Some Special Coalition Formation Games

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***Abstract*—We focus on a widely studied class of coalition formation games called *hedonic games*, with emphasis on stability concepts. We review some landmark results for *symmetric additively separable hedonic games* attempting to provide simple intuition for them. We then propose a new subclass of hedonic games, *shared preference hedonic games* which attempts to model some real life scenarios closely. We investigate some algorithmic properties of this game model in detail.**

I. Introduction

Coalition formation games are constructs in game theory, modelling outcomes of interactions between various players in a dynamic environment. They manifest themselves in almost all situations where agents are expected to group themselves according to their preferences. Hence it becomes important to analyse them from an algorithmic perspective and try to figure out answers to germane questions. Coalition formation games with *hedonic preferences* have been studied quite widely since their introduction by [1]. The main feature of these games is that every players payoff is dependent only on the coalition it is present in. This leads to a variety of interesting situations and tractable stable situations.

In recent times, interest in this field has focused on some basic algorithmic questions for some special classes of hedonic games. A lot of these are focused on the stability concepts involved in coalition formation games. In this work, we focus on arguably the most important of them: *core stability* and *Nash stability*. Though seemingly similar, these two concepts are quite distinct and presence (or lack thereof) of any of them has wide ranging implications for the game model. The research in this area mainly deals with EXISTENCE and COMPUTATION of these equilibrium concepts.

In this regard, *Symmetric Additively Separable Hedonic Games* strike out as a particularly interesting class of hedonic games. These games, referred to as SAS hedonic games, show some promising properties with regards to the stability concepts. We review them and provide some intuition for a key result involving SAS hedonic games.

We propose a new game model, which we call *Shared Preference Hedonic Game*. These games involve shared preferences over coalitions for all players. This is the case in a lot of cases in the practical domain. We investigate the EXISTENCE aspect of Nash equilibrium and core stable equilibrium and other similar properties.

II. Related Work

Hedonic games were introduced by *Drèze and Greenberg, 1980* [1]. Since then, a number of publications have demonstrated the remarkable efficacy of hedonic games in real life applications, like research team formation [2], group activity scheduling [3], coalition governments [4], etc. It is also noteworthy that hedonic games are generalizations of a number of (seemingly unrelated) classical problems like the stable marriage problem [5].

Banerjee et al., 2001 led the breakthrough in the study of symmetric additively separable hedonic games and provided a starting point for a lot of research in this area [6]. *Bogomolnaia and Jackson, 2002* demonstrated some nice properties of this model and provided useful insight into the structure of SAS hedonic games [7]. *Burani and Zwicker, 2003* were the source of the key result regarding Nash stability and core stability in SAS hedonic games [8], that we try to analyse. [9] and [10], among other related work, study the stability concepts of these games in detail.

III. Preliminaries

A. Hedonic Games

We adopt the conventions followed by [8]. In a coalition formation game, the agent set $N = \{1, 2, \ldots, n\}$ represents the set of players. A *coalition structure* π of set N into *coalitions* is a collection of subsets $\{C_i\}_{i=1}^{h}$ where $C_i \subseteq N\ \forall i$, such that every player is present in exactly one coalition and every player is assigned to some coalition. Formally, $\cup_{i=1}^{h} C_i = N$ and $C_i \cap C_j = \phi\ \forall i \neq j$. The coalition in π containing the player i is referred to as $\pi(i)$.

In general, in coalition formation games, the relative affinity of players to form coalitions is represented by *preference relations*. A higher preference indicates a players willingness to be associated with that particular coalition. Hence, in all such games, there exists a preference relation $\preceq_i$ for every player i, which is a reflexive, transitive and complete ordering over $\{C_i \mid C_i \subseteq N,\, i \in C_i\}$ such that

$$\pi \succeq_i \pi' \Leftrightarrow \pi(i) \succeq \pi'(i)$$

i.e. a player weakly prefers the coalition structure π over the coalition structure π' if and only if it weakly prefers the coalition $\pi(i)$ over $\pi'(i)$. We can similarly define a strict preference relation $\prec_i$ as well, with appropriate definitions.

