Exercises for Chapter 0

0.1.1

Given $v \in [0°, 90°]$.

a) Show that $\sin v = \sin(180° - v)$.

b) Show that $\cos v = -\cos(180° - v)$

0.1.2

Find the area of $\triangle ABC$ when

a) $\angle A = 60°$, $AB = 5$ and $AC = 7$.

b) $\angle B = 18°$, $AB = 4$ and $BC = 3$. $\left(\sin 18° = \frac{\sqrt{5}-1}{4}\right)$

c) $\angle A = 75°$, $\angle B = 60°$, $AC = \sqrt{6}$ and $BC = \sqrt{3} + 1$

0.1.3

a) Prove the area theorem.

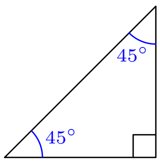
b) Prove the sine theorem.

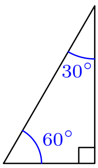
0.1.4

a) Show that $\cos 45° = \frac{\sqrt{2}}{2}$.

b) Show that $\sin 30° = \frac{1}{2}$.

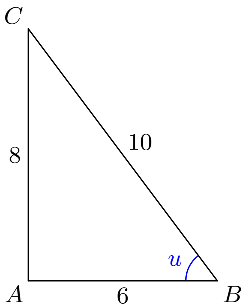
c) Show that $\cos 30° = \frac{\sqrt{3}}{2}$.





0.1.5 (1TV23D1)

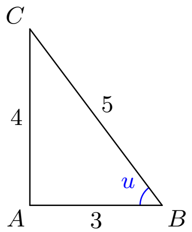
A right triangle has sides 6, 8, and 10. See the figure to the right.



Show that

$$(\sin u)^2 + (\cos u)^2 = 1$$

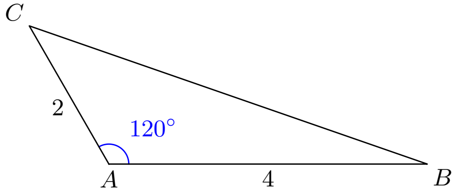
0.1.6 (1TH22D1)
Given the triangle to the right.
Show that



$$\frac{\sin u}{\cos u} = \tan u$$

0.1.7
Show that $\tan v = \dfrac{\sin v}{\cos v}$.

0.2.1 (1TH21D1)



Given the triangle above. Determine the length of side BC.

0.2.2
Given a triangle with sides a, b, and c and an inscribed circle with radius r. Explain why the area of the triangle is given as

$$\frac{1}{2}(a + b + c)r$$

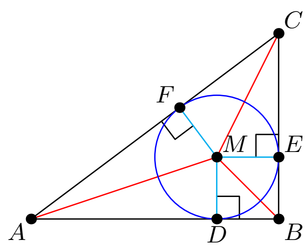
0.2.3

Let $a = BC$, $b = AC$, $c = AB$ and $DM = r$.

a) Show that $r = \frac{ac}{a+b+c}$.

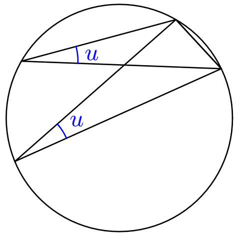
b) Show that $2r = a + c - b$.

c) Use the expressions from tasks a) and b) to find b^2 expressed by a and c. What is this formula called?



0.2.4

Explain why, from Rule **??**, it follows that two angles spanning the same arc are equal in size.



0.2.5

a) Show that Thales' theorem[1] follows from Rule **??**.

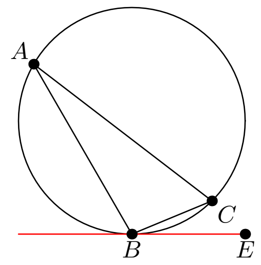
b) Given a right-angled triangle $\triangle ABC$ with hypotenuse AB. Which of $\Rightarrow$, $\Leftarrow$ and $\Longleftrightarrow$ should replace ??? below to describe the *inverse* case of Thales' theorem.

$$C = 90^\circ \quad ??? \quad AB \text{ is a diameter in the circumscribed circle of } \triangle ABC$$

[1] See MB.

