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Alternating current-driven graphene superlattices: Kinks, dissipative solitons, dynamic chaotization

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The possibility of the solitary electromagnetic wave formation in graphene superlattice subjected to the electromagnetic radiation is discussed. The chaotic behavior of the electron subsystem in graphene superlattice is studied by Melnikov method. Dynamic chaos of electrons is shown to appear for certain intervals of frequencies of incident electromagnetic radiation. The frequency dependence of the radiation critical amplitude which determines the bound of chaos appearance is investigated. The values of radiation frequency at which the critical amplitude increases indefinitely were found. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4926944]

In this article, we discuss the possibility of the solitary electromagnetic (EM) wave formation in graphene superlattice (SL) subjected to the sinusoidal electromagnetic radiation. The results are valid if radiation frequency is much more than plasma frequency. But, there is another requirement for radiation frequency, which is related with chaotic behavior of nonlinear system such as superlattice. Dynamic chaos of electrons is shown to appear for certain intervals of frequencies of incident radiation. In Sections III–V, we investigate the frequency dependence of the radiation critical amplitude which determines the bound of chaos appearance. Also, we found the values of radiation frequency at which the critical amplitude increases indefinitely.

I. INTRODUCTION

Tunability of graphene electronic and optical properties by external fields opens new opportunities in technological applications of material which are of particular importance.[1–6] So the effect of EM fields on the properties of the graphene structures is of high interest among the researchers now.[7–11] The dynamical modification of the graphene band structure under the high-frequency (HF) EM radiation was studied in Refs. 12–14. The quasi-classical theory of nonlinear EM response of graphene was developed in Refs. 9, 15, and 16, where the possible applications of graphene structures for generation of terahertz (THz) radiation were discussed.

Presently, among the different graphene structures, the special attention is paid to graphene superlattises (GSL).[17–27] Intensive investigations of electric and optical properties of GSL are also explained by its different possible experimental and technological applications.[6,9,28–31] Structures with SL are the suitable materials for generating of THz EM radiation[32–34] and for formation of solitary EM waves.[35,36] In turn, the study of solitary EM waves in SL is stimulated by possible applications in information processing, data transmission, and data storage.[37–39] However, the strong damping of solitary waves in semiconductor SL is an obstacle to the possibility of their practical use for the information transferring over the long distances. It explains the search of ways to amplify and stabilize the solitary EM waves.[40–42]

In Ref. 42, the HF EM radiation was supposed to use for stabilization of EM soliton shape. HF field has a significant effect on the dynamics of the electron subsystem. In particular, this field can lead to such effects as dynamic modification of SL electron spectrum,[12–14,42,43] and dynamic chaos in SL.[44–46] In Ref. 36, to observe the solitary EM waves (2π-pulses), the GSL was proposed to use. Results obtained in Refs. 14, 42, and 43 are valid if frequency of EM radiation ω is much more than plasma frequency. However, there is another requirement for frequency ω, which is related with chaotic behavior of nonlinear systems.[44–51] Such phenomenon should be taken into account in the process of stabilization of the nonlinear and solitary EM waves. The chaotic dynamics of the charge carriers induced by the alternate current in GSL with dissipation was suggested in Refs. 45 and 46.

Below, we investigate the possibility of dynamic chaotization in GSL subjected to the EM radiation. We find the conditions for amplitude and frequency of such radiation when the chaotic behavior of electrons takes place. The D'Alembert equation in this case differs from that of Refs. 45 and 46; therefore, the results obtained below are new. Moreover, unlike the SL based on AlGaAs/GaAs,[35] the non-additivity of GSL electron spectrum allows the possibility of the dynamic chaotization in the case when vector potential of incident EM wave $\mathbf{A}^{ext}$ is perpendicular to the GSL axis.

II. EFFECT OF HF ELECTRIC FIELD ON THE SHAPE OF SOLITARY EM WAVE IN GSL: BRIEF REVIEW

One of methods of GSL fabrication is based on the spatial modulation of the gap induced in graphene by substrate consisting of periodically alternating layers of any two crystals

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along the certain direction (for instance, SiO_2/h-BN (Ref. 17) or SiO_2/SiC (Ref. 21)). As shown in Refs. 21–23, the electron spectrum of such structure can be written in the form

$$\varepsilon(\mathbf{p}) = \sqrt{\Delta^2 + p_x^2 \upsilon_F^2 + \Delta_1^2 (1 - \cos p_y d)}, \tag{1}$$

where $d = 2\times 10^{-6}$ cm is GSL period, $\upsilon_F = 10^8$ cm/s is the velocity on the Fermi surface, xy is graphene plane, Oy is the layers interleaving direction (GSL axis), and parameters Δ, Δ_1 are set during the process of obtaining of the GSL and are determined by the transparency of barriers formed in graphene.

