THESIS TITLE: DEEPCUBE**A**, A STUDY OF REINFORCEMENT LEARNING ALGORITHMS FOR SOLVING THE RUBIK'S CUBE

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ABSTRACT

DEEPCUBEA: SOLVING THE RUBIK'S CUBE USING REINFORCEMENT LEARNING

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This thesis studies the DeepCubeA algorithm, which uses deep reinforcement learning to solve the Rubik's Cube. DeepCubeA effectively identifies the optimal solutions for the cube by using a neural network that was trained using an altered version of the A* search algorithm. The implementation shows how computers may solve challenging combinatorial puzzles without the need for human participation by utilizing recent developments in artificial intelligence. I have examined the algorithm's design, training procedure, and performance measures, in order to shed light on the algorithm's efficacy and possible uses in more general AI research.

Keywords: DeepCubeA; Rubik’s Cube; Deep Learning; Reinforcement Learning; Neural Networks

ABSTRAKT

DEEPCUBEA: ZGJIDHJA E KUBIT TË RUBIKUT PËRMES MËSIMIT PËRFORCUES

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Kjo tezë studion algoritmin DeepCubeA, i cili përdor Mësimin Përforcues të Thellë për të zgjidhur Kubin e Rubikut. DeepCubeA identifikon në mënyrë efektive zgjidhjet optimale për kubin duke përdorur një Rrjet Nervor Artificial që është trajnuar duke përdorur një version të ndryshuar të algoritmit të kërkimit A*. Implementimi tregon se si kompjuterët mund të zgjidhin puzzle të ndërlikuara kombinatorike pa nevojën e pjesëmarrjes njerëzore duke përdorur zhvillimet e fundit në inteligjencën artificiale. Gjatë kësaj teze unë kam shqyrtuar dizajnin e algoritmit, procedurën e trajnimit dhe masat e performancës për të hedhur dritë mbi efikasitetin e algoritmit dhe përdorimet e mundshme në kërkimin më të gjerë të inteligjencës artificiale.

Fjalë kyçe: DeepCubeA; Kubi i Rubikut; Mësimin i Thellë; Mësimi Përforcues; Rrjet Nervor Artificial

Dedicated to my family, friends and mentors who have supported me throughout this journey.

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AI	Artificial Intelligence
RL	Reinforcement Learning
NN	Neural Networks
A*	A Star Algorithm
DNN	Deep Neural Networks
DAVI	Deep Approximate Value Iteration
BWAS	Batch Weightet A* Search
GANs	Generative Adversarial Networks
VAEs	Variational Autoencoders
DRL	Deep reinforcement learning
PPO	Proximal Policy Optimization
A2C	Advantage Actor-Critic
A3C	Asynchronous Advantage Actor-Critic
MCTS	Monte Carlo tree search

CHAPTER 1

INTRODUCTION

The Rubik's Cube, a 3D combinatorial puzzle, has been a subject of fascination and study since its invention in 1974 by Ernő Rubik. Its complexity, poses a significant challenge for both human solvers and computational algorithms. Research has shown that solving the Rubik's Cube can enhance cognitive abilities (Johnson, 2019). The advent of machine learning, particularly reinforcement learning (RL), has opened new avenues for solving such puzzles.

DeepCubeA is a pioneering algorithm that utilizes deep RL to solve the Rubik's Cube. By training a neural network with a modified A* search algorithm, DeepCubeA can efficiently find optimal solutions, demonstrating the potential of AI in tackling complex problems without human intervention (Agostinelli, 2019).

1.1. Overview of the Rubik's Cube and Its Complexity

This iconic puzzle has become a global symbol of creativity, intellectual challenge, and problem-solving prowess. The standard Rubik's Cube is composed of 26 smaller cubes, commonly referred to as "cubies," which form a 3x3x3 arrangement. Each of these cubies can be rotated around three different axes, allowing for a vast number of possible configurations. The primary objective of the puzzle is to start from a scrambled state (Figure 1. ***Rubik's Cube Mechanism*** and manipulate the cube until each of the six faces displays a uniform color.

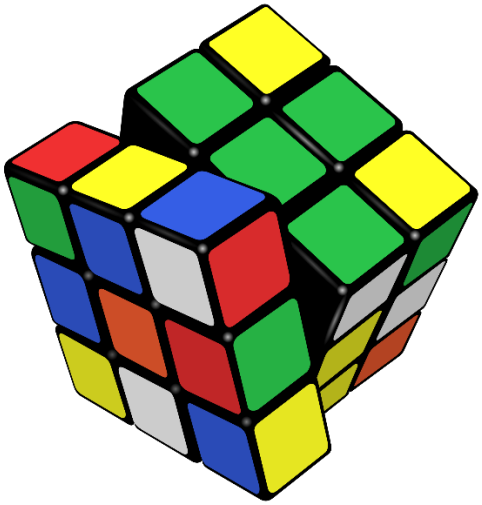

Figure 1. Rubik's Cube Mechanism ***(Boooyabazooka, 2008)***

The sheer number of possible configurations of the Rubik's Cube adds to its complexity and allure. As detailed by (Joyner, 2008), the cube can be arranged in 43,252,003,274,489,856,000 (approximately 4.3×10^19) distinct states. Despite this astronomical figure, every solvable configuration of the Rubik's Cube can be resolved in 20 moves or fewer, a phenomenon known as "God's Number" (Rokicki, 2010). This concept underscores the fascinating blend of difficulty and simplicity that characterizes the cube, revealing the underlying elegance of its design.

Solving the Rubik's Cube has intrigued a diverse array of individuals, from mathematicians and computer scientists to puzzle enthusiasts. Traditional methods for solving the cube, such as those devised by David Singmaster in the 1980s, employ a series of algorithms that decompose the problem into more manageable subproblems. These algorithms typically involve solving the cube layer by layer or reducing it to a simpler form before applying specific sequences of moves to achieve the solved state.

The field of artificial intelligence and machine learning has introduced novel approaches to solving the Rubik's Cube, leveraging modern computational power and sophisticated heuristic search techniques. Algorithms like Thistlethwaite's and Kociemba's (Kociemba, 1992) have been significantly refined and enhanced. In recent years, the DeepCubeA algorithm has demonstrated that deep reinforcement learning can

effectively solve the cube without any human intervention (Agostinelli, 2019). This algorithm's success illustrates the potential of AI to tackle complex, well-defined problem spaces.

The Rubik's Cube serves as an excellent benchmark for evaluating the performance of various algorithms due to its clearly defined problem space and explicit goal. Its complexity and the extensive solution space it presents continue to inspire ongoing research and innovation across multiple disciplines, including mathematics, computer science, and artificial intelligence. The cube's enduring appeal lies not only in its status as a challenging puzzle but also in its capacity to drive advancements in algorithmic theory and practice.

1.2. Background on Reinforcement Learning

Reinforcement learning (RL) is a pivotal subset of machine learning focused on how agents can learn to make decisions through interactions with an environment. Unlike supervised learning, which relies on labeled datasets, RL employs a trial-and-error approach to discover optimal behaviors by maximizing cumulative rewards. This methodology has its roots in behavioral psychology, where learning is driven by the outcomes of actions—either rewards or punishments.

1.2.1. Key Components of Reinforcement Learning

The core components of RL are states, actions, rewards, and policies, each playing a crucial role in guiding the learning process.

The state represents a specific situation or configuration in which the agent finds itself within the environment. Each state encapsulates all the relevant information needed for

the agent to make an informed decision. In the context of the Rubik's Cube, a state would correspond to a particular arrangement of the cube's smaller cubes or "cubies." This configuration determines the agent's perception of the problem at any given moment. States can be incredibly diverse and complex, especially in environments with numerous variables and possible configurations, such as the Rubik's Cube with its 43 quintillion possible states. The richness of the state space often poses a significant challenge in RL, requiring the agent to effectively learn and generalize across many different scenarios.

Actions are the set of all possible moves that an agent can take from any given state. In RL, each action transforms the current state into a new state, thereby driving the agent through its environment. For the Rubik's Cube, actions are defined as the rotations of the cube's faces. These rotations can be performed in various directions (clockwise, counterclockwise, and 180 degrees), and each move alters the cube's configuration. The set of possible actions in any given state must be comprehensive enough to allow the agent to navigate through all possible states. In complex environments, the number of available actions can be vast, necessitating sophisticated techniques for action selection and management to ensure efficient learning and decision-making.

