

FYS4150 - Project 3

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github.com/sindrerb/FYS4150-Collaboration/tree/master/Doc/Project3

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Abstract

I. INTRODUCTION

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II. METHODS

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III. IMPLEMENTATION

Every planet in our solar system is affected by the gravitational force from the sun and other nearby planets. Where the gravitational force is

$$F_G = G \frac{M_o M_\bullet}{r^2} \quad (1)$$

where M_o and M_\bullet are the masses of two arbitrary objects. Decomposing the force into Cartesian coordinates, the components may be represented

$$F_x = -\frac{GM_o M_\bullet x}{r^3}, F_y = -\frac{GM_o M_\bullet y}{r^3}, F_z = -\frac{GM_o M_\bullet z}{r^3}$$

By Newton's second law the relation between acceleration and position, we have the relation between the position of the planet and the total force component acting on the planet.

$$\frac{d^2x}{dt^2} = \frac{F_x}{M_\bullet}, \quad \frac{d^2y}{dt^2} = \frac{F_y}{M_\bullet}, \quad \frac{d^2z}{dt^2} = \frac{F_z}{M_\bullet}$$

Using the fact that Earth orbits the Sun in an almost circular motion [1], units of earth masses may be achieved by setting the centripetal force of the orbit equal to the gravitational force and solve for GM_\odot . Where M_\odot is the mass of the Sun.

$$GM_\odot = 4\pi^2 \frac{\text{AU}^3}{\text{Yr}^2} \quad (2)$$

Astronomical Units AU and years Yr are defined as the distance from Earth to the sun and the orbit time of Earth respectively. Equation 2 introduces Earth masses, Astronomical Units and Years as natural units for this system.

In total the system gets three sets of coupled differential equations per object in the system, which has to be solved to describe the system properly.

$$\frac{dx}{dt} = v_x \quad \wedge \quad \frac{dv_x}{dt} = \frac{F_x}{M_\bullet} \quad (3)$$

$$\frac{dy}{dt} = v_y \quad \wedge \quad \frac{dv_y}{dt} = \frac{F_y}{M_\bullet} \quad (4)$$

$$\frac{dz}{dt} = v_z \quad \wedge \quad \frac{dv_z}{dt} = \frac{F_z}{M_\bullet} \quad (5)$$

The system may be described with numerical methods by discretizing equations (6-8). Letting $t \rightarrow t_i = t_0 + idt, i \in \mathbb{N}$, $x \rightarrow x_i + 1 = x_i + v_i dt$ and $v \rightarrow v_i + 1 = v_i + a(x_i) dt$ be the new discretized relations, the derivative may be approximated by Taylor expansions.

$$\frac{dx}{dt} = v_x \quad \wedge \quad \frac{dv_x}{dt} = \frac{F_x}{M_\bullet} \quad (6)$$

$$\frac{dy}{dt} = v_y \quad \wedge \quad \frac{dv_y}{dt} = \frac{F_y}{M_\bullet} \quad (7)$$

$$\frac{dz}{dt} = v_z \quad \wedge \quad \frac{dv_z}{dt} = \frac{F_z}{M_\bullet} \quad (8)$$

All code and results are found in the GitHub repository

github.com/sindrerb/FYS4150-Collaboration/

IV. RESULTS AND DISCUSSION

Table 1: Table showing the three lowest computed eigenvalues λ with N mesh points, compared to the exact $\lambda_0 = 3, \lambda_1 = 7$ and $\lambda_2 = 11$.

N	λ_0	λ_1	λ_2
10	2.68672	6.11302	11.0574
50	2.98745	6.93692	10.8453
100	2.99687	6.98432	10.9617
200	2.99916	6.99610	10.9904
300	2.99961	6.99828	10.9958
400	2.99986	6.99903	10.9976
500	2.99993	6.99937	10.9986

V. SUMMARY AND CONCLUSION

REFERENCES

- [1] Sandra May. What is orbit? , 2010. [Online; accessed 04-October-2016].

Figure 1: *Graph over the number of iterations for a given set of mesh points N . The solid red line is proportional to N^2 as a comparison.*