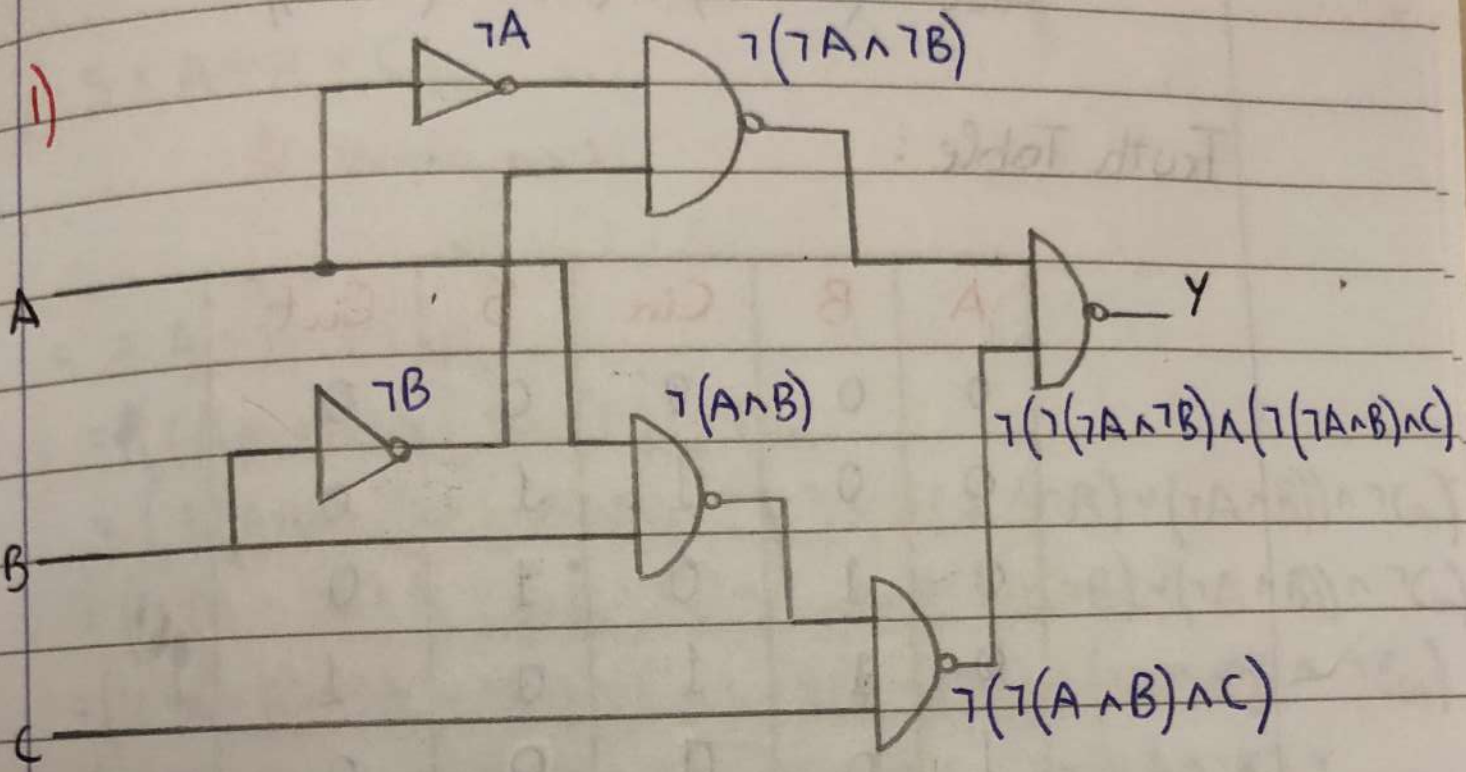
SHEET 8

1)