Rewards are numerical values received after taking an action in a particular state, serving as immediate feedback on the action's desirability. The reward function in RL is meticulously designed to reinforce actions that lead to favorable outcomes and discourage those that do not. In solving the Rubik's Cube, the reward might be structured to provide positive feedback for moves that lead the agent closer to the solved state and negative feedback for moves that do not contribute to progress or worsen the configuration. The reward function is pivotal in shaping the agent's learning process, as it directly influences the policy the agent develops. The challenge lies in designing a reward function that accurately reflects the long-term goals and complexities of the task.

A policy is a strategy employed by the agent to determine the next action based on the current state. The policy is essentially the agent's decision-making mechanism. It can be

deterministic, where a specific action is chosen for each state, or stochastic, where actions are chosen according to a probability distribution. The policy aims to maximize the expected cumulative reward over time, guiding the agent to achieve its long-term objectives. In the context of the Rubik's Cube, the policy would dictate which face to rotate and in which direction, based on the cube's current configuration. Developing an optimal policy is a fundamental goal in RL, requiring the agent to balance exploration (trying new actions to discover their effects) and exploitation (choosing actions that are known to yield high rewards). Advanced RL algorithms often employ sophisticated methods to approximate the optimal policy, especially in environments with large state and action spaces.

An additional crucial concept in RL is the balance between exploration and exploitation. Exploration involves trying out new actions to discover their effects and learn more about the environment, which can lead to discovering more effective strategies. Exploitation, on the other hand, involves using known actions that have yielded high rewards in the past to maximize immediate gains. Finding the right balance between these two strategies is essential for the agent to effectively learn and optimize its performance over time. For example, in the Rubik's Cube, while exploiting known successful moves is important to make progress, exploring new sequences of moves might uncover more efficient paths to the solved state.

The value function is another key concept in RL, which estimates the expected cumulative reward an agent can achieve from a given state (or state-action pair). There are two primary types of value functions: the state-value function, which evaluates the long-term reward of being in a particular state, and the action-value function (or Q-value), which assesses the expected reward of taking a specific action from a given state. The value function helps the agent to evaluate the potential benefits of different states and actions, guiding the learning process towards more rewarding paths. In the context of the Rubik's Cube, the value function would help the agent understand which

configurations are closer to the solution and which moves are likely to lead to those configurations.

1.2.2. The RL Process

The RL process is a dynamic and iterative cycle where an agent learns to make decisions through continuous interaction with its environment. This process involves several critical steps, each contributing to the agent's development of an optimal policy. Let's delve into the key stages of the RL process in greater detail.

The RL process begins with the agent exploring the environment randomly. During this phase, the agent is not yet knowledgeable about the environment and its dynamics, so it performs actions without a predefined strategy. The primary goal at this stage is to gather as much information as possible about the environment's structure and the consequences of various actions. For the Rubik's Cube, this might involve the agent performing random rotations of the cube's faces to observe how different moves affect its configuration.

As the agent interacts with the environment, it accumulates experience in the form of state-action-reward sequences. Each sequence comprises the current state (the cube's configuration), the action taken (a specific rotation), and the reward received (feedback indicating progress towards solving the cube). These sequences form the basis of the agent's learning, providing the data needed to understand the relationships between states, actions, and rewards.

A critical aspect of RL is balancing exploration and exploitation. Exploration involves trying new actions to discover their effects and learn more about the environment. This phase is essential for the agent to avoid local optima and to gather comprehensive knowledge about the environment's dynamics. Exploitation, on the other hand, involves selecting actions that are known to yield high rewards based on the agent's accumulated

experience. This balance ensures that the agent can efficiently learn and refine its strategy over time.

For the Rubik's Cube, balancing exploration and exploitation might mean that while the agent occasionally tries new sequences of moves to discover potentially better solutions (exploration), it also relies on previously learned successful moves to make steady progress (exploitation). Effective RL algorithms employ strategies like the epsilon-greedy approach, where the agent explores randomly with a probability epsilon and exploits the best-known actions otherwise.

As the agent receives rewards based on its actions, it updates its knowledge about the environment. This update process is typically guided by algorithms such as Q-learning and Temporal Difference (TD) learning.

Q-learning: Q-learning is a model-free RL algorithm that updates the value of state-action pairs (Q-values) based on the rewards received. The Q-value represents the expected cumulative reward of taking a specific action from a given state and following the optimal policy thereafter. The Q-learning update rule adjusts the Q-values to reduce the difference between predicted and actual rewards, thereby improving the agent's predictions over time.

For the Rubik's Cube, these algorithms help the agent predict the long-term benefits of various moves and adjust its strategy to maximize the cumulative reward. For example, the agent might learn that rotating a particular face often leads to configurations closer to the solved state and update its policy to favor such moves.

Over time, as the agent continues to explore, exploit, and update its knowledge, its policy converges towards the optimal policy. The optimal policy is the strategy that maximizes the cumulative reward over time, guiding the agent to achieve its goal in the most efficient manner. This convergence process may require extensive training, particularly in complex environments like the Rubik's Cube, where the state space is vast and intricate.

Training the agent involves numerous iterations of the RL process, with the agent repeatedly refining its policy based on new experiences and feedback. In the context of the Rubik's Cube, this might mean millions of simulated moves and rotations, gradually honing in on the most efficient sequences to solve the puzzle.

The convergence towards an optimal policy in complex environments is a significant challenge in RL. The Rubik's Cube, with its vast number of possible states, exemplifies such a challenge. The agent must navigate this enormous state space, learning effective strategies to transition from any given configuration to the solved state. This requires sophisticated exploration strategies, robust learning algorithms, and considerable computational resources.

1.2.3. Applications of Reinforcement Learning

RL has proven its versatility and effectiveness across a wide range of domains, revolutionizing various fields with its ability to learn optimal strategies through interaction with environments. Below are some notable applications of RL, each highlighting its transformative impact.

One of the most prominent applications of RL is in game playing, where algorithms have achieved remarkable success. RL algorithms like Deep Q-Networks (DQN) have attained superhuman performance in classic video games such as those in the Atari series. In these games, RL agents learn to play by maximizing scores through trial and error, refining their strategies over millions of iterations. For example, in the game of Breakout, the RL agent not only learned to play but also discovered an optimal strategy of creating tunnels through the bricks to reach the highest scores. Another significant achievement is AlphaGo, developed by DeepMind, which utilized RL to master the game of Go. By combining deep neural networks with advanced tree search techniques, AlphaGo defeated world champion Go players, showcasing the power of RL in handling

complex, strategic decision-making. Interestingly, AlphaGo made moves that were initially considered unconventional by human experts but later recognized as highly innovative, changing the way the game is played (Mnih et al., 2015; Silver et al., 2016).

In the field of robotics, RL is extensively used to enhance the capabilities of robots in performing various tasks. Path planning, a critical aspect of robotics, involves determining the optimal route for a robot to follow to reach a destination while avoiding obstacles. RL enables robots to learn efficient paths through environments by trial and error, often in simulated settings before real-world deployment. Additionally, RL is applied to manipulation tasks, where robots learn to interact with objects, such as picking and placing items with precision. For example, robotic arms in manufacturing settings can be trained using RL to handle delicate components without damaging them. Autonomous navigation is another area where RL excels, allowing robots to navigate complex and dynamic environments without human intervention. In some advanced applications, RL-trained robots have participated in rescue missions, exploring hazardous areas that are unsafe for humans (Kober et al., 2013). These applications highlight RL's ability to improve robotic autonomy and adaptability.

Recommendation systems benefit significantly from RL by optimizing content suggestions to enhance user engagement. Traditional recommendation algorithms often rely on static models that do not adapt to changing user preferences. RL, however, enables recommendation systems to learn and evolve based on user interactions. By modeling user preferences as a dynamic environment, RL algorithms can continuously update their recommendations to maximize long-term user satisfaction and engagement. For instance, RL can optimize the sequence of content presented to users on streaming platforms or e-commerce websites, leading to more personalized and relevant recommendations. Fun fact: Netflix and YouTube use RL-based algorithms to recommend shows and videos that keep users engaged for longer periods, tailoring content to individual tastes based on viewing history (Zhao et al., 2019).

