3.1)

a) $f(n) = 9n$ and $g(n) = 5n^3$

- $\lim\limits_{n \to \infty} \dfrac{f(n)}{g(n)} = \lim\limits_{n \to \infty} \dfrac{9n}{5n^3} = \lim \dfrac{9}{\underbrace{5n^2}_{\to \infty}} = 0$

It belongs to $f \in o(g)$ and $f \in O(g)$. It is not in the set of $f \in \omega(g)$, $f \in \Omega(g)$ and $f \in \Theta(g)$

- $\lim\limits_{n \to \infty} \dfrac{g(n)}{f(n)} = \lim\limits_{n \to \infty} \dfrac{5n^3}{9n} = \lim\limits_{n \to \infty} \dfrac{5n^2}{9} = \infty$

It belongs to $g \in \omega(f)$ and $g \in \Omega(f)$. It is not in the set of $g \in o(f)$, $g \in O(f)$ and $g \in \Theta(f)$

b) $f(n) = 9n^{0.8} + 2n^{0.3} + 14 \log n$ and $g(n) = \sqrt{n}$

leading term

- $\lim\limits_{n \to \infty} \dfrac{f(n)}{g(n)} = \lim\limits_{n \to \infty} \dfrac{9n^{0.8} + 2n^{0.3} + 14 \log n}{\sqrt{n}} = \lim\limits_{n \to \infty} \dfrac{\underline{9n^{0.8}} + 2n^{0.3} + 14 \log n}{\underline{n^{0.5}}}$

leading term

$= \lim\limits_{n \to \infty} 9n^{0.3} = \infty$

It belongs to $f \in \omega(g)$ and $f \in \Omega(g)$. It is not in the set of $f \in o(g)$, $f \in O(g)$ and $f \in \Theta(g)$.

leading term

- $\lim\limits_{n \to \infty} \dfrac{g(n)}{f(n)} = \lim\limits_{n \to \infty} \dfrac{\underline{\sqrt{n}}}{\underline{9n^{0.8}} + 2n^{0.3} + 14 \log n} = \lim\limits_{n \to \infty} \dfrac{1}{\underbrace{9n^{0.3}}_{\to \infty}} = 0$

leading term

It belongs to $g \in o(f)$ and $g \in O(f)$. It is not in the set of $g \in \omega(f)$, $g \in \Omega(f)$ and $g \in \Theta(f)$

c) $f(n) = \frac{n^2}{\log n}$ and $g(n) = n\log n$

• $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{\frac{n^2}{\log n}}{n \log n} = \lim_{n \to \infty} \frac{n}{(\log n)^2} = \lim_{n \to \infty} \frac{1}{\frac{2\log n}{n \ln 2}}$

$= \lim_{n \to \infty} \frac{n \ln 2}{2 \log n} = \lim_{n \to \infty} \frac{\frac{\ln 2}{2}}{\frac{1}{n \ln 2}} = \lim_{n \to \infty} \frac{n (\ln 2)^2}{2} = \infty$

– By using L'Hopital's Rule

It belongs to $f \in \omega(g)$ and $f \in \Omega(g)$. It is not in the set of $f \in o(g)$, $f \in O(g)$ and $f \in \Theta(g)$

• $\lim_{n \to \infty} \frac{g(n)}{f(n)} = \lim_{n \to \infty} \frac{n \log n}{\frac{n^2}{\log n}} = \lim_{n \to \infty} \frac{n \log^2 n}{n^2} = 0$

$n^2 \to \infty$

It belongs to $g \in o(f)$ and $g \in O(f)$. It is not in the set of $g \in \omega(f)$, $g \in \Theta(f)$ and $g \in \Omega(f)$

d) $f(n) = (\log(3n))^3$ and $g(n) = 9 \log n$

• $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{(\log(3n))^3}{9 \log n} = \lim_{n \to \infty} \frac{\frac{3(\log(3n))^2}{n \ln 2}}{\frac{9}{n \ln 2}} =$

$\lim_{n \to \infty} \frac{(\log(3n))^2}{3} = \infty$

– By using L'Hopital's Rule

It belongs to $f \in \Omega(g)$ and $f \in \omega(g)$. It is not in the set of $f \in o(g)$, $f \in O(g)$ and $f \in \Theta(g)$

• $\lim_{n \to \infty} \frac{g(n)}{f(n)} = \lim_{n \to \infty} \frac{9 \log n}{(\log(3n))^3} = \lim_{n \to \infty} \frac{9 \log n}{\log^3(3n)} = 0$

It belongs to $g \in o(f)$ and $g \in O(f)$. It is not in the set of $g \in \omega(f)$, $g \in \Theta(f)$ and $g \in \Omega(f)$