Exercises for Chapter 0

0.1.1

Use the definition of the derivative to show that for the function $f(x) = \frac{1}{x}$, $f'(x) = -\frac{1}{x^2}$.

0.1.2

Differentiate the expressions

a) $5x^3$ b) $-8x^6$ c) $\frac{3}{7}x^7$ d) $-x^{\frac{2}{3}}$ e) $x^{\frac{9}{7}}$

0.1.3

Differentiate the expressions

a) $2e^x$ b) $-30e^x$ c) $8\ln x$ d) $-4\ln x$

0.1.4

Explain how you can rewrite expressions in the form $\frac{1}{x^k}$ so that you can use (**??**) to differentiate the expressions.

0.1.5

Differentiate the expressions (Hint: See Exercise 0.1.4)

a) $\dfrac{5}{x^2}$ b) $\dfrac{7}{x^{10}}$ c) $-\dfrac{2}{9x^7}$ d) $\dfrac{3}{11x^{\frac{8}{5}}}$

0.1.6

Differentiate the functions

a) $g(x) = 3x^3 - 4x + \frac{1}{x}$ b) $f(x) = x^2 + \ln x$ c) $h(x) = \ln x + x^2 + 2$
d) $a(x) = x^2 + e^x$ e) $p(x) = e^x + \ln x$

0.1.7

Differentiate the expressions with respect to x.

a) $ax^2 + bx + c$ b) $7x^5 - 3ax + b$ c) $-9qx^7 + 3px^3 + b^3$

0.2.1

Differentiate the functions

a) $f(x) = x\sqrt{1-2x}$ b) $p(x) = 3xe^{2x}$ c) $h(x) = 3x^2 \ln x$
d) $k(x) = \sqrt{4x^2 - 5}$ e) $f(x) = x^3\sqrt{2x-1}$ f) $q(x) = \frac{x^3}{x^2-2}$
g) $f(x) = (x^2+2)^7$ h) $h(x) = \frac{x}{e^{x^2}}$

0.2.2

Solve **Gruble ??** using L'Hopital's rule.

Gruble 1

(R1V22D1)
A function f is given by

$$f(x) = \begin{cases} x^2 + 1 & , \quad x < 2 \\ x - t & , \quad x \geq 2 \end{cases}$$

a) Determine the number t so that f becomes a continuous function. Remember to justify your answer.

b) Decide if f is differentiable at $x = 2$ for the value of t you found in part a).

Gruble 2

Use the definition of the derivative to find the derivative function for $f_1(x) = x$, $f_2(x) = x^2$, $f_3(x) = x^3$.

Let $f_n(x) = x^n$ for $n \in \mathbb{N}$. Use what you have found to suggest an expression for $f_n'(x) = x^n$.

Gruble 3

(T1H23D1)

The function f is given by

$$f(x) = x^3 - 3x^2 - x + 4$$

Determine the equation of the tangent to f at the point $(1, f(1))$.

Gruble 4

Prove that (**??**) is valid.

Gruble 5

Prove that $(a^x)' = a^x \ln a$.

Gruble 6

a) Show that

> If the derivative function of $f(x)$ is continuous for $x \in [a, b]$, then f is continuous for $x \in (a, b)$.

Hint: Use (**??**).

b) Use the result from part a) to explain why all polynomial functions are continuous for all x.