

## SHEET 7

$$7.1) \varphi(A, B, C, D, E) = m_0 + m_2 + m_4 + m_6 + m_9 \\ + m_{10} + m_{13} + m_{14} + m_{15} + m_{16} + m_{17} + m_{21} + m_{26} \\ + m_{28} + m_{30} + m_{31}$$

a) Prime implicants of  $\varphi$

• Numbers into binary representation

$$m_0 \rightarrow 00000$$

$$m_2 \rightarrow 00010$$

$$m_4 \rightarrow 00100$$

$$m_6 \rightarrow 00110$$

$$m_9 \rightarrow 01001$$

$$m_{10} \rightarrow 01010$$

$$m_{13} \rightarrow 01101$$

$$m_{14} \rightarrow 01110$$

$$m_{15} \rightarrow 01111$$

$$m_{16} \rightarrow 10000$$

$$m_{17} \rightarrow 10001$$

$$m_{21} \rightarrow 10101$$

$$m_{26} \rightarrow 11010$$

$$m_{28} \rightarrow 11100$$

$$m_{30} \rightarrow 11110$$

$$m_{31} \rightarrow 11111$$

• Classify the minterms



Group	Minterm	Pattern
0 1's	$m_0$	00000
1 1	$m_2$	00010
	$m_4$	00100
	$m_{16}$	10000
	$m_6$	00110
2 1's	$m_9$	01001
	$m_{10}$	01010
	$m_{17}$	10001
	$m_{13}$	01101
3 1's	$m_{14}$	01110
	$m_{21}$	10101
	$m_{26}$	11010
	$m_{28}$	11100
4 1's	$m_{15}$	01111
	$m_{30}$	11110
5 1's	$m_{31}$	11111



Minterm	Pattern	Used	Minterm	Pattern	Used	Minterms	Patterns
$m_0$	00000	✓	$m_{0,2}$	000-0	✓	$m_{0,2,4,6}$	00--0
			$m_{0,4}$	00-00	✓	$m_{0,4,2,6}$	00--0
			$m_{0,16}$	-0000	-		
$m_2$	00010	✓	$m_{2,6}$	00-10	✓	$m_{2,6,10,14}$	0--10
$m_4$	00100	✓	$m_{2,10}$	0-010	✓	$m_{2,10,8,14}$	0--10
$m_{16}$	10000	✓	$m_{4,6}$	001-0	✓		
			$m_{16,17}$	1000-	-		
$m_6$	00110	✓	$m_{6,14}$	0-110	✓	$m_{10,14,26,30}$	-1-10
$m_8$	01001	✓	$m_{8,13}$	01-01	-	$m_{10,26,14,30}$	-1-10
$m_{10}$	01010	✓	$m_{10,14}$	01-10	✓		
$m_{17}$	10001	✓	$m_{10,26}$	-1010	✓		
			$m_{17,21}$	10-01	-		
$m_{13}$	01101	✓	$m_{13,15}$	011-1	-	$m_{14,15,30,31}$	-111-
$m_{14}$	01110	✓	$m_{14,15}$	0111-	✓	$m_{14,30,15,31}$	-111-
$m_{21}$	10101	✓	$m_{14,30}$	-1110	✓		
$m_{26}$	11010	✓	$m_{26,30}$	11-10	✓		
$m_{28}$	11100	✓	$m_{28,30}$	111-0	-		
$m_{15}$	01111	✓	$m_{15,31}$	-1111	✓		
$m_{30}$	11110	✓	$m_{30,31}$	1111-	✓		
$m_{31}$	11111	✓					



No more combinations can be done



There are 10 prime implicants:

- $m_{0,16} = (\neg B \wedge \neg C \wedge \neg D \wedge \neg E)$
- $m_{16,17} = (A \wedge \neg B \wedge \neg C \wedge \neg D)$
- $m_{9,13} = (\neg A \wedge B \wedge \neg D \wedge E)$
- $m_{17,21} = (A \wedge \neg B \wedge \neg D \wedge E)$
- $m_{13,15} = (\neg A \wedge B \wedge C \wedge E)$
- $m_{28,30} = (A \wedge B \wedge C \wedge \neg E)$
- $m_{0,2,4,6} = (\neg A \wedge \neg B \wedge \neg E)$
- $m_{2,6,10,14} = (\neg A \wedge D \wedge \neg E)$
- $m_{10,14,26,30} = (B \wedge D \wedge \neg E)$
- $m_{14,15,30,31} = (B \wedge C \wedge D)$

b) The essential prime implicants

	$m_0$	$m_2$	$m_4$	$m_6$	$m_9$	$m_{10}$	$m_{13}$	$m_{14}$	$m_{15}$	$m_{16}$	$m_{17}$	$m_{21}$	$m_{26}$	$m_{28}$	$m_{30}$	$m_{31}$	Comment
$m_{0,16}$	✓									✓							
$m_{16,17}$										✓	✓						
$m_{9,13}$					✓		✓										Essential
$m_{17,21}$											✓	✓					Essential
$m_{13,15}$							✓		✓								
$m_{28,30}$														✓	✓		Essential
$m_{0,2,4,6}$	✓	✓	✓	✓													Essential
$m_{2,6,10,14}$		✓		✓		✓		✓									
$m_{10,14,26,30}$						✓		✓					✓			✓	Essential
$m_{14,15,30,31}$								✓	✓							✓	Essential
	✓	✓	✓	✓	✓	✓	✓	✓	✓	-	✓	✓	✓	✓	✓	✓	Coverage of essential



The essential prime implicants:

$$m_{9,13}$$

$$m_{1,7,21}$$

$$m_{28,30}$$

$$m_{0,2,4,6}$$

$$m_{10,14,26,30}$$

$$m_{14,15,30,31}$$

Either  $m_{0,16}$  or  $m_{16,17}$

Minimal boolean expressions

$$\begin{aligned} \bullet \quad \varphi'(A, B, C, D, E) = & (\neg A \wedge B \wedge \neg D \wedge E) \vee \\ & (A \wedge \neg B \wedge \neg D \wedge E) \vee (A \wedge B \wedge C \wedge \neg E) \vee \\ & (\neg A \wedge \neg B \wedge \neg E) \vee (B \wedge D \wedge \neg E) \vee (B \wedge C \wedge D) \\ & \vee (\neg B \wedge \neg C \wedge \neg D \wedge \neg E) \end{aligned}$$

$$\begin{aligned} \bullet \quad \varphi'(A, B, C, D, E) = & (\neg A \wedge B \wedge \neg D \wedge E) \vee \\ & (A \wedge \neg B \wedge \neg D \wedge E) \vee (A \wedge B \wedge C \wedge \neg E) \vee \\ & (\neg A \wedge \neg B \wedge \neg E) \vee (B \wedge D \wedge \neg E) \vee (B \wedge C \wedge D) \\ & \vee (A \wedge \neg B \wedge \neg C \wedge \neg D) \end{aligned}$$