a) Boolean expression

$$Y = \neg(\neg(\neg A \wedge \neg B) \wedge \neg(\neg(A \wedge B) \wedge C))$$

1) b) Derive the DNF

$$\neg(\neg(\neg A \wedge \neg B) \wedge \neg(\neg(A \wedge B) \wedge C))$$
$$= \neg((A \vee B) \wedge \neg((\neg A \vee \neg B) \wedge C)))$$
$$= \neg((A \vee B) \wedge \neg(\neg A \vee \neg B) \vee \neg C))$$
$$= \neg((A \vee B) \wedge (A \wedge B \vee \neg C))$$
$$= (\neg A \wedge \neg B) \vee ((\neg A \vee \neg B) \wedge C)$$

$$DNF = (\neg A \wedge \neg B) \vee (\neg A \wedge C) \vee (\neg B \wedge C)$$

2) $S = A \dot{\vee} B \dot{\vee} C_{in}$

$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$

Truth Table:

A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

a) Both functions as a disjunction of product terms

We know:

$A \dot{\vee} B = \neg(A \wedge B) \wedge (A \vee B)$

$= (\neg A \vee \neg B) \wedge (A \vee B)$

$$= (A \wedge \neg B) \vee (\neg A \wedge B)$$

$S = A \dot{\vee} B \dot{\vee} C_{in}$

⇓ Write as:

$$S = A \dot{\vee} B \dot{\vee} C_{in}$$
$$= ((A \wedge \neg B) \vee (\neg A \wedge B)) \vee C_{in}$$
$$= (\neg((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in})$$
$$= ((\neg(A \wedge \neg B) \wedge \neg(\neg A \wedge B)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in})$$
$$= (((\neg A \vee B) \wedge (A \vee \neg B)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in})$$
$$= (((\neg A \wedge (A \vee \neg B)) \vee (B \wedge (A \vee \neg B))) \wedge C_{in}) \vee ((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in})$$
$$= (((\neg A \wedge \neg B) \vee (B \wedge A)) \wedge C_{in}) \vee (((A \wedge \neg B) \vee (\neg A \wedge B)) \wedge \neg C_{in})$$
$$= (\neg A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (\neg A \wedge B \wedge C_{in})$$

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$
$$= (A \wedge B) \vee (C_{in} \wedge ((A \wedge \neg B) \vee (\neg A \wedge B)))$$
$$= (A \wedge B) \vee ((C_{in} \wedge (A \wedge \neg B)) \vee (C_{in} \wedge (\neg A \wedge B)))$$
$$= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))$$
$$= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))$$

$= (A \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \wedge (B \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B)))$

$= (A \vee (C_{in} \wedge A \wedge \neg B) \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B) \vee B \vee (C_{in} \wedge \neg A \wedge B))$

$= (A \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee B \vee (C_{in} \wedge A \wedge \neg B))$

$= (A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B))$

$= ((A \vee C_{in}) \wedge (A \vee \neg A) \wedge (A \vee B)) \wedge ((B \vee C_{in}) \wedge (B \vee A) \wedge (B \vee \neg B))$

$= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in}) \wedge (B \vee A)$

$= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in})$

$= (A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B)$

b) Both functions as a conjunction of sum terms

- Construct CNF by looking at the table:

$S = A \dot{\vee} B \dot{\vee} C_{in}$

$= (\neg A \vee \neg B \vee C_{in}) \wedge (A \vee B \vee C_{in}) \wedge (A \vee \neg B \vee \neg C_{in}) \wedge (\neg A \vee B \vee \neg C_{in})$

$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$

$= (A \wedge B) \vee (C_{in} \wedge ((A \wedge \neg B) \vee (\neg A \wedge B)))$

$= (A \wedge B) \vee ((C_{in} \wedge (A \wedge \neg B)) \vee (C_{in} \wedge (\neg A \wedge B)))$

$= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))$

$= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))$

$= (A \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \wedge (B \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B)))$

$= (A \vee (C_{in} \wedge A \wedge \neg B) \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B) \vee B \vee (C_{in} \wedge \neg A \wedge B))$

$= (A \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee B \vee (C_{in} \wedge A \wedge \neg B))$

$= (A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B))$

$= ((A \vee C_{in}) \wedge (A \vee \neg A) \wedge (A \vee B)) \wedge ((B \vee C_{in}) \wedge (B \vee A) \wedge (B \vee \neg B))$

$= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in}) \wedge B \vee A)$

$= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in})$

c) Both functions using only not ($\neg$) and not- and ($\uparrow$) operations.