GSL is suggested to be in the field of two EM waves. One of these waves is the sinusoidal HF radiation incident upon the graphene plane. Vector potential of this wave is: $\mathbf{A}^{\text{ext}} = (0, -(E_0/\omega)\sin\omega t, 0)$, where E_0 is the amplitude of electric field of HF wave. Another wave is the solitary EM wave which propagates along the axis Ox. It turns out that the shape of resulting solitary wave depends on the amplitude of HF radiation.[14] Such feature is absent in semiconductor SL.[42] In Ref. 14, the shape of resulting solitary EM wave was determined by using the renormalized d'Alembert equation which was obtained by averaging over the HF electric field period. The latter is valid if the frequency of HF field ω is much more than plasma frequency ω_{pl}.

According to Ref. 14, the type of these waves in GSL exposed to the HF electric field depends on the magnitude of the parameter $h = 4\Delta_1^2 G(a)/\omega^2 D(a)$, where[14]

$$D(a) = J_0(a) + \frac{a^2}{8}\sum_{k=0}^{\infty}\frac{(-1)^k}{(k+1)^2}\left[J_k\left(\frac{a}{2}\right) + J_{k+2}\left(\frac{a}{2}\right)\right]^2,$$

$$G(a) = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}J_k^2(a)}{k^2},$$

$J_k(a)$ is the Bessel function of integer order, and $a = edE_0/\omega$ is the dimensionless amplitude of HF EM wave. If amplitude a is such that $|h| < 1$, then the solitary wave is the 2π-pulse. If the $h > 1$, then two types of EM pulses are possible in GSL: the 2π-pulse and 0π-pulse. In the case when $D(a) = 0$, π-pulse is formed in the GSL. When $h < -1$, then other two types of EM pulses become feasible in GSL: $2\varphi_0$-pulse and $2(\pi - \varphi_0)$-pulse, where $\varphi_0 = \arccos(-h^{-1})$. Thus, the pulse area (topological charge multiplied by 2π) depends on the HF field amplitude as a consequence.

In Ref. 41, the automodel solution of renormalized d'Alembert equation was shown to exist for certain intervals of the amplitude a. This situation corresponded to the amplification of solitary waves which was explained by the fact that GSL became the medium with population inversion. The amplification of EM pulse takes place until the energy taken away from the population-inverted medium by solitary wave per unit time is equal to the energy lost per unit time due to dissipation. Dissipation, which is obstacle to the amplification of EM pulse, is due both to scattering of charge carriers by inhomogeneities of GSL and with absorbtion by interminiband transitions. To take into account the dissipation in GSL which is related with inter-mini-band transitions of charge carriers, the current density j_y^{d} induced by such transitions was added in the right side of the d'Alembert equation.

Current density j_y^{d} was found with the help of the next considerations. In the case of weak dissipation, EM energy dissipated by the unit of surface square per unit of time ($j_y^{\text{d}}E_y$) is proportional to the energy flux density of the EM wave ($E_y^2/4\pi$): $j_y^{\text{d}}E_y = \alpha E_y^2/4\pi$. Here, $E_y = -(\partial A_y/\partial t)$ is intensity of the electric field of the wave, and α is the coefficient of inter-mini-band absorption of EM radiation. Hence, $j_y^{\text{d}} = -\sigma\partial A_y/\partial t$, where $\sigma = \alpha/4\pi$ is the constant.[53]

Periodicity of electron velocity in the narrow conduction band is known to be a decisive factor in formation of solitons in SL.[35] The presence of HF radiation leads not only to a dynamical narrowing of the miniband as in SL based on AlGaAs/GaAs,[42] but also changes the structure of Brillouin zone so that Bloch oscillations of electrons becomes anharmonic. As a consequence, in particular, HF radiation affects the area of solitons in GSL. If the amplitude of HF field a is the zero of function $D(a)$, then the period of reciprocal lattices decreases in two times in comparison with the case of absence of HF radiation. In such situation, the π-pulse propagates in ideal GSL instead of 2π-pulse. A similar effect should be expected in the GSL with dissipation.

