Homework 7

Problem 7.1 Quicksort with Partition Versions

d) We get:

The average time for Lomoto partition is 171186 microseconds.

The average time for Hoare partition is 15683 microseconds.

The average time for Median of Three partition is 107815 microseconds.

Hoare partition is more efficient than Lomuto partition because it does three times fewer swaps on average. It also creates partitions even when all values are equal. Hoare partition degrades to O (n^2) when the input array is already sorted while Lomuto partition degrades to O (n^2) even when the input array is already sorted or when the array has all equal elements. In conclusion, the worst case for both partitions is O (n^2). Meanwhile, with the Median of Three the worst case is avoided. Time complexity for sorted or reverse data is O (n logn). There is a constant increase in time to select the pivot, but due to the fact that we are not going to compare with the first and last element, that increase in time is partly compensated. Median of Three partition improves the runtime behavior by 3 – 10%.

Problem 7.2 Modified Quicksort

b)

- The best case happens when the two pivots divide the array into 3 equal parts. in total we have n elements. We have recursive occurrences T(n/3) for the 3 subarrays. The partition function is called 3 times.

  $$T(n) = 3\,T(n/3) + O(n)$$

  Let's solve it using Master's Method:

  a = 3 $\quad$ $n^{\log_b a} = n^{\log_3 3}$

  b= 3 $\quad$ $n^{\log_3 3} = n^1$

  f(n) = O(n)

  It is the second case of Master Theorem. Therefore, the solution is O(n logn).

- The worst case happens when the pivots are positioned at the opposite ends, one in the start and the other in the end of the array, after we do the partition. The first and last partition have 0 elements while the middle partition has n – 2 elements:

$$T(n) = T(n-2) + O(n)$$

Recurrences :

$$\begin{aligned} T(n) &= T(n-2) + O(n) \\ &= T(n-4) + 2O(n) \\ &= T(n-6) + 3O(n) \\ &= T(n-8) + 4O(n) \\ &\quad \ldots \\ &\quad \ldots \\ &= T(n-n) + (n/2)O(n) \end{aligned}$$

Consequently:

$$\begin{aligned} T(n) &= T(n-2) + O(n) \\ &= O(n^2) \end{aligned}$$