SHEET 6

6.1) We should prove that the two elementary boolean functions $\rightarrow$ and $\neg$ are universal. Hence, we need to show that $\wedge, \vee$ and $\neg$ are produced with $\rightarrow$ and $\neg$

- $\uparrow$ (nand) is a universal function.

• $x \wedge y = (x \uparrow y) \uparrow (x \uparrow y)$

$x \to \neg y$ is equivalent to $\uparrow$ $\Rightarrow$ we can write $\wedge$ using $\to$ and $\neg$

$x \wedge y = (x \to \neg y) \to \neg (x \to \neg y$

The truth table :

x	y	$\neg y$	$x \to \neg y$	$\neg(x \to \neg y)$	$(x \to \neg y) \to \neg (x \to \neg y)$	$x \wedge y$
0	0	1	1	0	0	0
0	1	0	1	0	0	0
1	0	1	1	0	0	0
1	1	0	0	1	1	1

• $x \vee y = (x \uparrow x) \uparrow (y \uparrow y)$

$x \to \neg y$ is equivalent to $\uparrow$, $\Rightarrow$ we can write $\vee$ using $\to$ and $\neg$

$x \vee y = (x \to \neg x) \to \neg (y \to \neg y)$

The truth table

x	y	$\neg x$	$\neg y$	$x \to \neg x$	$y \to \neg y$	$\neg(y \to \neg y)$	$(x \to \neg x) \to \neg(y \to \neg y)$	$x \vee y$
0	0	1	1	1	1	0	0	0
0	1	1	0	1	0	1	1	1
1	0	0	1	0	1	0	1	1
1	1	0	0	0	0	1	0	1

- $\neg x = x \uparrow x$

$x \to \neg x$ is equivalent to $\uparrow$ ⇒ we can write $\neg$ using $\to$ and $\neg$

$$\neg x = x \to \neg x$$

The truth table:

x	$\neg x$	$x \to \neg x$
0	1	1
1	0	0

6.2)

a) $\varphi(A,B) = (\neg A \vee \neg B) \wedge (\neg A \vee B) \wedge (A \vee \neg B)$

$= (\neg A \vee (B \wedge \neg B)) \wedge (A \vee \neg B)$ (distributivity)

$= (\neg A \vee 0) \wedge (A \vee \neg B)$ (complementation)

$= \neg A \wedge (A \vee \neg B)$ (identity)

$= (\neg A \wedge A) \vee (\neg A \wedge \neg B)$ (distributivity)

$= 0 \vee (\neg A \wedge \neg B)$ (complementation)

$= \neg A \wedge \neg B$ (identity)

$= \neg (A \vee B)$ (de Morgan)

b) $\varphi(A,B,C) = (A \wedge \neg B) \vee (A \wedge \neg B \wedge C)$

$= A \wedge (\neg B \vee (\neg B \wedge C))$ (distributivity)

$= A \wedge \neg B$ (absorption)

c) $\varphi(A,B,C,D) = (A \vee \neg (B \wedge A)) \wedge (C \vee (D \vee C))$

$= (A \vee \neg B \vee \neg A) \wedge (C \vee (C \vee D))$ (commutativity, de Morgan)

$= (A \vee \neg A \vee \neg B) \wedge ((C \vee C) \vee D)$ (associativity, commutativity)

$= (1 \vee \neg B) \wedge (C \vee D)$ (idempotency, complementation)

$= 1 \wedge (C \vee D)$ (domination)

$= C \vee D$ (identity)

d) $\varphi(A,B,C) = (\neg(A \wedge B) \vee \neg C) \wedge (\neg A \vee B \vee \neg C)$

$= (\neg A \vee \neg B \vee \neg C) \wedge (\neg A \vee B \vee \neg C)$ (de Morgan)

$= (\neg A \vee \neg C \vee \neg B) \wedge (\neg A \vee \neg C \vee B)$ (commutativity)

$= (\neg A \vee \neg C) \vee (\neg B \wedge B)$ (distributivity)

$= (\neg A \vee \neg C) \vee 0$ (complementation)

$= \neg A \vee \neg C$ (identity)

$= \neg(A \wedge C)$ (de Morgan)

e) $\varphi(A,B) = (A \vee B) \wedge (\neg A \vee B) \wedge (A \vee \neg B) \wedge (\neg A \vee \neg B)$

$= (B \vee (A \wedge \neg A)) \wedge (\neg B \vee (A \wedge \neg A))$ (distributivity)

$= (A \wedge \neg A) \vee (B \wedge \neg B)$ (distributivity)

$= 0 \vee 0$ (complementation)

$= 0$

6.3)

P	Q	R	S	$\neg P$	$\neg P \vee Q$	$\neg Q$	$\neg Q \vee R$	$\neg R$	$\neg R \vee S$	$\neg S$	$\neg S \vee P$	$(\neg P \vee Q) \wedge (\neg Q \vee R)$	$(\neg P \vee Q) \wedge (\neg Q \vee R) \wedge (\neg R \vee S)$	$(\neg P \vee Q) \wedge (\neg Q \vee R) \wedge (\neg R \vee S) \wedge (\neg S \vee P)$
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	0	0	1	1	0
0	0	1	0	1	1	1	1	0	0	1	1	1	0	0
0	1	0	0	1	1	0	0	1	1	1	1	0	0	0
1	0	0	0	0	0	1	1	1	1	1	1	0	0	0
1	1	0	0	0	1	0	0	1	1	1	1	0	0	0
1	0	1	0	0	0	1	1	0	0	1	1	0	0	0
1	0	0	1	0	0	1	1	1	1	0	1	0	0	0
0	1	1	0	1	1	0	1	0	0	1	1	1	0	0
0	1	0	1	1	1	0	0	1	1	0	0	0	0	0
0	0	1	1	1	1	1	1	0	0	0	0	1	0	0
1	1	1	0	0	1	0	1	0	0	1	1	1	0	0
1	1	0	1	0	1	0	0	1	1	0	1	0	0	0
1	0	1	1	0	0	1	1	0	1	0	1	0	0	0

0	1	1	1	1	1	0	1	0	1	0	0	1	1	0
1	1	1	1	0	1	0	1	0	1	0	1	1	1	1

a) There are two rows which result is 1, so there are two interpretations of the variables P, Q, R, S that satisfy φ

b) Where the result is 1, there we obtain a DNF. Row 1 and row 16 satisfy φ as their result is 1. In order to write the formula for φ in DNF for each interpretation, we negate the variables which are 0 and then combine them with $\wedge$. After that, we combine the parentheses with $\vee$

DNF: $(\neg P \wedge \neg Q \wedge \neg R \wedge \neg S) \vee (P \wedge Q \wedge R \wedge S)$