In Ref. 41, the expression which describes dissipative soliton was found. Such localized formation as it propagates removes energy in GSL with population inversion and dissipates it as a result of losses on the interminiband transitions.[54] Unlike Ref. 42, the area of dissipative soliton in GSL can differ from π and be regulated by changing of amplitude of HF radiation.[41] If $-1 < h^{-1} < 1$, then propagation of both kink and antikink is possible in GSL. In this case, the areas of EM pulses depend on the amplitude of HF field a and are equal $\pm\arccos(-h^{-1})$.

One of the manifestations of existence of the dissipative soliton in GSL can be the photon drag effect, i.e., the appearance of the current along the direction of the wave propagation.[35] The charge dragged by dissipative soliton in GSL was found in Ref. 41 to be $Q \sim \mp Q_0 G(1 \pm h^{-1})^2$. Here, the upper sign is chosen for antikink and the lower sign is chosen for kink, $Q_0 = en_0\upsilon_F^2\Delta_1^4/\sigma\omega^2\Delta^2$ (n_0 is the concentration of charge carriers).

III. VECTOR POTENTIAL OF EXTERNAL EM RADIATION IS PARALLEL TO THE GSL AXIS

The results described above and in Refs. 14 and 41 are valid if frequency of HF radiation ω is much more than plasma frequency. However, there is another requirement for frequency ω, which is related with chaotic behavior of electron subsystem of SL.[44,45] Further, we investigate the conditions for amplitude a and frequency ω of HF radiation when the dynamic chaotization of the electronic subsystem in ac-driven GSL takes place.

GSL is suggested to be obtained by graphene deposited on a banded substrate formed by periodically alternating layers of any two crystals along the axis Oy. Vector potential of nonlinear EM wave propagating in GSL with miniband (1) along the axis Ox is equal $\mathbf{A} = (0, A, 0)$. Besides, GSL is supposed to be subjected to the external sinusoidal EM radiation with potential $\mathbf{A}^{\text{ext}}$. Current density arising through the GSL axis is $j_y = j_y^{\text{d}} + j_y^{\text{f}}$. Here, the term $j_y^{\text{d}} = -\sigma \partial A_y/\partial t$ is the current density taking into account dissipation processes, where σ is phenomenological constant.[53] Dissipation is due to both scattering of charge carriers by inhomogeneities of SL and absorption by interminiband transitions. The term j_y^{f} is the current density induced by the specified EM fields:[41]

$$j_y^{\text{f}} = -e\sum_{\mathbf{p}} V_y(\mathbf{p} + e\mathbf{A} + e\mathbf{A}^{\text{ext}})f_0(\mathbf{p}), \tag{2}$$

where $f_0(\mathbf{p})$ is the equilibrium state function and $V_y = \partial\varepsilon/\partial p_y$ is the velocity of charge carriers along the GSL axis. The electron gas is assumed to be nondegenerate, so the function $f_0(\mathbf{p})$ is the Boltzmann state function. Also electron average free path is assumed to be much less than radiation wavelength. This allows us to neglect the dependence of the intensities of EM fields and state function on the coordinates. In this section, vector potential $\mathbf{A}^{\text{ext}}$ of sinusoidal EM radiation is supposed to be parallel to the GSL axis. So we have: $\mathbf{A}^{\text{ext}} = (0, -(E_0/\omega)\sin\omega t, 0)$. At low temperatures ($T \ll \Delta,\ \Delta_1$), summation over the momenta in (2) leads to the next result:

$$j_y^{\text{f}} = -\frac{\omega_{\text{pl}}^2 \sin(\varphi - a\sin\omega t)}{4\pi e d\sqrt{1 - 4\lambda\cos(\varphi - a\sin\omega t)}}. \tag{3}$$