Autonomous vehicles represent a cutting-edge application of RL, where it is used for decision-making in dynamic and uncertain environments. Self-driving cars must navigate complex traffic scenarios, make split-second decisions, and ensure passenger safety. RL algorithms help these vehicles learn optimal driving strategies by simulating various driving conditions and scenarios. Through extensive training, RL enables self-driving cars to improve their performance in tasks such as lane keeping, obstacle avoidance, and adaptive cruising. The ability to learn from experience and adapt to new situations makes RL an invaluable tool for advancing autonomous driving technology. For example, companies like Waymo and Tesla utilize RL to enhance the capabilities of their autonomous vehicles, enabling them to handle a wide range of driving conditions from city traffic to highway cruising (Sallab, 2017).

In the specific context of solving the Rubik's Cube, RL demonstrates its prowess in tackling intricate combinatorial problems. The DeepCubeA algorithm, for example, leverages deep learning to manage the high-dimensional state space and complex reward structures of the Rubik's Cube. By training on numerous scrambled configurations, DeepCubeA learns to solve the cube efficiently, finding solutions in a minimal number of moves. This application showcases RL's potential to address challenging problems that require sophisticated planning and optimization. Impressively, DeepCubeA can solve the Rubik's Cube in an average of just 20 moves, a feat that even experienced human solvers struggle to achieve consistently (Agostinelli, 2019). The success of DeepCubeA illustrates how RL can be applied to other complex puzzles and combinatorial tasks, opening new avenues for research and practical applications. Fun fact: the Rubik's Cube has over 43 quintillion possible configurations, yet it can always be solved in 20 moves or less, a concept known as "God's Number".

1.3. Introduction to DeepCubeA

DeepCubeA is an advanced algorithm that uses deep learning and RL to solve the Rubik's Cube from any scrambled state. This novel approach combines neural network predictive power with search algorithms' strategic exploration capabilities. The algorithm employs a neural network trained on self-play to predict the most promising moves, which are then evaluated by a modified A* search algorithm to determine the best solution path. It is a notable development in the use of AI to combinatorial puzzles because of its capacity to effectively traverse the large state space of the Rubik's Cube. This strategy not only shows how deep learning and conventional search techniques may work together to solve complex problems, but it also creates new avenues for problem solving.

1.4. Problem Statement and Objectives

Despite the advancements in AI and machine learning, solving the Rubik's Cube efficiently from any scrambled state remains a challenging task due to the vast state space and the complexity of the solution paths. The DeepCubeA algorithm represents a significant step forward by integrating deep learning with RL to address these challenges. However, a thorough analysis of its architecture, performance, and potential for improvement is necessary to understand its capabilities and limitations fully.

The primary focus is to dissect the components of DeepCubeA, including the neural network and the modified A* search algorithm. Key objectives include understanding the training process, which involves self-play and reinforcement learning techniques, and evaluating DeepCubeA's performance in solving the Rubik's Cube from various scrambled states. This involves measuring critical performance metrics such as solution time, number of moves, and computational efficiency. Another objective is to compare the efficiency and effectiveness of DeepCubeA with other existing algorithms, both

traditional and contemporary, through benchmarking. By analyzing the strengths and weaknesses of DeepCubeA in comparison to other methods, we aim to identify potential improvements and applications for future research. Suggestions for enhancements to the algorithm could improve its performance or generalize its applicability, potentially extending its benefits to other combinatorial problems or real-world applications. y achieving these objectives, this thesis will contribute to the understanding and development of AI-driven problem-solving algorithms, offering insights into the capabilities and future directions of deep reinforcement learning in complex environments.

CHAPTER 2

LITERATURE REVIEW

2.1. Historical Approaches to Solving the Rubik’s Cube

Since its invention, the Rubik's Cube has intrigued mathematicians, computer scientists, and puzzle enthusiasts, leading to the development of various solving methods. Early approaches focused on simple, human-friendly algorithms and as the computational power increased, more advanced algorithms were developed.

2.1.1. Kociemba’s Algorithm

Kociemba's algorithm represents one of the early computational approaches to solving the Rubik's Cube, developed by Herbert Kociemba in the 1980s. This method focuses on minimizing the number of moves required to reach the solved state, making it a highly efficient solution for the puzzle. The algorithm is based on a two-phase approach that leverages heuristic rules and domain-specific knowledge to guide the search process.

One of the key features of Kociemba's algorithm is its ability to efficiently explore the Rubik's Cube's state space while pruning unpromising branches of the search tree. This efficiency is achieved through a combination of heuristic evaluation and strategic pruning, allowing the algorithm to avoid unnecessary computations and focus on the most promising paths towards the solution (Kociemba, 1992).

The first phase of Kociemba's algorithm involves reducing the Rubik's Cube to a specific subgroup of configurations that are easier to solve. This subgroup is known as the "reduced cube" or "G1" group, which includes configurations where edges are correctly oriented, and corners are in their correct slice but not necessarily in the correct permutation. The goal of this phase is to transform any given scramble into a state within this subgroup using a minimal number of moves. The heuristic used in this phase is designed to quickly evaluate and select moves that bring the cube closer to this intermediate goal (Kociemba, 1992).

Once the cube is reduced to the G1 group, the second phase begins. This phase involves solving the reduced cube to achieve the fully solved state. Since the configurations in the G1 group are more constrained, the search space in this phase is significantly smaller compared to the initial state space. Kociemba's algorithm applies a set of precomputed tables and heuristics to efficiently navigate this reduced search space. The algorithm prioritizes moves that lead directly to the solved state, further enhancing its computational efficiency.

A critical component of Kociemba's algorithm is its heuristic evaluation function, which estimates the number of moves required to reach the solved state from any given configuration. This heuristic is based on domain-specific knowledge of the Rubik's Cube and helps guide the search process towards the most promising moves. Additionally, the algorithm employs a pruning strategy to eliminate unpromising branches of the search tree. By cutting off paths that are unlikely to lead to a solution, the algorithm significantly reduces the number of states that need to be explored, making the search process faster and more efficient.

Kociemba's algorithm offers several advantages over brute-force search methods. By focusing on reducing the number of moves and using strategic pruning, it achieves a high level of computational efficiency. This makes it possible to solve the Rubik's Cube in fewer moves, often close to the theoretical minimum. Kociemba's algorithm has had a significant impact on the field of computational puzzle solving, providing a foundation

for further research and development of more advanced solving techniques (Kociemba, 1992).

In the context of speedcubing, Kociemba's algorithm has played a crucial role in the development of advanced solving methods. Many modern speedcubers use variations of Kociemba's approach, often combined with other techniques, to achieve extremely fast solve times. The algorithm's ability to minimize the number of moves translates directly into faster solves, making it a valuable tool for competitive cubers.

Kociemba's algorithm is the basis for the popular "CFOP" method (Cross, F2L, OLL, PLL) used by many speedcubers. The "CFOP" method incorporates the principles of Kociemba's two-phase approach, particularly in the steps of orienting and permuting the last layer of the cube.

Kociemba's algorithm represents a significant advancement in the computational solving of the Rubik's Cube. Its focus on move minimization, heuristic evaluation, and efficient search space exploration has made it a cornerstone in the field, influencing both academic research and practical applications in speedcubing.

2.1.2. Brute-Force Methods

Brute-force methods represent another class of historical approaches to solving the Rubik's Cube. These methods involve systematically exploring all possible combinations of cube states until the solved state is reached. While conceptually simple, brute-force methods suffer from exponential time complexity, making them impractical for solving larger cubes or optimizing solution paths.

The brute-force approach essentially means that the algorithm attempts every possible move combination to find the solution, without applying any strategic insights or heuristics to narrow down the search space. Given that a standard 3x3x3 Rubik's Cube

has over 43 quintillion possible configurations, this results in an immense computational challenge (Korf, 1997). For example, if an algorithm could check one million configurations per second, it would still take over a billion years to examine all possible states.