Definition 1. A *coalition formation game with purely hedonic preferences* (or simply a *hedonic game*) consists of $(N, \{\preceq_i\}_{i=1}^{n})$, i.e. a set of players and preference relations associated with each of them, with a player's preference of a coalition depending only on the players in that particular coalition.

B. Stability Concepts

We will now look over two key stability concepts in the setting of hedonic games.

Definition 2. A coalition structure π is *Nash stable* if $\forall i \in N$ and $\forall C \in \pi \cup \phi$ such that $C \neq \pi(i)$, the following holds:

$$\pi(i) \succeq_i C \cup \{i\}$$

As is apparent from the defintion, in a Nash stable partition, no agent can profit by switching to some other coalition *unilaterally*.

Definition 3. A coalition structure π is *core stable*, if $\nexists C \subseteq N$, such that C is non-empty and $C \succ_i \pi(i)\ \forall i \in C$.

A non-empty coalition C is called a *blocking coalition* or a *deviating coalition* for the partition π if $C \succ_i \pi(i) \forall i \in C$. If a coalition structure π is core stable, then there exists no blocking coalition for π.

Note that core stability and Nash stability are inde- pendent concepts, with neither of them implying the other. [11] Nash stability deals with deviation of a single player to any other coalition, while core stability deals with deviation of a block of players, which necessarily come together after deviating. Hence, a partition may be Nash stable, since no player can profitably deviate unilaterally, but may not be core stable, since a group of players can come together to benefit each other. Similarly, a partition may be core stable, since no group of players can come together to benefit each other, but may not be Nash stable, since a player can do better by switching to some other coalition unilaterally.

IV. Symmetric additively Separable Hedonic Games

In general, it is not guaranteed that a Nash stable or a core stable partition will exist in a hedonic game. Hence, a lot of recent work has been focused on studying classes of hedonic games for which Nash stable and/or core stable partitions exist. Symmetric additively separable hedonic games is an interesting class of games which is intuitive to understand and has some remarkable properties [9]–[11]. The class is defined in the following paragraphs.

Definition 4. In a hedonic game with *separable preferences*, for all players $i, j \in N$ and $C_i \subseteq N$ such that $i \in C_i$, $j \notin C_i$, the following holds:

$$C_i \preceq_i C_i \cup \{j\} \Leftrightarrow \{i\} \preceq_i \{i, j\}$$

and

$$C_i \prec_i C_i \cup \{j\} \Leftrightarrow \{i\} \prec_i \{i, j\}$$

In simple terms, if a player i prefers being with the player j than being alone, then keeping everything else constant, a player will prefer to be in the coalition containing j rather than being in the same coalition, without j. In essence, the preference of the player i is now *separated* on the rest of the players.

Definition 5. In a hedonic game with *additively separable preferences*, $\forall i \in N$, there exists a real valued function $v_i : N \longrightarrow \mathbb{R}$ for all $i \in N$, such that the following holds:

$$C_i \preceq_i D_i \Leftrightarrow \sum_{j \in C} v_i(j) \leq \sum_{j \in D} v_i(j)$$

and

$$C_i \prec_i D_i \Leftrightarrow \sum_{j \in C} v_i(j) < \sum_{j \in D} v_i(j)$$

In addition to the preference relation of each player being separable, the constraint of the additive property imposes a quantification on the preference that each player has on the other players, through the real valued functions v_i.

We can model every additively separable hedonic game by a complete weighted directed graph $G \equiv (N, E)$, where the vertex set is the set of players N and the weight $w(i, j)$ of the edge $i \rightarrow j$ is given by $w(i, j) = v_i(j)$. The payoff that player i has on virtue of being in a coalition C is just the sum of the weights of the outgoing edges of i to all players in C, i.e. $\sum\limits_{j \in C} v_i(j)$, which is the same as the expression in definition 5. Note that we consider $v_i(i) = 0\ \forall i \in N$.

