Artificial Intelligence Methods

Assignment 3

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1 Exercise 1 - Umbrella World

The *umbrella world* can be described in terms of a Hidden Markov Model (HMM), consisting of a set of unobserved variables denoted $\mathbf{X_t}$, as well as a set of observable variables denoted $\mathbf{E_t}$ for a given time-slice $\mathbf{t}$.

$$X_t \in \{Rainy, \neg Rainy\}$$
$$E_t \in \{Umbrella, \neg Umbrella\}$$

The *dynamic model* $\mathbf{P(X_t \mid X_{t-1})}$, sometimes called the *transition model* describes the probability of transitioning to one state from a given state. This model can be represented as a matrix where the each row corresponds to a given state to depart from and each of the columns corresponds to a given state to arrive at. In this exercise the first row corresponds to the weather being rainy and the second row correspond to the weather being sunny. The same applies for the columns (For further explanation, see p. 482, figure 14.2 in *Artificial Intelligence. A Modern Approach 4th*).

$$\begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

The *observation model* $\mathbf{P(E_t \mid X_t)}$, sometimes called the *sensor model* describes the probability of of a event occurring given a state of the associated hidden variable $\mathbf{X_t}$. This model can also be represented as a matrix where each row corresponds to a given state and each column corresponds to each evidence. In this exercise the first row represents the weather being rainy and the second row represents the weather being sunny. The first and second column though represents the event *umbrella* and *not umbrella* respectively.

$$\begin{bmatrix} 0.9 & 0 \\ 0 & 0.2 \end{bmatrix}$$

There are three main assumptions in this model.
Firstly, we encounter a problem when defining the transition model. A state in the network $\mathbf{X_t}$ seems to be depending on all its parents so that the number of variables approaches infinity. We also then make a *Markov Assumption* which states that for a first-order Markov process, a current state only depends on its previous state. Thus, any given state is conditionally independent of its parents.

$$\mathbf{P(X_t \mid X_{0:t-1}) = P(X_t \mid X_{t-1})}$$

Secondly, we assume that the probability distribution for a transition to a state is stationary. That is the probability of one day being rainy is the same for all $\mathbf{t}$ steps. Therefore we do not need to explicitly make a new distribution for each step in the sequence of state transitions. Thus, $\mathbf{P}(\mathbf{X_t} \mid \mathbf{X_{t-1}}) = \mathbf{k}$ is a fixed number.

Finally, we again make a *Markov Assumption*, but this time related to the observation model. The assumption is that a given observation only depends on its associated state. For the formula underneath: notice that the indexing differs as the events are indexed starting with 1 while states are 0-indexed.

$$\mathbf{P}(\mathbf{E_t} \mid \mathbf{X_{0:t-1}}, \mathbf{E_{1:t-1}}) = \mathbf{P}(\mathbf{E_t} \mid \mathbf{X_t})$$

I would say the assumptions are reasonable regarding the Markov process itself. However the domain of the weather being rainy or sunny could certainly depend on the past week for instance. Still, the model can be somewhat accurate for specific places on earth where there seldom rains n-days in a row and so fourth. And since modeling this particular problem as a *first-order* Markov process makes the calculations easier to compute I would of course consider the assumptions reasonable;)

2 Exercise 2 - Filtering and *Forward* Operation

I have implemented *filtering* using the forward operation in the HMM for the *umbrella world*. The source code for this have been included in the delivery for this assignment along with this PDF-report. The implementation has been verified by calculating $\mathbf{P}(\mathbf{X2} \mid \mathbf{e_{1:2}})$, where $\mathbf{e_{1:2}}$ is the evidence that the umbrella was used both on day 1 and day 2. The desired result is that the probability of rain at day 2 (after the observations) is 0.883, and I also got this answer. Though in my implementation I had to compute an array of probabilities of both true and false due to the nature of matrix-vector multiplication, so the answer were taken as the first entry of the resulting array.

```
● sindrethronaes@Sindres-MacBook-Pro-2 O3 % python3.9 assignment3.py
  0.883357041251778 is the probability for rain at day 2 given evidence of umbrella at day 1 and 2.
```

I have calculated the probability of rain at day 5 given the sequence of observations given in the assignment. Underneath follows the output from the terminal when calculating this probability as well as all normalized forward messages in the process.

```
sindrethronaes@Sindres-MacBook-Pro-2 O3 % python3.9 assignment3.py
------------------------------------
0.8673388895754848 is the probability for rain at day 5 given the sequence of evidences.
------------------------------------
----------FORWARD MESSAGES----------
----------
[0.81818182 0.18181818]
----------
[0.88335704 0.11664296]
----------
[0.19066794 0.80933206]
----------
[0.730794 0.269206]
----------
[0.86733889 0.13266111]
```