Exercises for Chapter 0

0.2.1

Given the function $f(x) = a(b - x)(c - x)$. Find the extremum point of f expressed in terms of b and c.

0.2.2

Given a quadratic function $f(x)$. Find the expression for f when

a) f has zeros at $x = 3$ and $x = -4$, and an extremal value of 5.

b) f has zeros at $x = -1$ and $x = 10$, and an extremal value of -100.

c) f has a zero at $x = 8$, and a peak at $(10, 9)$.

0.3.1

Find any horizontal and vertical asymptotes, and any intersections with the y-axis, for the functions.

a) $f(x) = \frac{4}{x-2}$ b) $g(x) = \frac{7}{x+3}$ c) $h(x) = \frac{x^2}{x^2-16}$

d) $j(k) = \frac{k-3}{k-2}$ e) $p(s) = \frac{s-8}{s}$

0.4.1

Find the inverse function g to f, and confirm that $g(f) = x$.

a) $f(x) = 3x$ b) $f(x) = -9x + 2$ c) $f(x) = \frac{5}{2}x - 7$

d) $f(x) = \frac{3}{x-5}$ e) $\sqrt{x}$ f) $\sqrt[3]{x}$ g) $\sqrt[4]{x+9}$

0.4.2

Find the inverse function g to f, and confirm that $g(f) = x$.

a) $f(x) = e^x + 2$ b) $f(x) = \ln(x + 5)$ c) $f(x) = \frac{1}{\ln(x)}$

0.4.3

The function $f(x) = a(2 - x - x^3)$ has an inverse function $g(y)$, and $g(490) = -4$. Find the value of a.

0.5.1

Given a polynomial function with extremal points a and b, which are the only extremal points of the function on the interval $[a, b]$. Explain why the function is injective on this interval.

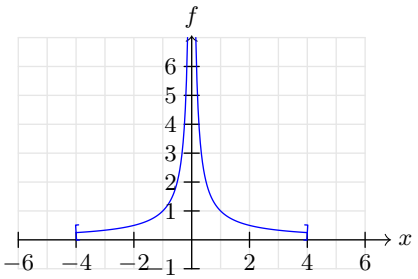
Gruble 1

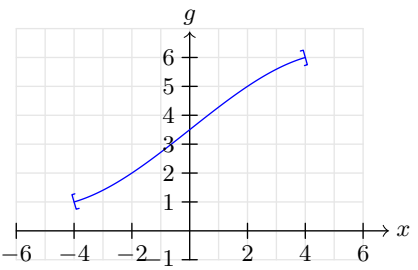
(R1V23D2)

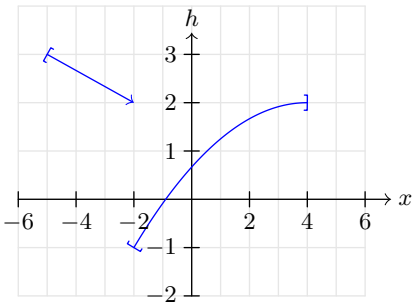
Below we have drawn the graphs of three functions f, g, h and k

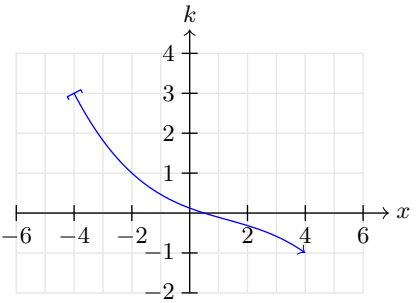
a) Determine and justify in each case whether the function has an inverse function.

b) Determine the domain of the inverse function in the cases where it exists.







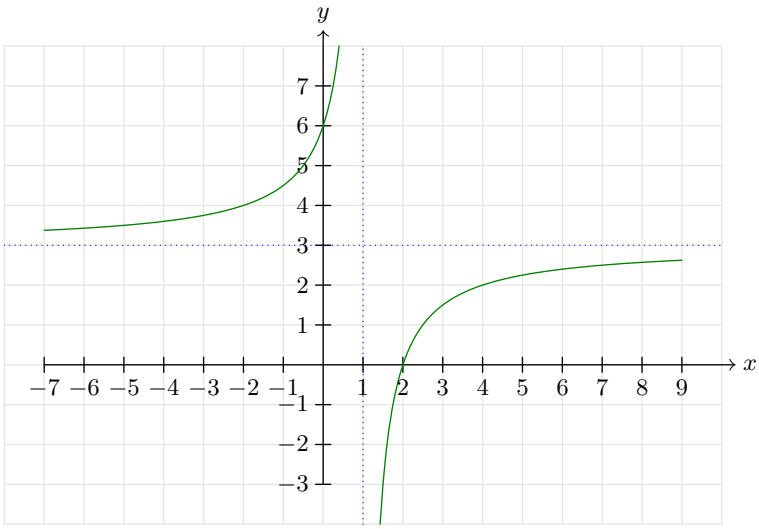


Gruble 2

(1TV23D1)

Below you see the graph of a rational function.

Determine $f(x)$. Remember to argue for why your answer is correct.



Gruble 3

Show that the function $f(x) = ax^2 + bx + c$ is convex if $a > 0$ and concave if $a < 0$.

Gruble 4

In the figure below, we have two parabolas. The green parabola is drawn by first reflecting the blue parabola across the horizontal line through the vertex, then shifting the parabolas so that they touch each other at a point B. A and C are located on the horizontal line through B, and D and E are along the same horizontal line.

Find the length of the segment AC, expressed in terms of s, when you know that $DE = 2s$.