Here, we define: $\varphi = edA$, $a = edE_0/\omega$, $\lambda = \Delta_1^2/4(\Delta^2 + \Delta_1^2)$, $\omega_{\text{pl}} = \sqrt{2\pi n_0 e^2 d^2 \Delta_1^2/a_0\sqrt{\Delta^2 + \Delta_1^2}}$ is the plasma frequency, n_0 is the surface concentration of charge carriers, and a_0 is graphene layer width. The resulting expression of current density j_y is substituted into D'Alembert equation: $\partial_t^2\varphi - \partial_x^2\varphi = 4\pi j_y$. Using the new argument: $\xi = (x - ut)/L_0$, where u is the nonlinear EM wave velocity, $L_0 = \sqrt{1 - u^2}/\omega_{\text{pl}}$, we derive for weak external radiation ($a \ll 1$)

$$\begin{aligned} -\frac{\mathrm{d}^2\varphi}{\mathrm{d}\xi^2} - 2\mu\frac{\mathrm{d}\varphi}{\mathrm{d}\xi} + \frac{\sin\varphi}{\sqrt{1 - 4\lambda\cos\varphi}} \\ = -aQ_{//}(\lambda, \varphi)\sin[\nu(\xi - \xi_x)]. \end{aligned} \tag{4}$$

Here, $\mu = 2\pi u\sigma/\omega_{\text{pl}}\sqrt{1 - u^2}$, $\nu = \omega L_0/u$, $\xi_x = x/L_0$,

$$Q_{//}(\lambda, \varphi) = \frac{\cos\varphi - 2\lambda(1 + \cos^2\varphi)}{(1 - 4\lambda\cos\varphi)^{3/2}}. \tag{5}$$

To find out the values of parameters a and ν when the chaos in the electron subsystem occurs, we use Melnikov method.[44,47–49] Thus, Melnikov function is written as following:

$$\begin{aligned} M_{//}(\xi_1) = \int_{-\infty}^{+\infty} \chi^{\text{s}}(-2\mu\chi^{\text{s}} + aQ_{//}(\lambda, \varphi^{\text{s}}) \\ \times \sin[\nu(\xi + \xi_1 - \xi_x)])\mathrm{d}\xi, \end{aligned} \tag{6}$$

where $\varphi^{\text{s}}(\xi)$ and $\chi^{\text{s}}(\xi) \equiv \mathrm{d}\varphi^{\text{s}}/\mathrm{d}\xi$ are the separatrix solutions of (4) in the absence of both dissipation ($\mu = 0$) and external radiation ($a = 0$). In this case, the solution of (4) is[36]

$$\xi = \int_{\pi}^{\varphi^{\text{s}}(\xi)} \frac{\mathrm{d}z}{g(\lambda, z)}, \quad \chi^{\text{s}} = g(\lambda, \varphi^{\text{s}}), \tag{7}$$

where

$$g(\lambda, z) = \sqrt{\frac{1}{\lambda}\left(\sqrt{1 - 4\lambda\cos z} - \sqrt{1 - 4\lambda}\right)}.$$

After substitution of (7) into (6) and some transformations, we have

$$M_{//}(\xi_0) = -2\mu G(\lambda) + aJ_{//}(\lambda, \nu)\sin\nu\xi_0. \tag{8}$$

Here, $\xi_0 = \xi_1 - \xi_x$, and we define the next integrals

$$\begin{aligned} G(\lambda) &= \int_0^{2\pi} g(\lambda, z)\mathrm{d}z, \\ J_{//}(\lambda, \nu) &= \int_0^{2\pi} Q_{//}(\lambda, z)\cos\left(\nu\int_{\pi}^{z}\frac{\mathrm{d}y}{g(\lambda, y)}\right)\mathrm{d}z. \end{aligned} \tag{9}$$

If Melnikov function (8) changes sign with the variable ξ_0 changing, then the stable manifold and unstable manifold on the plane $(\chi,\ \varphi)$ intersect ($\chi \equiv \mathrm{d}\varphi/\mathrm{d}\xi$). This leads to the appearance of chaotic dynamics in the suggested system.[44,47–49] Thus, in order for chaotic dynamics to be present, it is necessary that the values of amplitudes a and frequencies ν of external EM radiation satisfy the next criterion

$$\frac{2\mu G(\lambda)}{a|J_{//}(\lambda, \nu)|} < 1. \tag{10}$$

Using the inequality (10), we find the critical value of amplitude of external EM radiation above which would be a chaos

$$a_{\text{cr}} = \frac{2\mu G(\lambda)}{|J_{//}(\lambda, \nu)|}. \tag{11}$$

The dependence of critical amplitude a_{cr} on the frequency ν plotted numerically with the formula (11) for the values of parameters[21,22,49,53] $\mu = 10^{-3}$, $\Delta = 0.03$ eV, $\Delta_1 = 0.013$ eV is shown in Figure 1 (solid line).