Definition 6. In a hedonic game with *symmetric additively separable (SAS) preferences*, $v_i(j) = v_j(i)$ for all $i, j \in N$. For such a game, we can represent the preference using a global function $v : N \times N \longrightarrow \mathbb{R}$, such that $v(i, j) = v_i(j) = v_j(i)$.

In the equivalent definition of complete weighted graph, for SAS hedonic games, the directed graph has a symmetric weight function, i.e. $w(i, j) = w(j, i)$. Hence, we can reduce this to an undirected graph, such that for any edge $(i, j) \in E$, $w(i, j) = v(i, j)$ with v being the weight function.

Definition 7. In a hedonic game with *purely cardinal SAS preferences*, there exists a global weight function $d : N \longrightarrow \mathbb{R}$, such that the following holds:

$$v(i, j) = \begin{cases} d(i) + d(j) & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$

In case of purely cardinal SAS preferences, we can assign weights to the vertices in the graph G associated with the game. In this case, the weight associated with vertex i (corresponding to player i) is just $w(i)$. As with the case of SAS games, the graph is a complete weighted undirected graph. Here, the weight of any edge $(i, j) \in E$ is just $w(i, j) = d(i) + d(j)$ with d being the global weight function.

As shown by [8], a hedonic game with purely cardinal SAS preferences has a partition which is Nash and core stable. The proof of the same is given below.

Theorem 1. A hedonic game with *purely cardinal SAS preferences* has a partition which is Nash stable.

Proof: Arrange all the players in non-decreasing order of the global weight function values, $d(i)\ \forall i \in N$.

To construct the stable partition, let's start with all the players being in singleton coalitions $\{i\}_{i \in N}$. The main idea behind the proof is to construct a non-singleton coalition with some players and have all the other players in singleton coalitions.

We first observe that all the players i, such that $d(i) \geq 0$ have to be in the same coalition in a stable partition. If that is not the case, then the player with non-negative weight which is not present in the same coalition as the other such players, can simply switch his strategy to align with those players. This will obviously be more beneficial to all the players.

Also, we note that if a player i is present in the non-singleton coalition, then $\forall j \in N$ such that $d(j) > d(i)$, j is also present in the non-singleton coalition. If this is not the case, then simply replacing i with j in the non-singleton coalition benefits all the players in that coalition.

Hence, the method of constructing the non-singleton coalition is to start with the player i such that $d(i)$ is the maximum among all players and keep on adding players to this coalition in a non-decreasing order of values of $d(j)$. From our earlier observation, all the players with non-negative weights will get added to this partition. However, we will only add the negatively weighted players till some stopping criterion is reached.

For this partition to be a Nash stable partition, each player in the non-singleton coalition must be better off than being in a singleton coalition. Hence, to ensure Nash stability, the stopping criterion should be that we should keep adding the players to this coalition till the player being added strictly prefers being alone than joining the coalition. By construction, such a partition is Nash stable. ∎

Note: The partition constructed in the manner as discussed in the proof for theorem 1 is called the *top segment partition*. There appears to be no direct way to show that the top segment partition is a core stable partition as well. [8] show a convoluted way of proving the same. Also, the top segment partition is not necessarily a unique core stable partition.

V. Shared Preference Hedonic Games

A. Definition and core stability

While in the general setting of hedonic games, the preferences of the players over the coalitions may be different, it is quite common to observe situations in real life where the preference is an inherent property of coalitions, i.e. some coalitions appeal more to *all* the players. In such a case, we can have a common preference relation $\preceq$ over coalitions which is shared by all the players. This common prefernce relation is a transitive, reflexive and complete ordering over all coalitions $\{C \mid C \subseteq N\}$. This gives rise to a new class of hedonic games, which we propose and study in detail.