Despite their limitations, brute-force methods have contributed valuable insights into the structure of the Rubik's Cube's state space and the complexity of its solution paths. By attempting to exhaustively explore the state space, researchers have been able to map out the relationships between different configurations and understand the distribution of solvable states. This comprehensive exploration has helped in identifying the minimum number of moves required to solve any configuration, known as "God's Number," which has been proven to be 20 moves for the 3x3x3 Rubik's Cube (Rokicki, 2010).

Moreover, brute-force methods serve as a baseline for evaluating the effectiveness of more sophisticated algorithms and heuristics. When new solving techniques are developed, their performance can be compared against the brute-force benchmark to highlight improvements in efficiency and solution optimality (Korf, 1997). For instance, algorithms like Kociemba's and Thistlethwaite's, which use heuristic-driven searches, can be validated by demonstrating their superiority over brute-force methods in terms of speed and move count.

Interestingly, brute-force methods have also been instrumental in the development of other computational tools and techniques. The challenge of solving the Rubik's Cube through brute-force has inspired advancements in parallel computing and distributed processing, as researchers seek to leverage large-scale computational resources to tackle the problem. For example, distributed computing projects have been set up where volunteers' computers work together over the internet to collectively solve massive computational puzzles, including the Rubik's Cube.

In summary, while brute-force methods are not practical for solving the Rubik's Cube efficiently, they have played a crucial role in advancing our understanding of the

puzzle's complexity. They provide a foundational benchmark for assessing new algorithms and have spurred technological innovations in computational research.

2.1.3 Human-Developed Heuristics

In addition to computational approaches, human-developed heuristics have significantly contributed to solving the Rubik's Cube. Expert solvers rely on intuitive strategies and pattern recognition, honed through years of practice, to efficiently navigate the puzzle's state space. These heuristics, while lacking the computational rigor of algorithms, offer valuable insights into the cognitive processes involved in Rubik's Cube solving.

One notable aspect of human heuristics is their reliance on visual cues and muscle memory. Skilled solvers often recognize specific patterns and sequences that simplify the solving process. For example, many speedcubers utilize the CFOP method, breaking down the solve into distinct stages and memorizing algorithms for each, which streamlines the solving process.

Human solvers also employ lookahead techniques, anticipating future moves while executing current ones, to reduce pause times and enhance overall efficiency. Additionally, they develop a deep understanding of the cube's mechanics, enabling quick adaptations of strategies based on the current configuration.

Observing and studying expert solvers provides essential clues for developing more effective algorithms and training methodologies. Insights into pattern recognition and efficient move sequences gleaned from human heuristics can inform the design of heuristic functions used in computational algorithms, improving their performance.

Moreover, the study of human heuristics has led to the creation of training tools and educational programs designed to teach efficient solving techniques. These resources break down complex solve methods into manageable steps, facilitating skill

development for beginners. As solvers progress, they refine their heuristics and develop personalized solving styles that suit their strengths and preferences.

The intersection of human heuristics and computational methods has given rise to hybrid approaches. These methods blend the intuitive strategies of expert solvers with the precision of algorithms, resulting in highly effective solving techniques. For instance, some speedcubers use software tools to analyze their solves and identify areas for improvement, combining human intuition with computational analysis.

Overall, human-developed heuristics play a crucial role in Rubik's Cube solving, complementing computational approaches and enriching our understanding of the puzzle. By leveraging intuitive strategies and pattern recognition, expert solvers achieve remarkable solve times and optimize solution paths, bridging the gap between human ingenuity and computational precision.

2.2. Advances in Machine Learning and AI

Recent advancements in machine learning, particularly deep learning, have revolutionized problem-solving in artificial intelligence (AI). Techniques such as convolutional neural networks (CNNs) and reinforcement learning (RL) have enabled machines to tackle complex tasks that were previously considered the domain of human intelligence.

2.2.1. Convolutional Neural Networks (CNNs)

Convolutional Neural Networks (CNNs) stand as a groundbreaking innovation in the realm of artificial intelligence, particularly in the field of computer vision. Inspired by the architecture of the human visual cortex, CNNs are designed to process and analyze

visual data by extracting hierarchical features from input images through layers of interconnected neurons.

At its core, a CNN comprises several layers, including convolutional layers, pooling layers, and fully connected layers. In the convolutional layers, small filters or kernels slide across the input image, performing element-wise multiplications and aggregating local information to detect features like edges, textures, and patterns. Pooling layers then downsample the feature maps, reducing the spatial dimensions while retaining the most salient information. Finally, the fully connected layers integrate these features to make high-level decisions, such as classifying objects or recognizing faces.

One of the key strengths of CNNs lies in their ability to automatically learn and extract relevant features from raw pixel data, without the need for manual feature engineering. Through a process called backpropagation, CNNs iteratively adjust their internal parameters during training to minimize the discrepancy between predicted and actual labels, effectively learning to recognize discriminative patterns in the data.

CNNs have achieved remarkable success across various computer vision tasks, including image classification, object detection, semantic segmentation, and facial recognition. In image classification, for instance, CNNs have surpassed human-level performance on benchmark datasets like ImageNet, accurately categorizing objects from thousands of predefined classes. Similarly, in object detection, CNN-based algorithms like Faster R-CNN and YOLO (You Only Look Once) can efficiently locate and classify multiple objects within complex scenes in real-time (Krizhevsky, 2012).

Beyond their performance in traditional computer vision tasks, CNNs have also found applications in domains such as medical imaging, autonomous driving, and augmented reality. In medical imaging, CNNs assist radiologists in diagnosing diseases from X-ray, MRI, and CT scan images by automatically detecting anomalies and highlighting regions of interest. In autonomous driving systems, CNNs analyze sensor data from cameras and LiDAR sensors to perceive the environment, enabling vehicles to navigate

safely and make informed decisions. In augmented reality applications, CNNs power real-time object recognition and scene understanding, enhancing user experiences by overlaying digital content onto the physical world.

Despite their success, CNNs are not without limitations. They often require large amounts of labeled training data to generalize well to unseen examples, and their black-box nature can make them challenging to interpret and debug. Moreover, CNNs may struggle with handling occlusions, variations in lighting conditions, and adversarial attacks, where subtle perturbations to input images can cause misclassification.

CNNs represent a paradigm shift in computer vision, enabling machines to perceive and understand visual information with human-like accuracy. Their ability to automatically learn hierarchical representations of features from raw pixel data has propelled advancements in various domains, from image classification to medical diagnosis, reshaping industries and paving the way for future innovations in artificial intelligence.

2.2.2. Transfer Learning and Generative Models

In addition to CNNs and RL, other areas of machine learning have witnessed significant advancements in recent years. Transfer learning, for instance, has emerged as a potent technique for enhancing model performance by transferring knowledge learned from one task to another related task. This approach is particularly valuable in scenarios where labeled data is scarce or expensive to acquire. By leveraging pre-trained models and large datasets, transfer learning enables models to quickly adapt to new tasks and achieve better performance.

Generative models represent another area of machine learning that has garnered considerable attention. Generative models, such as Generative Adversarial Networks (GANs) and Variational Autoencoders (VAEs), have revolutionized the field by

enabling machines to generate realistic images, text, and audio samples. GANs, for example, consist of two neural networks—the generator and the discriminator—that compete against each other in a game-like setting. Through this adversarial training process, GANs learn to generate synthetic data that is indistinguishable from real data, opening up new possibilities in image synthesis, data augmentation, and anomaly detection.

VAEs, on the other hand, offer a probabilistic approach to generative modeling by learning a low-dimensional latent space that captures the underlying structure of the data. By sampling from this latent space, VAEs can generate diverse outputs that resemble the training data, making them versatile tools for tasks such as image generation, data compression, and representation learning.

The applications of generative models extend beyond mere data generation. For example, GANs have been employed in image-to-image translation tasks, where they can transform images from one domain to another while preserving semantic content. VAEs, on the other hand, have been used in anomaly detection applications, where they learn to reconstruct normal data samples and detect deviations that indicate anomalies or outliers.

