0.6097141
Learn_advice	0.2636646	0.3617093	0.06433246	0.6261727

As can be seen above, all degree centralizations are relatively low, suggesting that in all four networks, generally, the majority of lawyers have the same number of links (in work, friendship, and advice-giving). In networks '***ADVICE***' and '***LEARN ADVICE***' the degree centralizations are equal to ~0.26 which is relatively low, suggesting fewer attorneys may be more connected than others. '***FRIEND***' has the lowest degree of centralization which shows that friendships are more evenly distributed. Closeness centralization measures the extent to which some nodes are closer to all others in the network. The values of the networks '***ADVICE***' and '***LEARN ADVICE***' are both ~0.36, a moderate level of centralization, which indicates that there exist some lawyers who are more centrally located and can reach others quickly. Our two other networks show NaN values of closeness centralizations because of detached components, therefore the algorithm was unable to measure the closeness centrality.

The betweenness centralizations will show us the extent to which nodes/attorneys can control information flow in each network. All four networks show very low values of betweenness centralization, indicating an even distribution of control over the information flow in each of them. The network '***FRIEND***' shows the highest value of ~0.11, suggesting that very few attorneys have control over the information flow (though the value is still extremely low). Eigenvector centralizations will measure how much influence within the network is concentrated in a few nodes. These values have resulted higher than in the others, indicating that the influence is concentrated on fewer individuals in the law firm, with the '***FRIEND***' network having the highest value of ~0.71. Our other networks resulted in ~0.63 for '***ADVICE***' and '***LEARN ADVICE***' and ~0.61 for '***WORK***'. All networks show values high enough to assume that influence is concentrated in a small number of individuals in the law firm.

In summary, the results of '***ADVICE***' and '***LEARN ADVICE***' show that a few key individuals or teams might be at the centre of interactions in these networks. Friendships, as seen in the '***FRIEND***' network, show a low degree of centralization and a highly concentrated influence on a few number of attorneys. Overall, all four relationships are relatively evenly distributed in regard to information flow, connections, and accessibility, though there is a high concentration of influence on particular attorneys. This could suggest that most attorneys have the same access to information and socialisation, but only a few individuals are pivotal in the internal dynamics of the firm.

In delving into the social tapestry of the Lazega Law Firm's network, a nuanced portrait of connectivity emerges, intricately woven by the threads of locale, legal specialty, and educational lineage. The vibrancy of connections within the Boston hub outshines its counterparts, suggesting a dynamic where proximity catalyses a rich exchange of counsel and collaboration. Within the legal battlegrounds, litigators seem to forge a denser web of alliances compared to their corporate law counterparts, an echo of the hands-on, collaborative spirit that litigation demands.

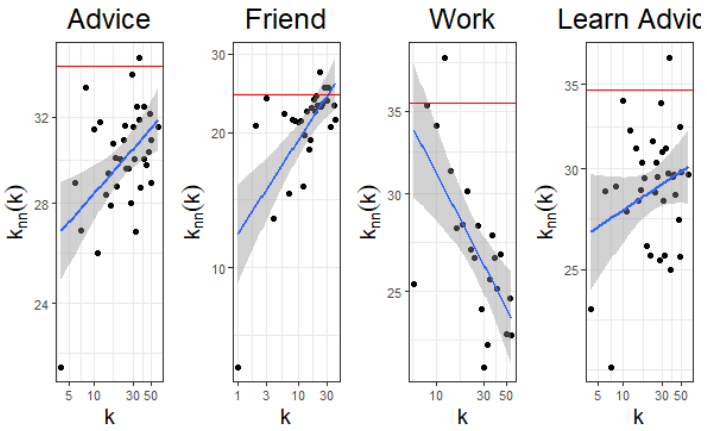
Yet, perhaps most compelling is the revelation of alma mater influence, where the bonds formed in the hallowed halls of prestigious institutions translate into a robust network within the

firm's ecosystem. This pattern is emblematic of the subtle yet potent role that shared educational experiences play in shaping professional landscapes. Such insights offer a lens through which the firm can view its internal network not just as a static entity, but as a living organism that both influences and is influenced by the multifaceted identities of its members. These observations serve not merely as a map of existing connections but as a beacon, guiding strategies to cultivate a culture of inclusivity and collective growth. By acknowledging the undercurrents that guide the formation of professional relationships, the firm can navigate towards fostering a milieu where collaboration thrives across all levels, practices, and backgrounds.

