FYS4150 - Project 3

Vegard Rønning & Heine H. Ness & Sindre R. Bilden

University of Oslo

vegard.ronning@fys.uio.no ; h.h.ness@fys.uio.no ; s.r.bilden@fys.uio.no

github.com/sindrerb/FYS4150-Collaboration/tree/master/Doc/Project3

October 19, 2016

Abstract

I. Introduction

Earth has been orbitin the Sun for billions of years. Even with several hundred natural satellites in a complex scheme of orbits is our solar system a stable physical system. Although the system has been stable over a long time, is is convenient to simulate the system to understand how it will react in different situations. Simulations gives also an usefull source of information about the fundaments of our solar system. This projects will build a numerical solver based on the gravitatonal force between two massive objects in combination with Newton's second and third law of motion. The methods tested for the solver is Euler's method and the Verlet velocity method, both methods commonly used for solving differential equations. The solver will be controlled by solving simple systems and compared to analytical results beforde introduced to more advanced systems where analytical solutions are impossible to find. Further, relativistic corrections will be implemented in the simulator to reduce unwanted effects such as perihelion distortion. The physics and numerical methods used in the project are described in more detail in section II. How theese are combined into a simulator is described in section III, and the output form test and experiments are described in section IV. A summary of the most important findings are found in section V.

II. Methods

III. Implementation

Every planet in our solar system is affected by the gravitational force from the sun and other nearby planets. Where the gravitational force is

$$F_G = G\frac{M_\circ M_\bullet}{r^2} \tag{1}$$

where $M_\circ$ and $M_\bullet$ are the masses of two arbitrary objects. Decomposing the force into Cartesian coordinates, the components may be represented

$$F_x = -\frac{GM_\circ M_\bullet x}{r^3}, F_y = -\frac{GM_\circ M_\bullet y}{r^3}, F_z = -\frac{GM_\circ M_\bullet z}{r^3}$$

By Newton's second law the the relation between acceleration and position, we have the relation between the position of the planet and the total force component acting on the planet.

$$\frac{d^2x}{dt^2} = \frac{F_x}{M_\bullet}, \qquad \frac{d^2y}{dt^2} = \frac{F_y}{M_\bullet}, \qquad \frac{d^2z}{dt^2} = \frac{F_z}{M_\bullet}$$

Using the fact that Earth orbits the Sun in an almost circular motion [1], units of earth masses may be achieved by setting the centripetal force of the orbit equal to the gravitational force and solve for $GM_\odot$. Where $M_\odot$ is the mass of the Sun.

$$GM_\odot = 4\pi^2\frac{\mathrm{AU}^3}{\mathrm{Yr}^2} \tag{2}$$

Astronomical Units AU and years Yr are defined as the distance from Earth to the sun and the orbit time of Earth respectively. Equation 2 introduces Earth masses, Astronomical Units and Years as natural units for this system.

In total the system gets three sets of coupled differential equations per object in the system, which has to be solved to describe the system properly.

$$\frac{dx}{dt} = v_x \qquad \wedge \qquad \frac{dv_x}{dt} = \frac{F_x}{M_\bullet} \tag{3}$$

$$\frac{dy}{dt} = v_y \qquad \wedge \qquad \frac{dv_y}{dt} = \frac{F_y}{M_\bullet} \tag{4}$$

$$\frac{dz}{dt} = v_v \qquad \wedge \qquad \frac{dv_z}{dt} = \frac{F_z}{M_\bullet} \tag{5}$$

The system may be described with numerical methods by discretizing equations (3-5). Letting $t \rightarrow t_i = t_0 + ih, i \in \mathbb{N}$, $x \rightarrow x_i$ and $v \rightarrow v_i$ be the new discretized relations with the initial conditions $x(t_0) = x_0$ and $v(t_0) = v_0$ where x_0 and v_0 is known, and $h = \frac{t_{max}-t_0}{N}$. The equations may be solved by Euler's method resulting in an algorithm with an error $\mathcal{O}(h^2)$ at each calculation. Here $a_{k,i} = \frac{F_k(k_i)}{M_\bullet}$ where k represents an axis label.

```
x[i+1]=x[i]+h*xV[i]
y[i+1]=y[i]+h*yV[i]
z[i+1]=z[i]+h*zV[i]
xV[i+1]=xV[i]+h*xA[i]
yV[i+1]=yV[i]+h*yA[i]
zV[i+1]=zV[i]+h*zA[i]
```

Or with the more stable Verlet Vocity method reducing the error to $\mathcal{O}(h^3)$ for a calculation.

```
x[i+1]=x[i]+h*xV[i]+0.5*h*xA[i]
y[i+1]=y[i]+h*yV[i]+0.5*h*yA[i]
z[i+1]=z[i]+h*zV[i]+0.5*h*zA[i]
xV[i+1]=xV[i]+0.5*h*h*(xA[i+1]+xA[i])
yV[i+1]=yV[i]+0.5*h*h*(yA[i+1]+yA[i])
zV[i+1]=zV[i]+0.5*h*h*(zA[i+1]+zA[i])
```

These algorithms were implemented in a script that simulates the dynamics of the Sun and Earth.