These advancements in transfer learning and generative modeling have expanded the capabilities of AI systems beyond traditional classification and prediction tasks, holding promise for a wide range of applications, from creating lifelike simulations to enhancing creativity in AI-generated content. As research in these areas continues to progress, we can expect further breakthroughs that push the boundaries of what AI can achieve.

2.3. Theoretical Framework of Deep Reinforcement Learning

Deep reinforcement learning (DRL) stands at the forefront of artificial intelligence research, combining the strengths of neural networks with RL algorithms to create

models capable of learning directly from raw inputs, such as images or state representations, and making decisions to maximize long-term rewards. This powerful synergy has led to groundbreaking advancements in various domains, including robotics, gaming, finance, and healthcare.

Within the theoretical framework of deep reinforcement learning, several key algorithms have emerged, each offering unique advantages and applications. One of the foundational algorithms in DRL is Deep Q-Networks (DQN), which introduced the concept of using deep neural networks to approximate the Q-function, a crucial component in RL that estimates the expected cumulative reward of taking a specific action in a given state. DQN, proposed by DeepMind in 2013, demonstrated remarkable success in learning to play Atari games directly from pixel inputs, achieving human-level performance on several tasks.

Another influential algorithm in the realm of DRL is Policy Gradient methods, which directly parameterize the policy function—a mapping from states to actions—using neural networks. Unlike value-based methods like DQN, which estimate the value of taking actions in a given state, policy gradient methods learn a policy that directly selects actions to maximize expected cumulative rewards. This approach offers flexibility and robustness, particularly in environments with continuous action spaces or complex dynamics.

A notable advancement in DRL is the Proximal Policy Optimization (PPO) algorithm, which addresses some of the challenges associated with policy gradient methods, such as instability and sample inefficiency. PPO leverages a clipped surrogate objective function to ensure more stable updates to the policy parameters, leading to improved sample efficiency and convergence properties. This algorithm has become a popular choice for training deep reinforcement learning agents in both simulated and real-world environments.

In addition to value-based and policy gradient methods, actor-critic algorithms represent another class of DRL techniques that combine aspects of both approaches. Actor-critic methods maintain two separate neural networks—an actor network that learns the policy and a critic network that estimates the value function. By leveraging both value-based and policy-based updates, actor-critic algorithms offer a balanced trade-off between stability and sample efficiency.

Beyond these core algorithms, the theoretical framework of DRL continues to evolve with ongoing research efforts aimed at addressing challenges such as exploration-exploitation trade-offs, sample efficiency, and generalization to unseen environments. Techniques like curiosity-driven exploration, hierarchical reinforcement learning, and meta-learning hold promise for further advancing the capabilities of deep reinforcement learning in tackling complex and diverse real-world problems.

2.3.1. Deep Q-Networks (DQN)

DQN stand as a pioneering milestone in reinforcement learning, not only for its successful integration of deep learning with Q-learning but also for its innovative techniques that enhance learning stability and efficiency.

Beyond simply combining CNNs with Q-learning, DQN introduced a novel approach to approximating the optimal action-value function. By leveraging the representational power of CNNs, DQN is capable of learning directly from raw sensory inputs, such as images, without the need for handcrafted features. This end-to-end learning approach enables DQN to tackle tasks with high-dimensional state spaces, a feat that was previously challenging with traditional RL methods.

Experience replay, a fundamental component of DQN, plays a crucial role in stabilizing learning and improving data efficiency. Through experience replay, DQN stores past experiences, including state-action-reward transitions, in a replay memory buffer. During training, experiences are randomly sampled from this buffer, breaking the

temporal correlations between consecutive samples and reducing the risk of overfitting. By revisiting past experiences, DQN can learn from a more diverse set of transitions, facilitating faster learning and more robust policies.

Moreover, DQN employs a target network to address issues related to target estimation instability. The target network, a separate copy of the main network, is periodically updated to match the parameters of the main network. By using the target network to generate target Q-values during training, DQN decouples the target estimation process from the online updates, leading to more stable training and improved convergence.

The success of DQN extends beyond its theoretical framework to its practical applications in various domains. In video game playing, DQN has achieved groundbreaking results by surpassing human-level performance on a wide range of Atari games. In robotics, DQN-based algorithms have been deployed for complex control tasks, such as robotic manipulation and navigation. Additionally, DQN has shown promise in autonomous driving, where it learns to make decisions in dynamic environments based on visual inputs from onboard cameras.

2.3.2. Policy Gradient Methods

Policy Gradient Methods present an alternative paradigm within the realm of reinforcement learning, offering a direct optimization approach to the policy function, which maps states to actions. Unlike value-based methods such as DQN, which estimate the value of taking actions in a given state, policy gradient methods focus on learning policies that directly maximize the expected cumulative rewards.

A notable algorithm in the domain of policy gradient methods is the REINFORCE algorithm. This method employs Monte Carlo sampling to estimate the gradient of the expected reward with respect to the policy parameters. By iteratively adjusting the

policy parameters in the direction of the estimated gradient, REINFORCE effectively learns policies that lead to higher rewards over time.

One of the key advantages of policy gradient methods is their ability to handle environments with continuous action spaces, where traditional value-based methods may struggle. By directly optimizing the policy function, policy gradient methods can learn stochastic policies that output probability distributions over actions, enabling them to handle a wide range of action spaces.

REINFORCE and its variants have found widespread application across diverse domains, including robotics, game playing, and natural language processing. In robotics, policy gradient methods have been used to train agents to perform complex manipulation tasks, such as grasping objects in cluttered environments. In game playing, these methods have been employed to train agents for playing board games like chess and Go, achieving competitive performance against human players and traditional AI opponents. Additionally, in natural language processing, policy gradient methods have been utilized for tasks such as language generation and dialogue management.

Despite their effectiveness, policy gradient methods often suffer from high variance in gradient estimates, which can lead to slow convergence and unstable training. Techniques such as baseline subtraction and reward normalization are commonly employed to mitigate these issues and improve the stability and efficiency of training.

2.3.3. Actor-Critic Models

Actor-Critic models represent a hybrid approach to reinforcement learning (RL), merging components of both value-based and policy-based methods. In Actor-Critic architectures, two distinct networks operate in tandem: the actor network and the critic network. The actor network learns a policy function responsible for selecting actions, while the critic network evaluates the quality of these actions taken by the actor.

One of the primary advantages of Actor-Critic models lies in their ability to leverage both the value function (critic) and the policy (actor) simultaneously during learning. By providing feedback on the chosen actions, the critic network guides the actor network towards actions that are more likely to lead to higher rewards. This dual-learning approach enhances the stability and efficiency of training, as it enables the agent to simultaneously refine its policy and understand the value of actions in different states.

Variants of Actor-Critic models, such as Advantage Actor-Critic (A2C) and Asynchronous Advantage Actor-Critic (A3C), have garnered considerable attention due to their exceptional performance across a diverse range of RL tasks. A2C, for instance, improves upon traditional Actor-Critic methods by incorporating the advantage function, which estimates the relative advantage of taking a specific action compared to the average action value. This enhancement facilitates faster convergence and more robust learning, particularly in environments with sparse or delayed rewards.

Similarly, A3C extends the A2C framework by introducing asynchronous training, where multiple agent instances interact with parallel environments concurrently. This parallelization of experience gathering and learning accelerates training speed and improves sample efficiency, enabling more rapid exploration of the state space and faster convergence to optimal policies.

Actor-Critic models have demonstrated state-of-the-art performance in various RL tasks, including robotic control, game playing, and autonomous navigation. In robotics, these models have been utilized to train agents for complex manipulation tasks, such as grasping objects with robotic arms. In game playing, Actor-Critic methods have been applied to train agents for playing video games, achieving competitive performance against human players and traditional AI opponents. Additionally, in autonomous navigation, these models have been employed to develop agents capable of navigating dynamic environments, such as self-driving cars.