Assortativity Forces

Research for assortativity forced in any of the 4 given graphs was plotted by calculating the Degree Correlation Function for estimation of the Correlation Exponent using the means of a linear model. Assortativity by degree was negative for the advisory and working networks, with friendship being slightly positive. That would mean that there is a tendency to create more links between actors working or advising each other, within the same group. For friendship, the actors were just slightly more likely to create bonds with lawyers outside of their work circle. All four graphs show a strong correlation tendency, positive for the *'**ADVICE**'* and *'**FRIENDSHIP**'* network and negative for the *'**WORK**'* network.

Graph 9 - DCF - Correlation Exponent Estimation.



By this estimation we can state that advice is given most likely outside the most usual working circle, advice tends to be told more to lawyers with whom nodes do not show up as much with. The line is positive for the friendship network and negative for the co-working network as predicted by the previous statistic. At last we calculated confidence intervals for all of the networks. First two graphs (*'**ADVICE**'* and *'**FRIENDSHIP**'*) have above 0 confidence intervals, but very close to 0 signifying a weak sign of assortativity in the direction explained by the previous linear model. The trend of negative ties was kept for the *'**WORK**'* network, *'**LEARN ADVICE**'* network contained a 0 in the interval therefore we can conclude there were no significant patterns making the nodes more likely to connect.

Network Randomness and Structures

A full analysis has been led to explore the structure and dynamic of the four networks available within the law firm. The objective of this part of the analysis was to get a clear comprehension of the complexity and structure of these networks, especially the community structures, centrality measures, small-world properties, and identification of key nodes playing crucial roles within the internal networking of the firm.

To understand network structures and also to identify both non-random patterns as well as trends, a corresponding random network has been generated for each of the four empirical networks. These were created using the Erdős-Rényi (ER) model for random networks, with N nodes in each empirical model retained. The probability of the existence of an edge p for each random network has been determined using the formula:

$$p = 2E/n(n-1)$$

where E denotes the number of edges in the empirical network. In this case, the expected number of edges in random networks of the same degree distribution will be equal to that of the empirical networks and therefore, meaningful comparison of their structures can be performed.

The nodes in the ***'ADVICE'*** network have a very similar average degree of 25.13 as compared to 25.46 in the random network. These values imply a dense network where one individual is likely to seek advice from many colleagues, implying a high interconnection and potential sharing of information within the firm. In the friend network, the average degree was 16.20 while the random equivalent had a slightly lower average of 14.96. This suggests a cohesive social environment yet with less cohesion than the advice network which could be explained by the fact that in this network, the relationships are more selective and possibly based on trust and common interests. The degree of the '***WORK'*** network is comparatively low at 21.30 in average while that of a random network is relatively higher at 22.42. The closeness in these values points to a robust workflow network where the distribution of collaborative work tasks is extensive but not as indiscriminate as in a random network.

The clustering coefficient for the ***'ADVICE'*** network being 0.478 is significantly more than that of the random network which was 0.367 indicating advice seeking behaviour within this group is not random but is focused or targeted towards particular clusters. We can therefore assume that advice could be seeked by individuals in the company to be functional or hierarchical to specific groups of employees. The '***FRIEND'*** network has a clustering coefficient of 0.449 compared to its much higher random equivalent at 0.215. The difference between the measures reflects there being significantly many tight-knit social circles in the firm as friendships most probably since their creation depends on the background, interests or common experience area of each individual. For the ***'WORK'*** network, the clustering coefficient is 0.307 compared to the random network's 0.367. The unexpected finding where the random network comes out with higher clustering suggests that the ***'WORK'*** network may have specific, well defined workflows that do not necessarily put it within a tightly-clustered pattern but could be dictated by either project requirements or departmental boundaries.