Definition 8. A hedonic game is called a *shared preference hedonic game* if all the players share a common preference relation over coalitions.

Now since this preference relation is present over all coalitions, we can assign weights to coalitions via the weight function $w : 2^N \longrightarrow \mathbb{R}$ such that

$$w(C) \leq w(D) \Leftrightarrow C \preceq D$$

and

$$w(C) < w(D) \Leftrightarrow C \prec D$$

As is evident from the following sections, this class of game has some very special properties.

Theorem 2. Every shared preference hedonic game always admits a core stable partition.

Proof: We provide a constructive proof of such a core stable partition. We start with an empty partition. Arrange the coalitions in non-increasing order of the value of $w(C)$. Add the highest weighted coalition to the partition, C_1^0. If there are more than one coalition with the highest weights, choose any one of them. Now, consider the game formed by the players $N \setminus C_1^0$, with the same preference function w. In this game, the set of coalitions will be a strict subset of the set of coalitions of the game of N players. Hence, we can use the same preference function for this game. In this game, we can again obtain the highest weighted coalition, C_1^1. Add this coalition to the increasing partition. We repeat this procedure of removing the players already present in the partition from the player set and choosing the highest weighted coalition, till we have exhausted the player set. We obtain a partition $(C_1^0, C_1^1, \ldots, C_1^k)$ through this procedure.

We will now argue for the core stability of the partition hence formed, using the principle of mathematical induction on the coalitions C_1^i. For the base case, since the players in the coalition C_1^0 are already in their most preferred coalition, they won't be a part of any blocking coalition. Hence, we only consider the players in $N \setminus C_1^0$. Similarly, for the induction step, the induction hypothesis is that all the players in the sets $C_1^0, \ldots, C_1^{i-1}$ can not appear in any blocking coalition. In that case, by construction, we know that C_1^i is the coalition with the higest weight value in the game comprised of the remaining players. Hence, these players will not appear in any blocking coalition in the game of the remaining players. Now, since no player who does not appear in this game is in any blocking coalition of the original game (by induction hypothesis), the players in C_1^i are not in any blocking coalition in the game of all players in N. This completes the induction step and the proof. ∎

B. Convergence properties and Nash stability

We now take a detour and discuss some convergence properties in general abstract strategy games. A strategic form game is defined by a set of players N (such that $|N| = n$), a set of strategies for each player $\{S_i\}_{i \in N}$ and a set of utility functions $\{u_i\}_{i \in N}$ such that $\forall i \in N, u_i : S_1 \times S_2 \ldots \times S_n \longrightarrow \mathbb{R}$. A strategy profile $s \in S \equiv S_1 \times S_2 \ldots \times S_n$ is obtained by each player choosing a strategy in its strategy set, i.e. $s \equiv (s_1, s_2, \ldots, s_n)$ and $\forall i, s_i \in S_i$.

A *path* is a sequence $(s^1, s^2, \ldots)$ of strategy profiles such that $\forall k > 1$, s^{k-1} and s^k differ in strategy of exactly one player i. An *improvement path* is a path which is maximal and $\forall k > 1, \forall i \in N,\ u_i(s^{k-1}) < u_i(s^k)$, i.e. the player switching his strategy in successive strategy profiles does so only when there is a strict benefit from the switch to that player.

Definition 9 ([12]). A strategic form game has the *finite improvement property (FIP)* if every improvement path in the game is finite.

Note: It follows from definition 9 that if a game has FIP, then there exists atleast one Nash equilibrium strategy profile in the game.

Definition 10 ([13]). A strategic form game is said to be *weakly acylic (WA)* if for every strategy profile s, there exists atleast one finite improvement path.

Note: It follows from definition 10 that there exists atleast one Nash equilibrium strategy profile in any WA game.