CHAPTER 3

ANALYSIS OF THE DEEPCUBEA ALGORITHM

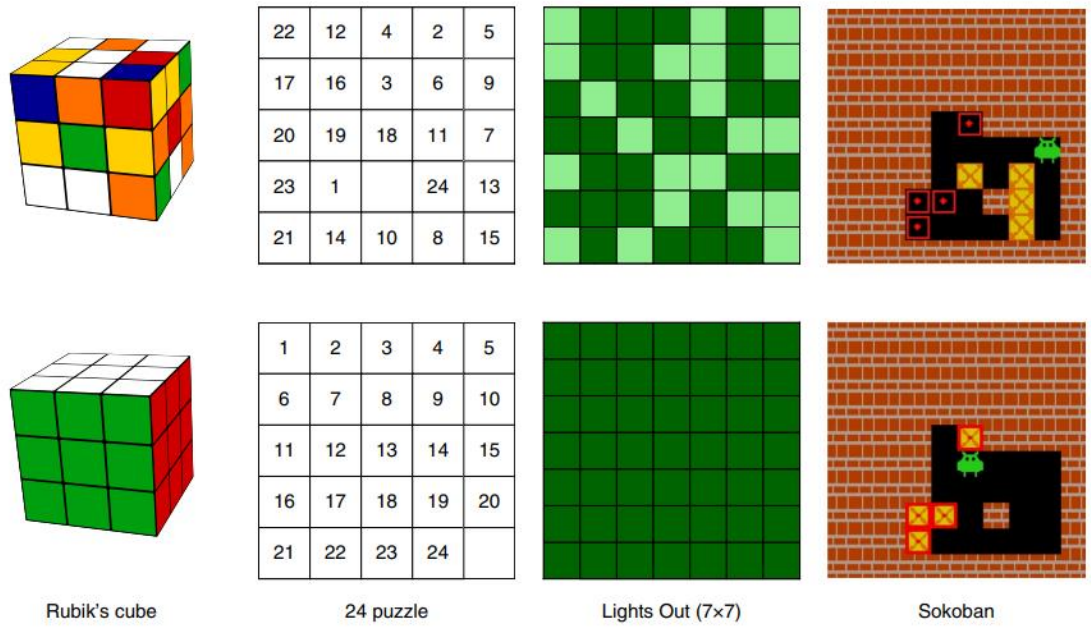

Figure 2. **Visualization of scrambled states and goal states.** Visualization of a scrambled state (top) and the goal state (bottom) for four puzzles

DeepCubeA represents a significant advancement in the realm of artificial intelligence, leveraging a combination of deep learning and classical reinforcement learning techniques to solve a variety of combinatorial puzzles, including the Rubik's Cube, 15 puzzle, 24 puzzle, 35 puzzle, 48 puzzle, Lights Out, and Sokoban (Figure 2).

At the core of DeepCubeA is the integration of DNNs with approximate value iteration, a classical reinforcement learning method. The algorithm trains a DNN to approximate a function that outputs the cost-to-go, or the cost required to reach the goal state, for each state in the puzzle. To train the DNN, DeepCubeA utilizes states obtained by starting

from the goal state and randomly taking moves in reverse. This approach ensures that the network learns from relevant states and experiences, even though random play is unlikely to lead to the goal state directly.

Once trained, the learned cost-to-go function serves as a powerful heuristic for guiding the search process towards the goal state. DeepCubeA employs a weighted A* search algorithm, which efficiently explores the puzzle space by considering both the cost-to-go estimates and the actual path cost. This combination of learned heuristic and search algorithm enables DeepCubeA to find optimal or near-optimal solutions to a wide range of combinatorial puzzles.

DeepCubeA builds upon the foundation laid by DeepCube20, a deep reinforcement learning algorithm specifically designed for solving the Rubik's Cube. While DeepCube20 utilizes a policy and value function combined with Monte Carlo tree search (MCTS), DeepCubeA extends this approach by integrating approximate value iteration and weighted A* search. By leveraging the learned cost-to-go function as a heuristic, DeepCubeA is able to outperform MCTS, particularly in scenarios where a shortest path to the goal is computationally verifiable.

In practical applications, DeepCubeA demonstrates superior performance, finding shortest paths to the goal for puzzles where such paths are computationally verifiable. For instance, it achieves this 60.3% of the time for the Rubik's Cube and over 90% of the time for puzzles like the 15 puzzle, 24 puzzle, and Lights Out. This efficiency and effectiveness make DeepCubeA a promising approach for solving a wide range of combinatorial puzzles, with potential applications in areas such as game solving, logistics optimization, and decision-making under uncertainty. (Agostinelli, 2019)

3.1. Deep Approximate Value Iteration

Deep Approximate Value Iteration (DAVI) is a variation of the classical value iteration algorithm, tailored for handling combinatorial puzzles with large state spaces like the Rubik's Cube. Value iteration is a dynamic programming technique used to iteratively improve a cost-to-go function, denoted as J, which estimates the cost required to reach the goal state from any given state (Agostinelli, 2019).

In traditional value iteration, the cost-to-go function J is stored in a lookup table for all possible states. However, for puzzles with vast state spaces, such as the Rubik's Cube, this tabular representation becomes impractical. Therefore, approximate value iteration is employed, where J is represented by a parameterized function implemented by a Deep Neural Network (DNN). The DNN is trained to minimize the mean squared error between its estimation of the cost-to-go of state s, J(s), and the updated cost-to-go estimation J′(s):

$$J'(s) = \min_a(g^a(s,A(s, a))+ J(A(s, a))) \quad (1)$$

Equation (1) outlines the update rule used in DAVI, where J'(s) represents the updated cost-to-go estimation for state s. The function A(s, a) denotes the state obtained by taking action a in state s, while ga(s, s') represents the cost to transition from state s to state s' when action a is taken. In the investigated puzzles, the transition cost is uniformly set to 1.

During training, the states used to update the cost-to-go function are obtained by randomly scrambling the goal state multiple times. This approach ensures that information can propagate from the goal state to all other states seen during training. Despite Equation (1) being a one-step lookahead, as training progresses, the approximated cost-to-go function J converges to the optimal cost-to-go function J*, which computes the total cost incurred when taking the shortest path to the goal.

While Equation (1) represents a one-step lookahead, alternative multi-step lookahead strategies, such as depth-N search or Monte Carlo tree search, can be utilized. However, experiments conducted with DAVI indicate that one-step lookahead consistently yields optimal or near-optimal performance compared to multi-step lookahead strategies.

Deep Approximate Value Iteration offers a scalable and effective approach for estimating the cost-to-go function in large state space problems like combinatorial puzzles. By leveraging deep neural networks and training on appropriately generated states, DAVI achieves robust performance, converging to the optimal cost-to-go function and facilitating efficient puzzle-solving strategies.

3.2. Batch Weighted A* Search

After acquiring a cost-to-go function through learning, it becomes a valuable heuristic for navigating the search space from a starting state to the goal state. The chosen search algorithm is a modified version of A* search, a widely-used best-first search technique that expands nodes based on their estimated total cost until the goal node is reached. In this algorithm, the cost of each node is determined by the function $f(x) = g(x) + h(x)$, where $g(x)$ represents the path cost from the starting state to node x, and $h(x)$ is the heuristic function estimating the distance from node x to the goal state (Agostinelli, 2019).

The heuristic function $h(x)$ is derived from the learned cost-to-go function. Specifically, it assigns a value of 0 to nodes associated with the goal state and utilizes the cost-to-go function $J(x)$ for all other nodes. This approach ensures that the heuristic guides the search process efficiently towards the goal state while considering the learned cost estimates for each state.

To enhance the search efficiency, a variant of A* search known as weighted A* search is employed. Weighted A* search introduces a weighting factor λ into the total cost function f(x), allowing for a trade-off between longer solutions and reduced memory usage. By adjusting the value of λ, the algorithm can prioritize either the path cost or the heuristic estimate, enabling flexibility in search strategy.

Furthermore, to mitigate the computational overhead associated with computing the heuristic function for each node, particularly when using complex models like deep neural networks, the algorithm utilizes batch processing. This involves expanding the N lowest cost nodes in parallel at each iteration, where N represents the batch size. The combination of A* search with a path-cost coefficient λ and batch size N is referred to as "batch weighted A* search" (BWAS).