$S = A \dot{\vee} B \dot{\vee} C_{in}$

$S = (\neg A \wedge B \wedge C_{in}) \vee (A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in})$

$= \neg\neg((\neg A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}))$

$= \neg(\neg(\neg A \wedge \neg B \wedge C_{in}) \wedge \neg(A \wedge B \wedge C_{in}) \wedge \neg(A \wedge \neg B \wedge \neg C_{in}) \wedge \neg(\neg A \wedge B \wedge \neg C_{in}))$

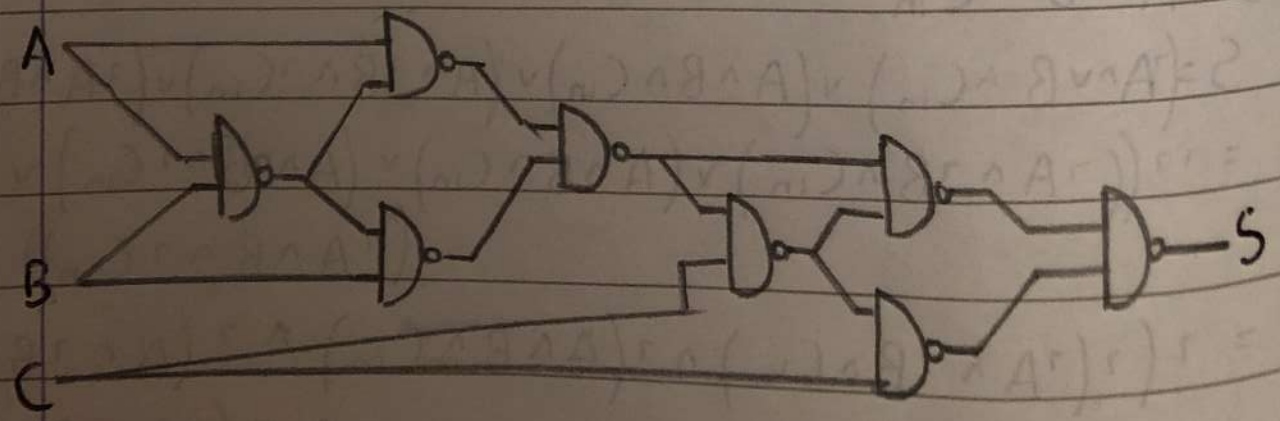
$$= (\neg(\neg A \wedge \neg B \wedge C_{in}) \uparrow \neg(A \wedge B \wedge C_{in}) \uparrow \neg(A \wedge \neg B \wedge \neg C_{in}) \uparrow \neg(\neg A \wedge B \wedge \neg C_{in}))$$

$$= (\neg A \uparrow \neg B \uparrow C_{in}) \uparrow (A \uparrow B \uparrow C_{in}) \uparrow (A \uparrow \neg B \uparrow \neg C_{in}) \uparrow (\neg A \uparrow B \uparrow \neg C_{in})$$

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$
$$= (A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B)$$
$$= \neg\neg((A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B))$$
$$= \neg(\neg(A \wedge B) \wedge \neg(C_{in} \wedge A) \wedge \neg(C_{in} \wedge B))$$
$$= \neg((A \uparrow B) \wedge (C_{in} \uparrow A) \wedge (C_{in} \uparrow B))$$
$$= ((A \uparrow B) \uparrow (C_{in} \uparrow A) \uparrow (C_{in} \uparrow B))$$
$$= (A \uparrow B) \uparrow (C_{in} \uparrow A) \uparrow (C_{in} \uparrow B)$$

d) Draw

- $S = A \dot{\vee} B \dot{\vee} C_{in}$



$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \vee B))$