Definition 11. A function $p : S \longrightarrow \mathbb{R}$ is called a *generalized ordinal potential function* for the game, if the following holds:

$$u_i(s', s_{-i}) - u_i(s, s_{-i}) > 0 \implies p(s', s_{-i}) - p(s, s_{-i}) > 0$$

The generalized ordinal potential function or potential function, tracks the change in payoff when one player deviates. Hence, it is quite useful in arriving at Nash equilibrium, which is a local minimum of the potential function. A finite game has FIP iff it has a generalized ordinal potential function [12].

Observation: It follows from definitions 9 and 10 that $FIP \implies WA$. Hence, if a game admits no Nash equilibrium profile, then neither does the game have FIP nor is it WA.

In the setting of hedonic games, a unilateral change of strategy by a player is equivalent to a player switching from one coalition to another (possibly empty) coalition. Hence, the definition of path and improvement path are inherited in their equivalent forms in case of hedonic games. Similarly, FIP and WA can be defined in case of hedonic games as well.

Theorem 3. A Nash stable partition need not exist in every shared preference hedonic game.

Proof: We prove by providing an example of a shared preference game with no Nash stable partition. Consider a simple game consisting of two players $N = \{a, b\}$. In this game, let us consider the weight function on coalitions w such that $w(\{a\}) < w(\{a, b\}) < w(\{b\})$. In this case, it is quite easy to see that there is no Nash stable partition. ■

Corollary 1. Using the previously stated observation and theorem 3, it is evident that in general, a shared preference hedonic game are not WA and do not have FIP. Hence, shared preference hedonic games are not potential games.

Also, as an aside, shared preference hedonic games are *not embeddable* in the setting of separable hedonic games. This can be shown very easily by creating a weight ordering on coalitions which violates the conditions in definition 4.

C. Zero weighted singleton sets

We now discuss a restriction on the class of shared preference hedonic games, which is of particular interest to us. In this class, apart from following the conditions in definition 8, we enforce the condition that all singleton sets, i.e. sets containing a single player, have *zero weight* and all the other sets have *strictly positive* weights.

If we account for offsetting the value of the weight function by a constant for all the coalitions, the final game model has a shared preference on all the coalitions, which is common to all the players. In this shared preference, singleton sets have the lowest preference, i.e. no player prefers being alone than being in a coalition with other players.

Definition 12. In addition to the conditions in definition 8, i.e. there is a shared preference on all the coalitions and a weight function $w : 2^N \longrightarrow \mathbb{R}$ can be constructed to emulate this preference relation, the following also holds in a shared preference game with zero weight singleton sets:

$$w(\{i\}) < w(C) \; \forall i \in N \text{ and } C \neq \{j\} \text{ for some } j \in N$$

Note: In this restricted model, theorem 2 still holds, since it states that *every* shared preference hedonic game admits a core stable partition and the construction is independent of the actual preference between coalitions.

However, we now see that the construction used in theorem 3 does not work anymore in this setting, since it was a proof by giving counter example. Hence, the questions of existence of a Nash stable partition, WA and FIP have to be explored independently for this restricted game model.

Observation: The *grand coalition*, i.e. the coalition containing all the players together is a *trivial* Nash stable partition in the restricted shared preference hedonic games.

Proof: To prove that the grand coalition N is a Nash stable partition, we need to show that no player benefits by unilaterally deviating from the coalition. We know that on unilateral deviation, a player will end up in a singleton coalition. However, by definition 12, a singleton coalition is bound to have weight strictly less than the grand coalition. Hence, no profitable deviation exists for any player. ■

Since there is a trivial Nash stable partition in every instance of the restricted class, the topic of interest becomes the existence of *non-trivial* Nash stable partition and determining whether this class is WA and or has FIP.

Theorem 4. The restricted class of shared preference hedonic games is not WA.

Proof: We will provide an example of such a game which has some partition such that all improvement paths starting from that partition are not finite in length.

Consider a game consisting of eight players, $N = \{a, b, c, d, e, f, g, h\}$. We first note that we are only interested in the relevant partitions where no two singleton coalitions exist. In such partitions, the union of two singleton coalitions will always be a strictly beneficial decision for both the players, according to definition 12.