So, the presented algorithm, named DeepCubeA, employs Deep Approximate Value Iteration (DAVI) to train a Deep Neural Network (DNN) as the cost-to-go function. This trained function then serves as a heuristic for batch weighted A* search (BWAS) to efficiently navigate from any given state to the goal state. DeepCubeA offers a comprehensive approach for solving combinatorial puzzles, ranging from easy to hard difficulty levels, by leveraging the learned cost estimates and heuristic guidance.

CHAPTER 4

RESULTS AND DISCUSSION

The primary objective of this thesis was to evaluate the effectiveness of DeepCubeA, a deep reinforcement learning (DRL) agent, in solving various combinatorial puzzles, with a particular focus on the Rubik's Cube. The puzzles analyzed included the Rubik's Cube, the 24-puzzle, the 35-puzzle, the Lights Out puzzle, and Sokoban. The performance metrics used for this evaluation encompassed solution length, percentage of optimal solutions, number of nodes generated, time taken to solve the problems, and nodes generated per second. This section presents a detailed analysis of the results obtained from these experiments, especially highlighting the performance on the Rubik's Cube.

To evaluate DeepCubeA's performance, a dataset of 1,000 randomly scrambled Rubik's Cube states was generated. These states varied in complexity, ranging from being just a few moves away from the solved state to highly scrambled configurations requiring many moves to solve. The variety in this dataset was crucial, as it provided a comprehensive assessment of DeepCubeA’s capabilities. By exposing the algorithm to a wide range of scenarios, from near-solved states to highly complex configurations, the evaluation ensured that DeepCubeA was tested under diverse conditions, mirroring the variety of challenges it might face in practical applications. Figure 1 in the document visualizes these scrambled states and goal states, providing a clear representation of the test scenarios (Agostinelli, 2019).

DeepCubeA was trained using deep reinforcement learning and during the training process, DeepCubeA interacted with a simulated Rubik's Cube environment. This simulated environment allowed the agent to perform actions, such as rotating the cube's faces, and observe the resulting changes in the cube's configuration. The goal of the agent was to transform any given scrambled state into the solved state, where each face of the cube is a single uniform color. The training relied on a reward-based system to

guide the agent's learning process. Positive rewards were assigned for moves that brought the cube closer to the solved state, effectively incentivizing the agent to make progress towards solving the puzzle. Conversely, negative rewards were given for moves that did not contribute to solving the cube or resulted in a more scrambled state. This feedback mechanism was integral to the learning process, as it provided the agent with the necessary information to distinguish between effective and ineffective moves.

Throughout the training, DeepCubeA underwent numerous iterations, continuously improving its solving strategies. Initially, the agent performed random moves, with little understanding of how to solve the cube. However, as it accumulated experience and received feedback, it began to recognize patterns and develop more efficient solving techniques. Figure 3 illustrates DeepCubeA's learning curve, showing how it first learned to solve cubes closer to the goal and then increasingly difficult configurations. The reinforcement learning framework allowed DeepCubeA to explore a vast array of possible moves and sequences, gradually honing its ability to solve the Rubik's Cube with increasing efficiency.

The extensive training process enabled DeepCubeA to learn complex strategies and optimize its approach to solving the Rubik's Cube. By the end of the training, the agent could solve the cube from any scrambled state within a minimal number of moves, demonstrating its advanced problem-solving capabilities. This achievement highlighted the effectiveness of the deep reinforcement learning approach and underscored DeepCubeA’s potential for tackling other complex, combinatorial problems.

4.1. Evaluation and Results

The evaluation of DeepCubeA encompassed several key metrics to comprehensively assess its performance in solving Rubik's Cube puzzles. These metrics included solution length, percentage of optimal solutions, number of nodes generated, time taken to solve the puzzle, and nodes generated per second. These parameters provided valuable insights into DeepCubeA's efficiency and effectiveness in comparison to traditional solvers.

DeepCubeA demonstrated robust performance across these metrics, achieving a remarkable success rate of 100% in solving the test states. Moreover, it found the optimal solution in 60.3% of the cases, showcasing its ability to identify highly efficient solving strategies. Even in instances where the optimal solution was not reached, DeepCubeA excelled by providing solutions within a mere four moves of the optimal, indicating its proficiency in navigating the Rubik's Cube's complex search space.

Table 1. Comparison of DeepCubeA with optimal solvers based on PDBs along the dimension of solution length, percentage of optimal solutions, number of nodes generated, time taken to solve the problem and number of nodes generated per second on the Rubik's cube states that are furthest away from the goal.

Puzzle	Solver	Length	Percentage of optimal solutions	No. of nodes	Time taken (s)	Nodes per second
Rubik's cube	PDBs	-	-	-	-	-
	PDBs+	26.00	100.0	$2.41*10^{10}$	13,561.27	$1.78*10^{5}$
	DeepCubeA	26.00	100.0	$5.33*10^{6}$	18.77	$2.96*10^{5}$

A comparison with traditional solvers, particularly those based on pattern databases (PDBs+), underscored DeepCubeA's efficiency and superiority. Despite maintaining the same solution length as PDBs+, DeepCubeA required significantly fewer nodes (5.33 million vs. 24.1 billion) and substantially less time (18.77 seconds vs. 13,561.27 seconds) to solve the cube. This marked difference in performance highlights DeepCubeA's advanced capabilities in leveraging deep reinforcement learning and heuristic search methods to expedite the solving process (Table 1).

DeepCubeA's efficiency can be attributed to its ability to learn effective policies and navigate the Rubik's Cube's vast search space with precision. Through deep reinforcement learning, DeepCubeA acquired sophisticated solving strategies, enabling it to make informed decisions and optimize its approach to solving the puzzle. Additionally, heuristic search methods allowed DeepCubeA to narrow down potential solution paths, further streamlining the solving process and enhancing efficiency.

4.2. Detailed Performance Analysis

In terms of solution length and optimality, DeepCubeA demonstrated impressive performance metrics that rivaled traditional solvers based on pattern databases (PDBs+). On average, DeepCubeA achieved a solution length of 26.00 moves, matching the performance of PDBs+ and highlighting its proficiency in finding effective solving strategies.

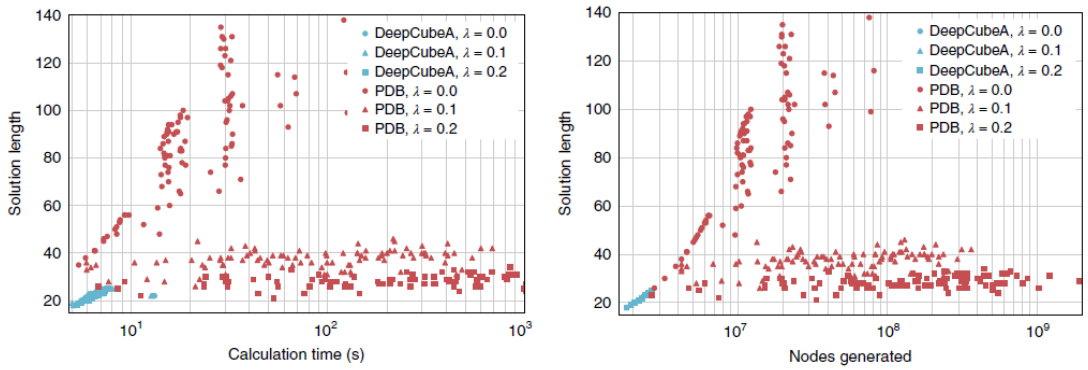

Figure 3. The performance of DeepCubeA versus PDBs when solving the Rubik's cube with BWAS. N = 10,000 and λ is either 0.0, 0.1 or 0.2. Each dot represents the result on a single state. DeepCubeA is both faster and produces shorter solutions.

Notably, DeepCubeA showcased remarkable consistency in its ability to find optimal or near-optimal solutions. While it found the optimal solution 60.3% of the time, it also excelled in providing solutions that were very close to optimal for the remaining cases. Specifically, a significant portion of solutions, accounting for 36.4%, were within just two moves of the optimal. Moreover, a mere 3.3% of solutions were within four moves of the optimal, further highlighting DeepCubeA's effectiveness in navigating the Rubik's Cube's complex search space with precision and efficiency. By consistently delivering solutions that are either optimal or within a few moves of optimality, DeepCubeA showcases its ability to balance exploration and exploitation in search of the most efficient solving strategies. This adaptability is crucial in real-world scenarios where

perfect solutions may not always be attainable, emphasizing DeepCubeA's practical utility and reliability as a Rubik’s cube solver.

