## 0.1 Monotonic Properties

Most function values vary. Descriptions of how functions vary are called descriptions of the functions' **monotonic properties**.

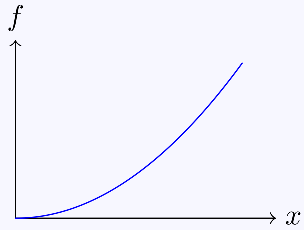
### 0.1 Increasing and Decreasing Functions

Given a function $f(x)$.

- $f$ is **increasing** on the interval $[a, b]$ if for all $x_1, x_2 \in [a, b]$ we have that

$$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2) \tag{1}$$

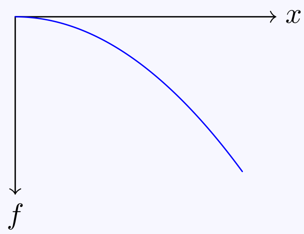
  If $f(x_1) \leq f(x_2)$ can be replaced with $f(x_1) < f(x_2)$, then $f$ is **strictly increasing** on the interval.




- $f$ is **decreasing** on the interval $[a, b]$ if for all $x_1, x_2 \in [a, b]$ we have that

$$x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2) \tag{2}$$

  If $f(x_1) \geq f(x_2)$ can be replaced with $f(x_1) > f(x_2)$, then $f$ is **strictly decreasing** on the interval.



## 0.2 Monotony Properties and The Derivative

Given $f(x)$ differentiable on the interval $[a, b]$.

- If $f' \geq 0$ for $x \in (a, b)$, then $f$ is increasing for $x \in [a, b]$
- If $f' \leq 0$ for $x \in (a, b)$, then $f$ is decreasing for $x \in [a, b]$

If respectively $\geq$ and $\leq$ can be replaced with $>$ and $<$, then $f$ is strictly increasing/decreasing.

### Example

Determine on which intervals $f$ is increasing/decreasing when

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 12x \qquad , \qquad x \in [0, 8]$$

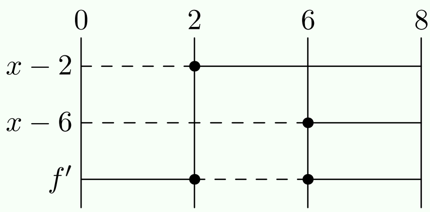
**Answer**

We have that

$$f'(x) = x^2 - 8x + 12$$

To clarify when $f'$ is positive, negative, or equal to 0, we do two things; we factorize the expression of $f'$ and draw a sign chart:

$$f'(x) = (x - 2)(x - 6)$$




The sign chart illustrates the following:

- The expression $x - 2$ is negative when $x \in [0, 2)$, equal to 0 when $x = 2$, and positive when $x \in (2, 8]$.
- The expression $x - 6$ is negative when $x \in [0, 8)$, equal to 0 when $x = 6$, and positive when $x \in (6, 8]$.
- Since $f' = (x - 2)(x - 6)$,

$$f' \geq 0 \text{ when } x \in (0, 2) \cup (6, 8)$$
$$f' = 0 \text{ when } x \in \{2, 6\}$$
$$f' \leq 0 \text{ when } x \in (2, 6)$$

This means that

$$f \text{ is increasing when } x \in [0,2] \cup [6,8]$$

$$f \text{ is decreasing when } x \in [2,6]$$

**0.3 Function domain on increasing/decreasing intervals**

Given a continuous function $f(x)$ strictly increasing/decreasing for $x \in [a,b]$. The domain of $f$ on this interval is then $[f(a), f(b))]$.