# FYS4150 - Project 3

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github.com/sindrerb/FYS4150-Collaboration/tree/master/Doc/Project3

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**Abstract**

## I. INTRODUCTION

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## II. METHODS

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## III. Implementation

Every planet in our solar system is affected by the gravitational force from the sun and other nearby planets. Where the gravitational force is

$$F_G = G\frac{M_\circ M_\bullet}{r^2} \tag{1}$$

where $M_\circ$ and $M_\bullet$ are the masses of two arbitrary objects. Decomposing the force into Cartesian coordinates, the components may be represented

$$F_x = -\frac{GM_\circ M_\bullet x}{r^3}, F_y = -\frac{GM_\circ M_\bullet y}{r^3}, F_z = -\frac{GM_\circ M_\bullet z}{r^3}$$

By Newton's second law the the relation between acceleration and position, we have the relation between the position of the planet and the total force component acting on the planet.

$$\frac{d^2x}{dt^2} = \frac{F_x}{M_\bullet}, \qquad \frac{d^2y}{dt^2} = \frac{F_y}{M_\bullet}, \qquad \frac{d^2z}{dt^2} = \frac{F_z}{M_\bullet}$$

Using the fact that Earth orbits the Sun in an almost circular motion [1], units of earth masses may be achieved by setting the centripetal force of the orbit equal to the gravitational force and solve for $GM_\odot$. Where $M_\odot$ is the mass of the Sun.

$$GM_\odot = 4\pi^2\frac{\mathrm{AU}^3}{\mathrm{Yr}^2} \tag{2}$$

Astronomical Units AU and years Yr are defined as the distance from Earth to the sun and the orbit time of Earth respectively. Equation 2 introduces Earth masses, Astronomical Units and Years as natural units for this system.

In total the system gets three sets of coupled differential equations per object in the system, which has to be solved to describe the system properly.

$$\frac{dx}{dt} = v_x \qquad \wedge \qquad \frac{dv_x}{dt} = \frac{F_x}{M_\bullet} \tag{3}$$

$$\frac{dy}{dt} = v_y \qquad \wedge \qquad \frac{dv_y}{dt} = \frac{F_y}{M_\bullet} \tag{4}$$

$$\frac{dz}{dt} = v_v \qquad \wedge \qquad \frac{dv_z}{dt} = \frac{F_z}{M_\bullet} \tag{5}$$

The system may be described with numerical methods by discretizing equations (3-5). Letting $t \rightarrow t_i = t_0 + ih, i \in \mathbb{N}$, $x \rightarrow x_i$ and $v \rightarrow v_i$ be the new discretized relations with the initial conditions $x(t_0) = x_0$ and $v(t_0) = v_0$ where $x_0$ and $v_0$ is known, and $h = \frac{t_{max}-t_0}{N}$. The equations may be solved by Euler's method resulting in an algorithm with an error $\mathcal{O}(h^2)$ at each calculation. Here $a_{k,i} = \frac{F_k(k_i)}{M_\bullet}$ where $k$ represents an axis label.

```
x[i+1]=x[i]+h*xV[i]
y[i+1]=y[i]+h*yV[i]
z[i+1]=z[i]+h*zV[i]
xV[i+1]=xV[i]+h*xA[i]
yV[i+1]=yV[i]+h*yA[i]
zV[i+1]=zV[i]+h*zA[i]
```

Or with the more stable Verlet Vocity method reducing the error to $\mathcal{O}(h^3)$ for a calculation.

```
x[i+1]=x[i]+h*xV[i]+0.5*h*xA[i]
y[i+1]=y[i]+h*yV[i]+0.5*h*yA[i]
z[i+1]=z[i]+h*zV[i]+0.5*h*zA[i]
xV[i+1]=xV[i]+0.5*h*h*(xA[i+1]+xA[i])
yV[i+1]=yV[i]+0.5*h*h*(yA[i+1]+yA[i])
zV[i+1]=zV[i]+0.5*h*h*(zA[i+1]+zA[i])
```

These algorithms were implemented in a script that simulates the dynamics of the Sun and Earth.

All code and results are found in the GitHub repository

github.com/sindrerb/FYS4150-Collaboration/

## IV. Results and discussion

**Table 1:** *Table showing the three lowest computed eigenvalues $\lambda$ with $N$ mesh points, compared to the exact $\lambda_0 = 3, \lambda_1 = 7$ and $\lambda_2 = 11$.*

| $N$ | $\lambda_0$ | $\lambda_1$ | $\lambda_2$ |
|---|---|---|---|
| 10 | 2.68672 | 6.11302 | 11.0574 |
| 50 | 2.98745 | 6.93692 | 10.8453 |
| 100 | 2.99687 | 6.98432 | 10.9617 |
| 200 | 2.99916 | 6.99610 | 10.9904 |
| 300 | 2.99961 | 6.99828 | 10.9958 |
| 400 | 2.99986 | 6.99903 | 10.9976 |
| 500 | 2.99993 | 6.99937 | 10.9986 |

## V. Summary and Conclusion

## References

[1] Sandra May. What is orbit? , 2010. [Online; accessed 04-October-2016].

**Figure 1:** *Graph over the number of iterations for a given set of mesh points $N$. The solid red line is proportional to $N^2$ as a comparison.*