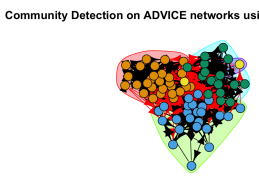
The average path length of the ***'FRIEND'*** network is 2.51 showing a higher separation compared to that of the random network that stands at 1.81. This distance can exist between any two people in the network since it might be because of the size of the firm or even existence of groups where there may not necessarily be direct linkage. The average path length in the `***WORK'*** network is 2.10, less than the `***FRIEND'*** network but still higher than the random network 1.68. This would imply that even if the firm has efficient workflow, it is not so freely interlinked as would be dictated by a random network, perhaps on account of the firm being hierarchically or departmentally organised. The ***'ADVICE'*** network had an average path length of 2.24, showing that advice channels are relatively direct yet not as much as those in the random network holding a value of 1.64. The little difference suggests that formal or informal procedures exist for seeking the advice within the firm.

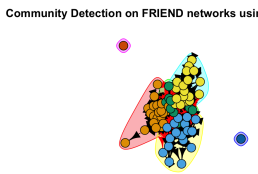
From the analysis of these metrics, it can be concluded that all the three networks are not likely to be random networks since all the three depict characteristics of small-world networks, high clustering coefficients combined with short average path lengths.
In other words, this is such a structure that suggests people are highly interrelated within particular groups or clusters but at the same time maintain relatively short paths across the overall network. These networks represent social (***'FRIEND'***), advisory (***'ADVICE'***) and professional (***'WORK'***) relationships within the form with their structure and dynamics. The small-world character implies efficient communication paths throughout the form. Information can quickly travel due to the short path lengths, aiding in effective collaboration and decision-making. On the influence of social structures: The different metrics of the ***'FRIEND'*** social network highlight that social relationships may be one of the significant factors of the firm's functioning, and probably results in a socialising impact on work related decisions and collaborations too.

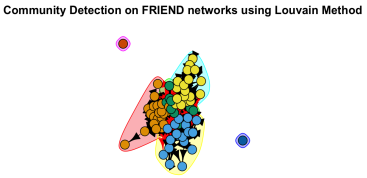
Community Detection

To further analyse whether the networks are random or not, community detection was performed on each of the networks. Community detection allows us to further uncover the structure of the network and to identify what type of relationships could be formed. The community detection algorithms explored were Edge Betweenness, Greedy Modularity, InfoMap and Louvain. After testing, and due to the fact that each network exhibits small-world characteristics and has high clustering coefficients, it was determined that the best community detection algorithm to use was the Louvain Modularity Algorithm because of its multi-step approach to optimising the modularity metric, which is optimal for these models that are small-worlds with high clustering coefficients. While Louvain has been recognised as the best algorithm to use, it must be noted that it only works with undirected networks, which meant there was a loss of information, as each of the networks had to be changed to undirected graphs. Below you can see that the visual representations of the communities within the '***ADVICE***' (graph 1), '***FRIEND***' (graph 2), and '***WORK***'(graph 3) networks reveal distinct clusters.

Graphs. 10-12 - Community Detection: Advice Network, Friendship Network (Louvain Algorithm), Work Network (Louvain Algorithm).







The '***ADVICE***' network has a modularity score of 0.286, and displays four main communities. Community 1, which is a prominent cluster potentially represents senior attorneys or specialists whose expertise is widely recognized. The presence of major advising figures is highlighted by the indicated hub nodes, which range from 8 to 15 within these communities. Smaller, more isolated clusters may represent niche areas of practice or junior attorneys who are less central in the advice-seeking network. The observed clusters in the undirected version of the advice network might, however, not accurately reflect the directional nature of advice-giving.