IV. VECTOR POTENTIAL OF EXTERNAL EM RADIATION IS PERPENDICULAR TO THE GSL AXIS

Unlike the SL based on AlGaAs/GaAs,[35] GSL has nonadditive electron spectrum (1). Therefore, external EM radiation with vector potential, which is perpendicular to the GSL axis, also can effect on the nonlinear wave propagation in GSL and on the chaotic behavior of electron subsystem. So, in this section, we consider the case when vector potential of external EM radiation is supplied along the axis Ox: $\mathbf{A}^{\text{ext}} = (-(E_0/\omega)\sin\omega t,\ 0,\ 0)$. At low temperatures ($T \ll \Delta,\ \Delta_1$), summation over the momenta in (2) gives

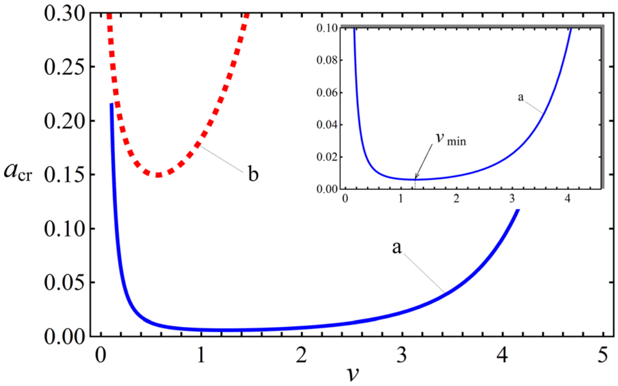

FIG. 1. The dependence of critical amplitude a_{cr} on the frequency ν. (a) Vector potential of external EM radiation is parallel to the GSL axis. (b) Vector potential of external EM radiation is perpendicular to the GSL axis. Inset: zoom in of the critical amplitude close to its first minimum in the case (a); ν_{min} is the first minimum point.

$$j_y^{f} = -\frac{\omega_{pl}^2 \sin\varphi}{4\pi e d\sqrt{1+\gamma a^2 \sin^2\omega t - 4\lambda\cos\varphi}}, \tag{12}$$

where $\gamma = v_F^2/d^2(\Delta^2+\Delta_1^2)$. For the values of parameters pointed above parameter γ has the order: $\gamma \sim 1$. Having substituted (12) into D'Alembert equation and used the variable ξ, we wrote for weak external radiation ($a^2 \ll 1$)

$$-\frac{d^2\varphi}{d\xi^2} - 2\mu\frac{d\varphi}{d\xi} + \frac{\sin\varphi}{\sqrt{1-4\lambda\cos\varphi}} = 2a^2 Q_\perp(\lambda,\varphi)\sin^2[\nu(\xi-\xi_x)]. \tag{13}$$

Here,

$$Q_\perp(\lambda,\varphi) = \frac{\sin\varphi}{4(1-4\lambda\cos\varphi)^{3/2}}. \tag{14}$$

Melnikov function is

$$M_\perp(\xi_0) = -\int_{-\infty}^{+\infty} \chi^s(2\mu\chi^s + 2a^2 Q_\perp(\lambda,\varphi^s)\sin^2[\nu(\xi+\xi_0)])d\xi. \tag{15}$$

After substituting separatrix solutions (7) into (15), we obtain

$$M_\perp(\xi_0) = -2\mu G(\lambda) - a^2 J_\perp(\lambda,\nu)\sin 2\nu\xi_0, \tag{16}$$

where

$$J_\perp(\lambda,\nu) = \int_0^{2\pi} Q_\perp(\lambda,z)\sin\left(2\nu\int_\pi^z \frac{dy}{g(\lambda,y)}\right)dz. \tag{17}$$

The next condition is seen from (16) to be satisfied the criterion of appearance of dynamic chaotization

$$\frac{2\mu G(\lambda)}{a^2|J_\perp(\lambda,\nu)|} < 1. \tag{18}$$

From the inequality (18), we find the critical value of amplitude of external EM radiation

$$a_{cr} = \sqrt{\frac{2\mu G(\lambda)}{|J_\perp(\lambda,\nu)|}}. \tag{19}$$

The dependence of critical amplitude a_{cr} on the frequency ν plotted numerically with the formula (19) for the values of parameters μ, Δ, Δ_1 pointed above is shown in Figure 1 (dashed line).