0.2.6

The red line is tangent to the circle. Show that $\angle BAC = \angle EBC$.



Gruble 1

(1TH21D1)
A triangle has a perimeter of 12, and one side of the triangle is 2. Determine the area of the triangle.

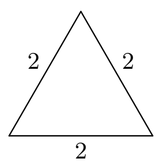
Gruble 2

(1TV21D1)
Sort the values in ascending order.

$$\sin 60^\circ \qquad \left(\frac{3}{4}\right)^{-1} \qquad \sin 160^\circ \qquad \lg 1$$

Gruble 3

An equilateral triangle has sides of length 2.



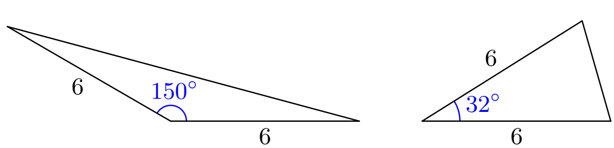
Use the triangle to show that

$$\cos 60^\circ = \frac{1}{2}$$

Gruble 4

Which of the two triangles has the larger area?

Remember to argue why your answer is correct.



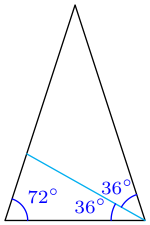
Gruble 5

Show that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Gruble 6

Show that $\sin 18^\circ = \frac{1}{4}(\sqrt{5} - 1)$. (Hint: See figure.)



Gruble 7

Prove the cosine theorem.

Gruble 8

Show that

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

It is sufficient to examine the case where v,$u \in [0°, 90°]$.

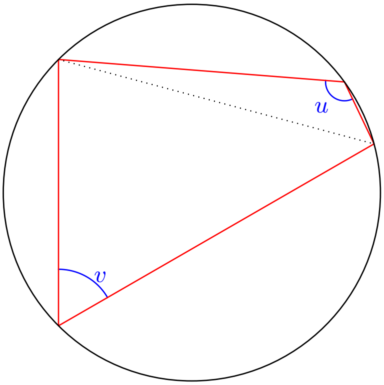
Gruble 9

Prove the converse case of Thales theorem (see Exercise 0.2.5).

Gruble 10

Show that

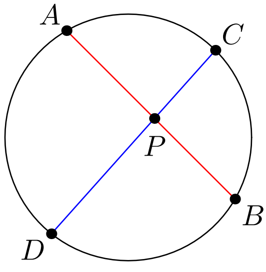
$$u = 180° - v$$



Gruble 11

Show that

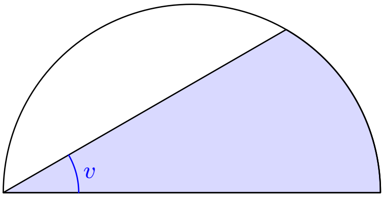
$$AP \cdot PB = DP \cdot PC$$



Note: This result is often called **the chord theorem**.

Gruble 12

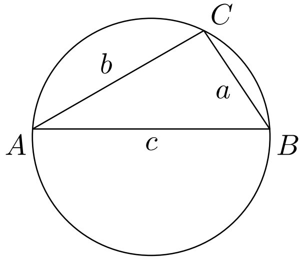
Let r be the radius of the semicircle. Express the area of the blue area in terms of v and r.



Gruble 13

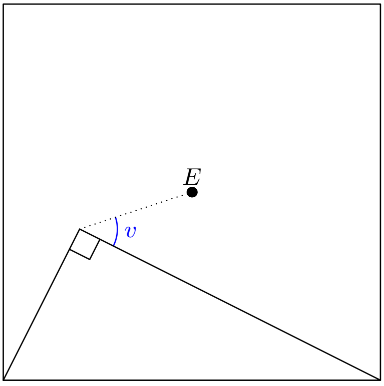
Let r be the radius of the circumscribed circle to $\triangle ABC$. Show that

$$r = \frac{abc}{4A_{\triangle ABC}}$$



Gruble 14

E is the midpoint of the square. Find the value of v.



Gruble 15

Show that

$$AB^2 = BC \cdot CD$$