With a modularity score of 0.372, the '***FRIEND***' network visualisation shows an overlapping of communities, indicative of social groups that may cross departmental lines, showing common interests or shared professional backgrounds. Isolated nodes might represent new members or those with more specialised roles within the firm. Given that friendships are typically reciprocal, the conversion to undirected in the friend network may have had minimal impact on the detection of overlapping communities. These communities likely still represent social groups formed around commonalities such as shared interests or backgrounds.

The '***WORK***' network, shows that there is the presence of multiple, interconnected communities suggests a collaborative environment with cross-functional teams, this can be reinforced with the modularity score of 0.395. However, the existence of isolated nodes or smaller clusters could point to certain individuals or groups that are either highly specialised or less integrated into the main workflow of the firm. While the presence of interconnected communities in the undirected network indicates a degree of collaboration, the original directed network would have provided deeper insights into how tasks are routed through the firm, who is initiating projects, and who is executing them.

All three networks show that there are strong internal linkages, and there is significant variation in the cohesiveness and density. However there is room for improvement for all networks, because there is a widespread lack of bridge nodes. Implementing this would potentially boost cross-functional collaboration and knowledge sharing within the firm.

Advice Network Inference Using The Exponential Family Random Graph Models (ERGM)

To present the ongoing relations between nodes and the attributes that explain the likelihood of creating connections amongst other actors in the '***ADVICE***' network, we have created an ERGM model. It contains explanations of node, dyadic, and relational effects layers. First, let us explain the numerical combinations on the node level, tenure has been proven a statistically significant factor in the model increasing the odds of a tie present for every other possible tie in the network, increasing the chances of a tie by 0.009 it is not a big factor, but still evidence that longer tenure in the company gives a greater chance of a link between actors. Age has been proved an insignificant factor. As for the links between actors of the same attribute, let us start with status. It has been proved a significant factor and the odds of a connection existing between two actors of the same class were found to be 1.09, signifying a higher chance of such a connection due to this factor. Partners are more likely to advise partners and associates are more likely to advise associates. Then as for practice, using nodemix we have found that the odds of an "advise" connection decrease on a statistically significant level for nodes of different classes, litigators are less likely to make connections with corporate lawyers and vice versa. What is interesting is that corporate lawyers are not significantly more likely to give advice to each other. Sex has been found to be a somewhat important factor in making ties, showing an increased tendency to advise actors of the same gender. The office has determined a statistically important factor increasing the odds of a link between the actors working within the same location of the law firm, increasing those odds by 1.14.

For this analysis, we also computed the centralities of every single actor in the network and added them as network attributes. We used the absolute difference method to explain its importance in the prediction of the network's structure. Closeness could not be included because ERGM does not take in missing values, and such was present for node V6 in the advice network. 3 other types of centralities were included - degree, betweenness, and eigenvector. All were found significant, with degree centrality being only slightly and the rest very important for the model. Betweenness centrality has been found a huge factor in increasing the odds of an "advise" connection between two nodes, with odds of 14.33. Eigenvector centrality brought those odds down by -3.13, making it less likely to give advice, given a higher eigenvector centrality. We also compared the structures of different graphs like the friendship or work networks, using both edgecov() and dyadcov() functions. This step checks if the structure of the '***ADVICE***' network can be predicted using the structure of the ***'FRIEND'*** and '***WORK***' networks. The structure of the work network has been found to be statistically significant, with positive odds of 1.85, meaning that a change for a tie increases for a tie present in the work network. In the friendship network, the mutual and upper triangle networks have been found to be significant, increasing the odds of a tie between nodes by 1.73 for mutual and 0.69 for upper. Those were the most important structural findings using the ERGM models. Out of all tried models, this one had the lowest AIC of 3393, meaning it best describes our '***ADVICE***' network structure, out of all tried ones. The output of the model is given here:

Table 3 - ERGM Model Estimation & Its Most Important Coefficients

```
ergm(formula = advice_net ~ edges + nodecov("tenure") + nodematch("status") +
    nodemix("practice") + nodematch("sex") + nodematch("office") +
    absdiff("degree_cent") + absdiff("betweenness_cent") + absdiff("egcent_cent")
+
    edgecov(work_net) + dyadcov(friend_net))

Maximum Likelihood Results:

                          Estimate Std. Error MCMC % z value Pr(>|z|)
edges                   -3.430193   0.154181      0 -22.248  < 1e-04 ***
nodecov.tenure           0.009140   0.003263      0   2.801 0.005093 **
nodematch.status         1.092839   0.100840      0  10.837  < 1e-04 ***
mix.practice.2.1        -0.994052   0.127733      0  -7.782  < 1e-04 ***
mix.practice.1.2        -1.239198   0.135349      0  -9.156  < 1e-04 ***
mix.practice.2.2         0.155568   0.115709      0   1.344 0.178791
nodematch.sex            0.312277   0.099016      0   3.154 0.001611 **
nodematch.office         1.140450   0.100744      0  11.320  < 1e-04 ***
absdiff.degree_cent      2.004694   1.099368      0   1.823 0.068228 .
absdiff.betweenness_cent 14.336191   2.557193      0   5.606  < 1e-04 ***
absdiff.egcent_cent      -3.133746   0.879486      0  -3.563 0.000366 ***
edgecov.work_net         1.851027   0.107307      0  17.250  < 1e-04 ***
dyadcov.friend_net.mutual  1.732850   0.241935      0   7.162  < 1e-04 ***
dyadcov.friend_net.utri    0.695952   0.183290      0   3.797 0.000146 ***
dyadcov.friend_net.ltri    0.500748   0.202026      0   2.479 0.013189 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

     Null Deviance: 6890  on 4970  degrees of freedom
 Residual Deviance: 3363  on 4955  degrees of freedom

AIC: 3393  BIC: 3490  (Smaller is better. MC Std. Err. = 0)
```

Complete Information

In delving into the social tapestry of the Lazega Law Firm's network, a nuanced portrait of connectivity emerges, intricately woven by the threads of locale, legal specialty, and educational lineage. The vibrancy of connections within the Boston hub outshines its counterparts, suggesting a dynamic where proximity catalyses a rich exchange of counsel and collaboration. Within the legal battlegrounds, litigators seem to forge a denser web of alliances compared to their corporate law counterparts, an echo of the hands-on, collaborative spirit that litigation demands.

Yet, perhaps most compelling is the revelation of alma mater influence, where the bonds formed in the hallowed halls of prestigious institutions translate into a robust network within the firm's ecosystem. This pattern is emblematic of the subtle yet potent role that shared educational experiences play in shaping professional landscapes. Such insights offer a lens through which the firm can view its internal network not just as a static entity, but as a living organism that both influences and is influenced by the multifaceted identities of its members.

These observations serve not merely as a map of existing connections but as a beacon, guiding strategies to cultivate a culture of inclusivity and collective growth. By acknowledging the undercurrents that guide the formation of professional relationships, the firm can navigate towards fostering a milieu where collaboration thrives across all levels, practices, and backgrounds.

Exploring the Dynamic Network of the Lazega Law Firm: A Study of Connectivity and Influence

A wide range of relationships inside the Lazegna Law Firm were exposed by the study. We examined the factors of geography, legal skill, and educational background of the individuals within the nodes. Based on this data, it is evident that litigators have the ability to establish a more notable network than those from the business sector. Even yet, there are still plenty of lively interactions even though the office is not as good as others. Because litigation work is collaborative in nature, lawyers who practise law tend to have stronger networks than business professionals.