V. ANALYTICAL CALCULATION OF MELNIKOV FUNCTION

If graphene deposited on a periodical substrate SiO_2/SiC then $\Delta = 0.03$ eV, $\Delta_1 = 0.013$ eV.[21,22] For such values the parameter λ has the order: $\lambda \sim 0.04$. If $\lambda \ll 1$, then in the absence of both dissipation ($\mu = 0$) and external radiation ($a = 0$), D'Alembert equation can be approximated with the double sine-Gordon equation. Separatrix solutions of such equation were given in Refs. 52 and 55, for instance. In this case, Melnikov function and bound of chaos appearance can be calculated analytically.

A. Vector potential of external EM radiation is parallel to the GSL axis

If $\lambda \ll 1$, the function (5) can be written approximately

$$Q_{//}(\lambda,\varphi) = \cos\varphi + 2\lambda\cos 2\varphi. \tag{20}$$

Substituting separatrix solutions[52] and (20) into (6), we obtain for Melnikov function

$$M_{//}(z_0) = -8\mu\left(\sqrt{1+2\lambda} + \frac{\Lambda}{\sqrt{2\lambda}}\right) + aM_1(\lambda,\Omega)\sin\Omega z_0, \tag{21}$$

where $z_0 = \xi_0\sqrt{1+2\lambda}$, $\Omega = \nu/\sqrt{1+2\lambda}$, $\Lambda = \ln(\sqrt{2\lambda} + \sqrt{1+2\lambda})$. Function $M_1(\lambda,\Omega)$ is equal

$$M_1(\lambda,\Omega) = -\frac{2\pi\Omega^2(1+2\lambda)}{\mathrm{ch}(\pi\Omega/2)}\cos\Omega\Lambda. \tag{22}$$

The bound of chaos appearance is determined by the critical amplitude

$$a_{cr} = \frac{4\mu}{\pi\Omega^2(1+2\lambda)|\cos\Omega\Lambda|}\left(\sqrt{1+2\lambda} + \frac{\Lambda}{\sqrt{2\lambda}}\right)\mathrm{ch}\left(\frac{\pi\Omega}{2}\right). \tag{23}$$

Note that if $\lambda = 0$, then, from (23) and (23), the result[49,50] is obtained. It can be seen from formula (23) that if the frequency Ω approaches to the value: $\Omega_{//} = \pi(2n+1)/2\Lambda$, where n is the integer number, then amplitude a_{cr} increases indefinitely. This result distinguishes the system described above by Eq. (4) from a systems described by the sine-Gordon equation.[44,48–51,56,57] In systems described by the sine-Gordon equation (in particular, in semiconductor SL based on AlGaAs/GaAs), amplitude a_{cr} has finite values for all frequencies from the same interval. Besides, the critical amplitude is seen to decrease when Ω increases up to the first minimum point Ω_{min}, where Ω_{min} has the order: $\Omega_{min} \sim 4/\sqrt{\pi^2+8\lambda}$ (in

Fig. 1 $\nu_{\min} = \Omega_{\min}\sqrt{1+2\lambda}$). This distinguishes using of external EM radiation instead of using of alternate current in GSL.[45]

B. Vector potential of external EM radiation is perpendicular to the GSL axis

In the case $\lambda \ll 1$, the function (14) can be written approximately

$$Q_{\perp}(\lambda,\varphi) = \frac{1}{4}(\sin\varphi + 3\lambda\sin 2\varphi). \tag{24}$$

Substituting separatrix solutions[52] and (24) into (15), we obtain for Melnikov function

$$M_{\perp}(z_0) = -8\mu\left(\sqrt{1+2\lambda} + \frac{\Lambda}{\sqrt{2\lambda}}\right) - a^2 M_2(\lambda,\Omega)\sin 2\Omega z_0. \tag{25}$$