All code and results are found in the GitHub repository

github.com/sindrerb/FYS4150-Collaboration/

IV. Results and discussion

Table 1: *Table showing the three lowest computed eigenvalues λ with N mesh points, compared to the exact $\lambda_0 = 3, \lambda_1 = 7$ and $\lambda_2 = 11$.*

N	λ_0	λ_1	λ_2
10	2.68672	6.11302	11.0574
50	2.98745	6.93692	10.8453
100	2.99687	6.98432	10.9617
200	2.99916	6.99610	10.9904
300	2.99961	6.99828	10.9958
400	2.99986	6.99903	10.9976
500	2.99993	6.99937	10.9986

V. Summary and Conclusion

References

[1] Sandra May. What is orbit? , 2010. [Online; accessed 04-October-2016].

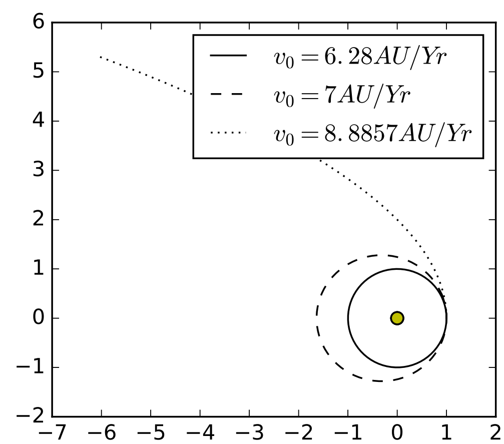

Figure 1: *Illustration of a set of earth initial velocities and their respective path.*

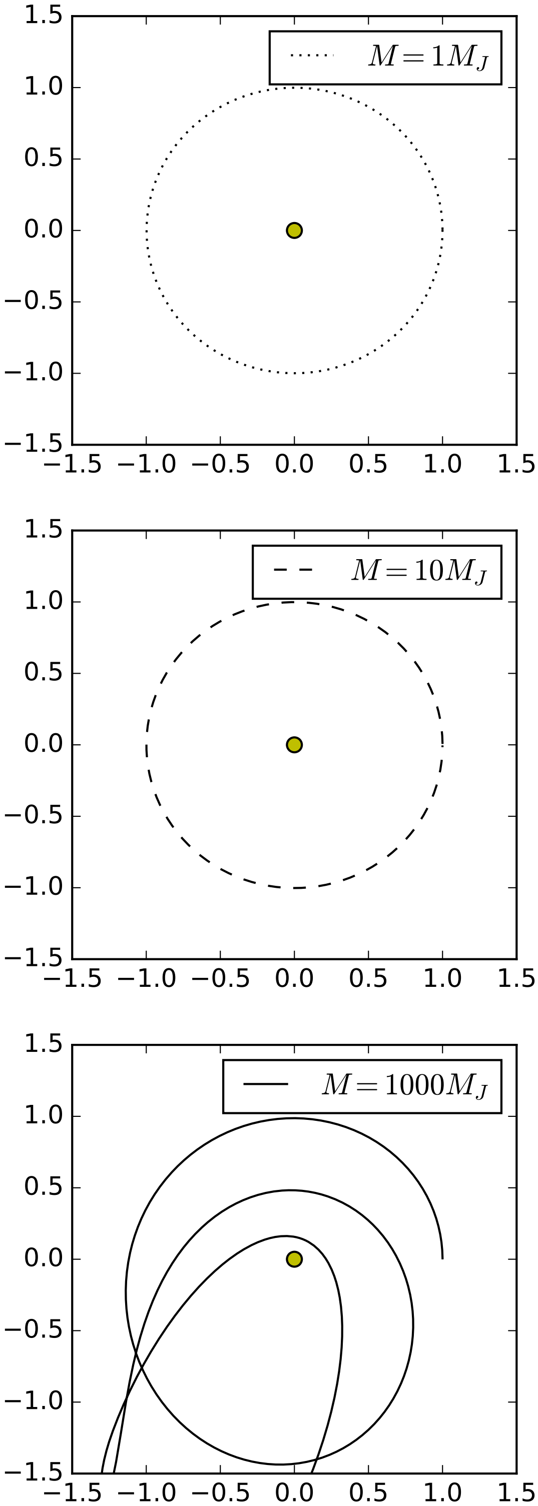

Figure 2: *Illustration of different paths of earth in a system with a stationary Sun at (0,0,0) and Jupiter (-5.2,0,0). M describes the Jupiter mass used in the simulation and* M_J *is the original Jupiter mass, length units given in AU*