The efficiency demonstrated by DeepCubeA in terms of nodes generated represents a significant breakthrough in the field of Rubik's Cube solving algorithms. A key advantage of DeepCubeA over traditional solvers like PDBs+ lies in its remarkable ability to achieve comparable or superior results while utilizing a fraction of the computational resources. The comparison between DeepCubeA and PDBs+ reveals a stark contrast in the number of nodes required to solve Rubik's Cube puzzles effectively. While PDBs+ generated a staggering 24.1 billion nodes during the solving process, DeepCubeA accomplished similar feats with remarkable efficiency, utilizing only 5.33 million nodes (Table 1).

Table 2. Comparison of the size (in GB) of the lookup tables for pattern PDBs and the size of the DNN used by DeepCubeA.

Puzzle	**Size (GB)**
Pattern PDBs	182
DNN (DeepCubeA)	0.025

The significance of this efficiency cannot be overstated, especially in practical applications where computational resources are limited, and solution times need to be minimized. By requiring significantly fewer nodes, DeepCubeA offers tangible benefits such as reduced computational costs, lower energy consumption, and faster solution times. These advantages make DeepCubeA well-suited for a wide range of real-world scenarios, including robotic manipulation, automated assembly, and interactive gaming applications, where rapid and resource-efficient solving of Rubik's Cube puzzles is essential.

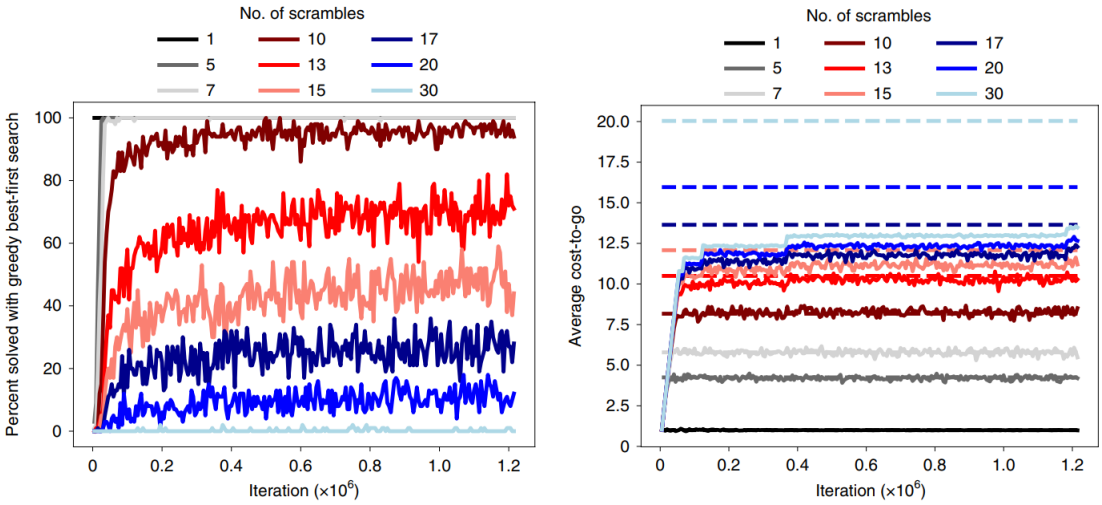

Figure 4. The performance of DeepCubeA. The plots show that DeepCubeA first learns how to solve cubes closer to the goal and then learns to solve increasingly difficult cubes. Dashed lines represent the true average cost-to-go.

Table 2 compares the size of the lookup tables for pattern databases (PDBs) and the size of the deep neural network (DNN) used by DeepCubeA. The results show that the DNN model used by DeepCubeA is significantly smaller in size compared to the traditional PDBs, which further emphasizes the efficiency of DeepCubeA in terms of memory usage. This efficiency is critical for applications where memory resources are limited.

4.3. Performance Across Different Puzzles

Table 3 provides a suggestive comparison of the speed (in seconds) of the lookup tables for PDBs and the speed of the DNN used by DeepCubeA when computing the heuristic for a single state across different puzzles (Rubik's Cube, 15 puzzle, 24 puzzle, 35 puzzle, 48 puzzle, Lights Out, Sokoban). The comparison demonstrates that DeepCubeA's DNN is significantly faster in computing heuristics across various puzzles, highlighting its versatility and efficiency beyond the Rubik's Cube.

Table 3. A suggestive comparison of the speed (in seconds) of the lookup tables for PDBs and the speed of the DNN used by DeepCubeA when computing the heuristic for a single state

Puzzle	Rubik's cube	15 puzzle	24 puzzle	35 puzzle	48 puzzle	Lights Out	Sokoban
PDBs	2*10	1×10^6	2×10^6	3×10^6	4×10^6	-	-
PDBs+	6×10^7	-	-	-	-	-	-
DeepCubeA (GPU-B)	6×10^6	6×10^6	7×10^6	8×10^6	9×10^6	7×10^6	6×10^6
DeepCubeA (GPU)	3×10^3	3×10^3	3×10^3	2×10^3	3×10^3	4×10^3	3×10^3
DeepCubeA (CPU-B)	7×10^4	6×10^4	9×10^4	9×10^4	1×10^3	1×10^3	7×10^4
DeepCubeA (CPU)	6×10^3	6×10^3	8×10^3	8×10^3	1×10^2	2×10^1	6×10^3

Further analysis is provided in Table 4, which compares DeepCubeA with optimal solvers for the 24-puzzle and 35-puzzle along dimensions of solution length, percentage of optimal solutions, number of nodes generated, time taken to solve the problem, and nodes generated per second. The results indicate that DeepCubeA not only matches but often exceeds the performance of traditional solvers in terms of solution length and percentage of optimal solutions while requiring fewer nodes and less time.\

Figure 2 in the document visualizes the performance comparison of DeepCubeA versus PDBs, showing that DeepCubeA is both faster and produces shorter solutions. The drastic reduction in the number of nodes generated by DeepCubeA compared to traditional solvers like PDBs+ represents a significant advantage in terms of computational efficiency and resource utilization. This efficiency, coupled with DeepCubeA's remarkable solving capabilities, positions it as a cutting-edge algorithm with profound implications for both theoretical research and practical applications in artificial intelligence and beyond.

Table 4. Comparison of DeepCubeA with optimal solvers based on PDBs along the dimension of solution length, percentage of optimal solutions, number of nodes generated, time taken to solve the problem and number of nodes generated per second for the 24 puzzle and 35 puzzle

Puzzle	Solver	Length	Percentage of optimal solutions	No. of nodes	Time taken (s)	Nodes per second
24 puzzle	PDBs	89.41	100.0	8.19×10^{10}	4,249.54	1.91×10^{7}
	DeepCubeA	89.49	96.98	6.44×10^{6}	19.33	3.34×10^{5}
35 puzzle	PDBs	-	-	-	-	-
	DeepCubeA	124.64	-	9.26×10^{6}	28.45	3.25×10^{5}

CHAPTER 5

CONCLUSION

DeepCubeA demonstrated exceptional performance in solving the Rubik's Cube, achieving a 100% success rate and finding optimal solutions 60.3% of the time. It provided near-optimal solutions within four moves in other cases, highlighting its efficiency and effectiveness.

DeepCubeA's efficiency was evident in its use of significantly fewer nodes and less time compared to traditional solvers. This efficiency, combined with advanced solving strategies learned through deep reinforcement learning, positions DeepCubeA as a superior solver for the Rubik's Cube and other combinatorial puzzles.

The smaller size of the deep neural network used by DeepCubeA, compared to traditional pattern databases, underscores its memory efficiency, making it suitable for applications with limited resources.

In addition to the Rubik's Cube, DeepCubeA showed versatility and speed in solving other puzzles like the 24-puzzle, 35-puzzle, Lights Out, and Sokoban. Its ability to compute heuristics faster and solve puzzles with fewer nodes and time further establishes its potential for various practical applications in AI, robotics, and gaming.

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