For this game, we consider a directed graph, where each node represents a (relevant) partition and there is an edge between two nodes $u \xrightarrow{i} v$, if there is a profitable deviation by player i in partition u which results in partition v. This forms a multipartite graph, with each set containing partition with similar structure in terms of number of players in each coalition, i.e. $\{8\}$, $\{1\}\{7\}$, $\{2\}\{6\}$, $\{3\}\{5\}$ and $\{4\}\{4\}$. An improvement path in the game is simply a path in this graph. All the irrelevant partitions only have an outgoing edge (profitable deviation) to the vertices of this graph and no incoming edges from the vertices of this graph.

We now consider the game instance where the weights of all the coalitions in the layer $\{2\}\{6\}$ are *low weighted* i.e. have weights equal to the same extremely low value which is more than weights of singleton coalitions, but less than weights of any other coalition. We can now construct a cycle using the vertices of the last two layers. We take the situation where we observe a single cycle using some nodes of the layers $\{3\}\{5\}$ and $\{4\}\{4\}$ only. This can be easily enforced by assigning suitable high weights to the coalitions, to ensure that there is no contradiction among the weights. One example of the weight assignment is such that:

$$w(\{a,b,c,d\}) < w(\{d,e,f,g,h\}) < w(\{a,b,c,e\}) < \\ w(\{c,d,f,g,h\}) < w(\{a,b,e,f\}) < w(\{b,c,d,g,h\}) < \\ w(\{a,e,f,g\}) < w(\{a,b,c,d,h\}) < w(\{e,f,g,h\})$$

This will create a cycle:

$$\{a,b,c,d\}\{e,f,g,h\} \xrightarrow{d} \{a,b,c\}\{d,e,f,g,h\} \xrightarrow{e} \\ \{a,b,c,e\}\{d,f,g,h\} \xrightarrow{c} \{a,b,e\}\{c,d,f,g,h\} \xrightarrow{f} \\ \{a,b,e,f\}\{c,d,g,h\} \xrightarrow{b} \{a,e,f\}\{b,c,d,g,h\} \xrightarrow{g} \\ \{a,e,f,g\}\{b,c,d,h\} \xrightarrow{a} \{e,f,g\}\{a,b,c,d,h\} \xrightarrow{h} \\ \{e,f,g,h\}\{a,b,c,d\}$$

For all the coalitions not involved in the ordering used in the construction, use low weights.

In this game, any path starting on any node in the layers $\{3\}\{5\}$ and $\{4\}\{4\}$ will end up in the cycle created above. Hence, there is no finite improvement path starting from these partitions. Hence, this game is not weakly acyclic.

Note that we skip the partitions with more than two coalitions. This is valid, since we only need to show that no improvement path from a particular strategy profile (in this case, any partition in the layers $\{3\}\{5\}$ and $\{4\}\{4\}$) is finite. ∎

Corollary 2. As a result of theorem 4 and the previously stated observation, the restricted class of games need not have FIP and hence are not potential games.

Theorem 5. A non-trivial Nash stable partition is not guaranteed to exist in the restricted class of shared preference hedonic games.

Proof: We use a similar construction as in the proof of theorem 4, with four players $N = \{a,b,c,d\}$ instead of eight. The layers in this case are i.e. $\{4\}$, $\{1\}\{3\}$ and $\{2\}\{2\}$. Here, there are no irrelevant partitions. Hence, the weight assignment we form are complete orderings on the set of coalitions. Similar to the weight assignment in the previous proof, we use any weight assignment for the coalitions such that it creates a cycle in the layers $\{1\}\{3\}$ and $\{2\}\{2\}$. One example of the weight assignment is such that:

$$w(\{c,d\}) < w(\{a,b,d\}) < \\ w(\{a,c\}) < w(\{b,c,d\}) < w(\{a,b\})$$

This will create a cycle:

$$\{a,b\}\{c,d\} \xrightarrow{d} \{a,b,d\}\{c\} \xrightarrow{a} \\ \{b,d\}\{a,c\} \xrightarrow{c} \{b,c,d\}\{a\} \xrightarrow{b} \{c,d\}\{a,b\}$$