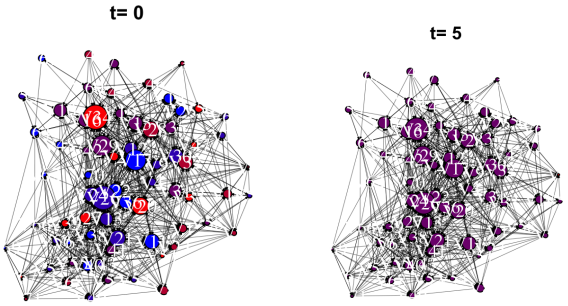
The study emphasises the significance of educational backgrounds as well. Prestigious university graduates foster a close-knit network within the company, demonstrating the enduring influence of common academic experiences on professional connections. These results show that the different identities of the firm's members have a continuous and dynamic impact on the network's evolution. Professional relationship insights serve as a roadmap for creating tactics that encourage diversity and group development, facilitating cooperation amongst practitioners, levels, and backgrounds.

Independent Cascade Model (ICM)

To measure the spread of innovation along the given networks, its diffusion, we created an pure imitation ICM model. It measures the internal-influence level, this means the cumulative adopters at a certain time can be the only influence for the new adopters. The corresponding results were found for each of the three basic networks. In the advisory network, starting with a randomly chosen node V21, the innovation diffusion stops after 4 time steps. The information first gets spread to the neighbours and with different steps those neighbours spread that innovation to other neighbouring actors. For the friendship network, the diffusion of that innovation takes over 7 time steps, the longest out of all models. This might be due to the lower number of links between actors in this network, making it harder for the innovation diffusion to happen. For the co-working network, the diffusion model takes 3 time steps, shortest out of all networks. However, through this network the number of nodes at the last step of innovation diffusion is 32, for the advice network it is 48. Therefore, it would be better for this process to happen on the advice network, since with just one more time step, the innovation spreads to 16 more actors.

DeGroot Model

The DeGroot model was used to see how 5 different opinions in the firm evolved during 6 iterations. The ***LEARN & ADVICE*** network was used as this is the only network with weights. Below are images from the beginning of the evolution (iteration 1; t=0) compared to the last iteration (iteration 6; t=5) . The assignment is set up in a way that higher opinion values get closer to the red end of the spectrum, and lower values towards the blue.



Graph 13 & 14 - De Groot Model.

Based on the visual change in the network from the first iteration to the sixth iteration, we can deduce that a consensus has been reached, because all of the nodes have changed from red or blue to purple which is seen in the second figure. Regarding whether the consensus reached is the optimum one or not, this would warrant external information being provided. Additionally, it does seem as though there is a bias, where those nodes with a higher degree were able to have more influence in changing the opinions of other nodes.

Conclusions

This detailed network analysis of the Lazega Law Firm has revealed a multifaceted landscape of professional relationships and social dynamics. The examination of the 'ADVICE', 'FRIEND', and 'WORK' networks, underpinned by methodologies like community detection and centrality measures, has unveiled the intricate ways in which these networks operate and interact.

Key findings highlight the influential role of geographical proximity, as seen in the vibrant connectivity of the Boston hub. This indicates that physical closeness can significantly enhance cooperative work and information exchange. The analysis also shows a clear distinction in networking patterns between litigators and corporate lawyers, with the former demonstrating a denser network. This could be reflective of the collaborative nature of litigation work.

Furthermore, the influence of educational backgrounds, especially connections formed in prestigious law schools, emerges as a significant factor in shaping the firm's professional environment. These alma mater ties not only persist but also thrive within the firm, indicating the lasting impact of shared educational experiences on professional networks. Despite the strong internal linkages, the analysis also identifies areas for improvement. The lack of bridge nodes across networks points to potential gaps in cross-functional collaboration and knowledge sharing. Addressing this could lead to a more integrated and dynamic workplace, fostering a culture that emphasises collective growth and inclusivity.

Overall, the Lazega Law Firm's networks are complex and dynamic, shaped by a confluence of factors like geographical location, legal specialisations, and educational ties. Moving forward, the firm can leverage these insights to enhance internal cooperation, promote knowledge exchange, and cultivate a more interconnected and inclusive professional environment.

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