Function $M_2(\lambda,\Omega)$ has the form

$$M_2(\lambda,\Omega) = \frac{\pi\Omega}{sh\pi\Omega}\sqrt{1+2\lambda}\left(\frac{\sin 2\Omega\Lambda}{2\sqrt{2\lambda}} - 3\Omega\sqrt{1+2\lambda}\cos 2\Omega\Lambda\right). \tag{26}$$

The critical amplitude of external EM radiation is

$$a_{\text{cr}} = \sqrt{\frac{8\mu}{|M_2(\lambda,\Omega)|}\left(\sqrt{1+2\lambda} + \frac{\Lambda}{\sqrt{2\lambda}}\right)}. \tag{27}$$

The critical amplitude (27) is seen to increase indefinitely if the frequency Ω approaches to value $\Omega_{\perp}$ determined from the equation

$$\frac{\text{tg}2\Omega_{\perp}\Lambda}{\Omega_{\perp}} = 6\sqrt{2\lambda(1+2\lambda)}.$$

Note that for $\lambda = 0.04$, the difference of the analytical calculations made by the formulas (23) and (27) from the numerical calculations (formulas (11) and (19) correspondingly) is $2\times 10^{-3}\%$–0.1%.

VI. CONCLUSION

The strong damping of solitary waves in SL is an obstacle to the possibility of their practical application for the information processing. This fact leads to the necessity of stabilization and amplification of solitary EM waves in SL. One of the ways to stabilize the solitary wave shape is using of the external EM radiation. However, the presence of alternate EM field can lead to the dynamic chaos in the electron subsystem of SL. For structure with nonadditivity spectrum (such as GSL), there are two cases which can be considered: (1) vector potential of external EM radiation is parallel to the GSL axis; (2) vector potential of external EM radiation is perpendicular to the GSL axis.

In the first case, it can be seen from formula (23) that radiation critical amplitude which determines the bound of chaos increases indefinitely when the frequency Ω approaches to value $\Omega_{//}$. This result distinguishes the system described above by Eq. (4) from a systems described by the sine-Gordon equation,[44,48–51,56,57] where the critical amplitude has finite values for all frequencies from the same interval. Besides, the critical amplitude is seen to decrease when Ω increases up to the first minimum point $\Omega_{\min}$. Due to this fact, the case of using of external EM radiation differs from the case of using of alternate current set in GSL.[45,46] Such distinction is on account of the following reasons. If alternate current density is set then the external perturbation depends on time only and does not depend on the state of the electron subsystem of GSL.[45,46] Perturbation which is in the right side of Eq. (4) and is equal to

$$g_x = -g_0 \sin[\nu(\xi - \xi_x)], \quad g_0 = aQ_{//}(\lambda,\varphi),$$

appears due to the interaction of HF field with electron subsystem of the GSL. As a consequence, it depends not only on time (ξ is the dimensionless time and $\nu = \sqrt{1+2\lambda}\Omega$ is dimensionless frequency) but on the instantaneous state of the system as well. Mathematically, it leads to the dependence of the perturbation amplitude g_0 on the dimensionalles potential φ of EM field formed in GSL. As a result, in the case of HF EM field, there is the self-consistent adjustment of the perturbation g_x, which is in the right side of Eq. (4) to the dynamical state of the electron subsystem. The latter leads to another dependence of the critical amplitude a_{cr} (the bound of the chaotic behavior) on Ω than that of the case of alternate current which is set.[45,46]

In the second case critical amplitude increases indefinitely when the frequency Ω is close to value $\Omega_{\perp}$. Moreover, the possibility of chaotic behavior of electron system in the second case distinguishes the structures with nonadditivity spectrum (such as structures based on graphene) from semiconductor SL based on AlGaAs/GaAs.

Thus, the possibility of appearance of dynamic chaos caused in electron subsystem of GSL by the presence of external EM radiation must be taken into account in the stabilization of solitary waves in GSL. For these purpose, the radiation which frequency is close to $\Omega_{//}$ or $\Omega_{\perp}$ are the best to use. Note that these values are determined by the parameters of band structure of GSL only. For $\lambda = 0.04$, the minimal numerical value of parameters $\Omega_{//}$ and $\Omega_{\perp}$ are: $\Omega_{//} = 5.6$ and $\Omega_{\perp} = 2.4$. These values correspond to frequencies: $7.8\times 10^{13}\,\text{s}^{-1}$ and $3.3\times 10^{13}\,\text{s}^{-1}$.

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