For all the coalitions not involved in the ordering used in the construction, use low weights. In this case, no partition in the layer $\{1\}\{3\}$ is Nash stable, since there is a profitable deviation to the grand coalition. Similarly, no partition in the layer $\{2\}\{2\}$ is Nash stable, since there is a profitable deviation to the non-low weighted coalitions in the layer $\{1\}\{3\}$. Also, any improvement path in using the nodes of the layers $\{1\}\{3\}$ and $\{2\}\{2\}$ either leads to the cycle formed in the earlier proof or to the grand coalition. Hence, the only Nash stable partition in this game is the trivial Nash stable partition, i.e. the grand coalition. ∎

Also, as an aside, the restricted class of shared preference hedonic games are *not embeddable* in the setting of separable hedonic games. This can be shown very easily by creating a weight ordering on coalitions which violates the conditions in definition 4.

VI. Discussion

We looked at the question of Existence of Nash stable partitions in the case of shared preference hedonic games and a restricted class with zero weights on singleton sets. The questions related to Computation still remain open. It appears that the question of determining whether a shared preference hedonic game admits a Nash stable partition or not is difficult. However, it is unknown whether it is NP-Complete or not. The question of determining whether a shared preference hedonic game with zero weights on singleton sets admits a non-trivial Nash stable partition or not is similar in structure.

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References

[1] Jacques H Dreze and Joseph Greenberg. Hedonic coalitions: Optimality and stability. *Econometrica: Journal of the Econometric Society*, pages 987–1003, 1980.

[2] José Alcalde and Pablo Revilla. Researching with whom? stability and manipulation. *Journal of Mathematical Economics*, 40(8):869–887, 2004.

[3] Andreas Darmann, Edith Elkind, Sascha Kurz, Jérôme Lang, Joachim Schauer, and Gerhard Woeginger. Group activity selection problem. In *Internet and Network Economics*, pages 156–169. Springer, 2012.

[4] Michel Le Breton and François Salanié. Lobbying under political uncertainty. *Journal of Public Economics*, 87(12):2589–2610, 2003.

[5] David Gale and Lloyd S Shapley. College admissions and the stability of marriage. *American mathematical monthly*, pages 9–15, 1962.

[6] Suryapratim Banerjee, Hideo Konishi, and Tayfun Sönmez. Core in a simple coalition formation game. *Social Choice and Welfare*, 18(1):135–153, 2001.

[7] Anna Bogomolnaia and Matthew O Jackson. The stability of hedonic coalition structures. *Games and Economic Behavior*, 38(2):201–230, 2002.

[8] Nadia Burani and William S Zwicker. Coalition formation games with separable preferences. *Mathematical Social Sciences*, 45(1):27–52, 2003.

[9] Haris Aziz, Felix Brandt, and Hans Georg Seedig. Stable partitions in additively separable hedonic games. In *The 10th International Conference on Autonomous Agents and Multiagent Systems-Volume 1*, pages 183–190. International Foundation for Autonomous Agents and Multiagent Systems, 2011.

[10] Haris Aziz and Florian Brandl. Existence of stability in hedonic coalition formation games. In *Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems-Volume 2*, pages 763–770. International Foundation for Autonomous Agents and Multiagent Systems, 2012.

[11] Haris Aziz and Rahul Savani. Hedonic games. *Handbook of Computational Social Choice*.

[12] Dov Monderer and Lloyd S Shapley. Potential games. *Games and economic behavior*, 14(1):124–143, 1996.

[13] H Peyton Young. The evolution of conventions. *Econometrica: Journal of the Econometric Society*, pages 57–84, 1993.